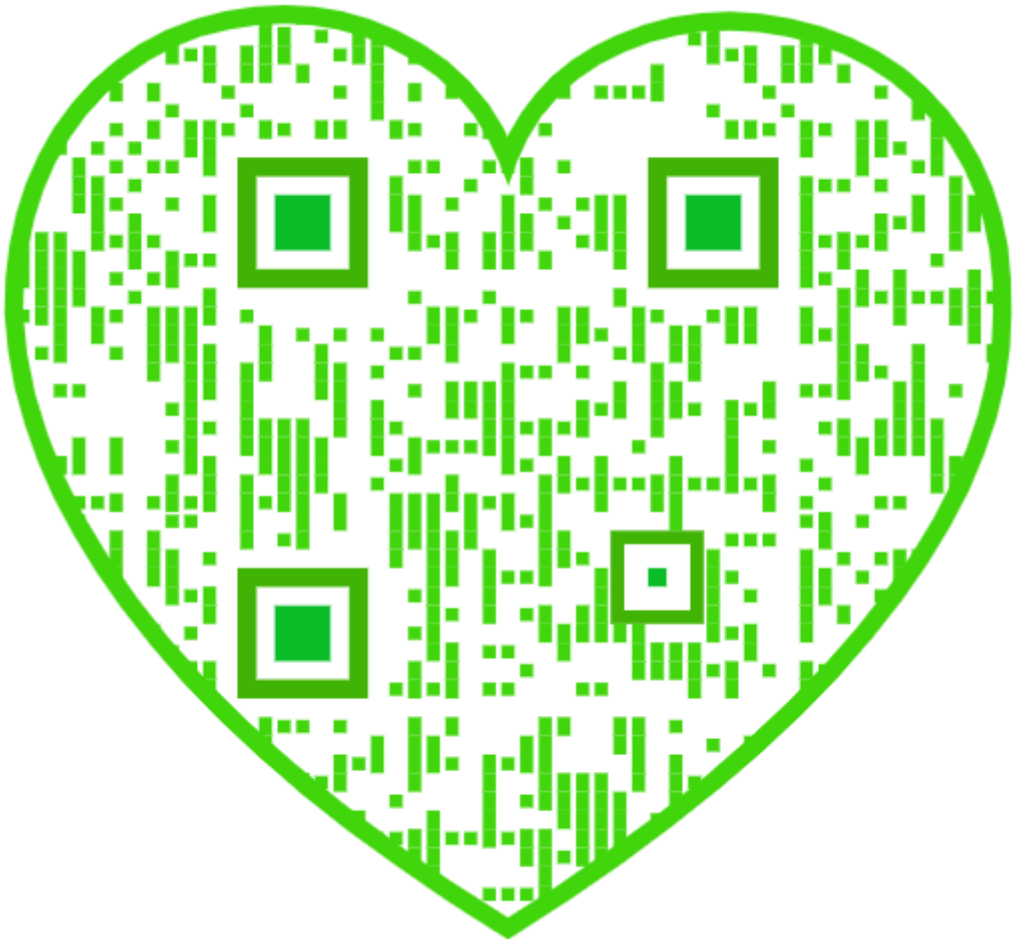

NAVIGATION
AND NAUTICAL ASTRONOMY

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NAUTICAL ASTRONOMY

BY

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CHAPLAIN AND NAVAL INSTRUCTOR, ROYAL NAVY; FORMERLY EXAMINER IN NAVIGATION AND NAUTICAL ASTRONOMY TO THE ROYAL NAVAL COLLEGE, GREENWICH

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PREFACE.

THE earlier portion of this book deals with that division of the subject which does not require a knowledge of astronomical definitions, nor, with the exception of Great Circle Sailing, of Spherical Trigonometry; the investigation of Raper's Rules for finding the distance from a Mountain Peak being inserted in the chapter on "Fixing a ship's position on a chart."

The second portion—the various necessary definitions having been stated and an explanation given of the method of constructing Nautical Astronomy diagrams—consists of the discussion of Time, Greenwich Date, Sextant, Altitudes and the rules for their correction; Longitude, Latitude, Chronometer and Compass Errors; Day's Work and Sumner's Method. This concludes the really practical portion, and then follows a chapter containing the *investigation* of the various corrections, Dip, Parallax, etc.; and of the errors produced in Longitude, Latitude, etc., by small errors in time or observation. These errors are treated geometrically, and are thus shortly and simply explained without the use of Differential Calculus (not admissible in an elementary work) or the long analytical methods which usually take its place.

The last chapter contains the investigation of Latitude by observation of the Pole Star; Equation of Equal Altitudes (the first method a short and simple one, much preferable to the ordinary method, which is, however, inserted as being familiar to many); Time at which the Sun will *dip* in a ship steaming at a high rate of speed; Interpolations; and a few other problems of interest in Nautical Astronomy.

A number of Miscellaneous Examples is added, which it is hoped will provide, in many cases, exercises for the student's ingenuity. The examples throughout the book are either

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original (in the case of the Practical examples almost entirely so) or such as have been set in the various examinations connected with the Royal Naval College, Greenwich.

It must add considerably to the utility of a work on Navigation if the student is able to actually make use of the *Nautical Almanac* when working out examples, instead of having the necessary data provided at the end of the questions. As, however, Nautical Almanacs go out of print, and it would make too bulky and expensive a book if the whole or a large portion of the Almanac were bound up with it, all the examples requiring its use have been composed for three months of the year 1895, viz. April, June, and December, which provide a considerable North and South Declination, and a medium Declination of the Sun, a sufficient variety for all practical purposes.

Those pages, therefore, or parts of pages which contain elements to which reference is made in the examples, viz. Sun's Declination, Equation of Time, Sidereal Time at Mean Noon; the Right Ascension, Declination, Semi-diameter, Horizontal Parallax, and Time of Meridian Passage of the Moon; the requisite Lunar Distances; the Right Ascension and Declination of the one Planet to which reference is made; together with the Pole Star Tables—are bound up at the end of the work.

The Right Ascension and Declination of Stars are not included, as those recorded in any Nautical Almanac are sufficiently near for all *practical* purposes: nor are any tables, such as Second Differences in Lunar Observations, which do not vary with the year.

Every care has been taken to ensure the correctness of the answers to the examples, but the notification of any errors that may be found will be gratefully received.

It will be noticed that no *definite rules* are given for the solution of the various problems in finding latitude, longitude, etc. The principle of any problem is explained, the necessary formulæ given, and examples fully worked in illustration, which should be sufficient, as all who have to make the calculations required in Navigation problems must be familiar with the use of Logarithms, and there would appear to be more *education* in working thus, than in mechanically following rules.

For convenience of reference, the necessary definitions are printed in italics and numbered consecutively, and the principal formulæ are collected at the end of their respective chapters. The Tables referred to are Inman's, which are in general use in the Royal Navy; but any set of Tables will suffice in almost every case, if it is remembered that the Log Haversine of Inman is exactly the same as the Log Sine Square of other Tables, while the Half Log Haversine is half the Log Sine Square.

Many Tables have also tables of Natural Sines, Cosines, etc., from which the Natural Versines may be obtained by subtracting the Cosines from unity.

N.B.—The word Log is omitted in the working of the examples, as it is easily understood that the numbers placed against the various Sines, Cosines, etc., or the natural numbers, are the requisite logarithms from the Tables.

The multiplication of methods of solving the various problems has been avoided as far as possible, the aim of the book being to provide the student of Navigation with methods which will enable him to understand the straightforward principles of Navigation, leaving it to himself to select afterwards, if he pleases, any of the various plans for shortening his calculations, of which so many are to be obtained.

The Author's thanks are especially due to Mr. H. B. Goodwin, of the Royal Naval College, Greenwich, and Mr. J. M. Pask, of H.M.S. "Britannia," from whom he has received valuable suggestions and advice.

F. C. S.

H.M.S. "Cambridge,"
June, 1896.

PREFACE TO THE SECOND EDITION.

A FEW alterations have been made. The chapter on Day's Work has been practically re-written; the examples of Sumner's Method have been re-worked so as to bring them into accordance with present day methods; and mention is made of a method of determining the position line when the Sun's Altitude near the Meridian is greater than 60°. This is due to Mr. W. D. Niven, C.B., Director of Studies at the Royal Naval College, Greenwich, who also supplies a method of determining the formula for Interpolation, based on the principles of Kinematics, a modification of a method proposed by the Author.

A few examples have been added at the end of Chapter XVII. to show how "errors" may be readily determined by the aid of "position lines."

The Double Altitude method of finding the latitude has been retained, as observations of this description are still on the list of those required at the yearly examination of Midshipmen of the Royal Navy. Moreover, it is advisable to have a method of finding latitude which is independent of the error of the chronometer.

Every effort has been made to correct misprints and errors, and it is hoped that few now remain.

A number of miscellaneous examples has been added, selected from those recently set at the Royal Naval College.

F. C. S.

November, 1902.

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NAVIGATION
AND NAUTICAL ASTRONOMY.

CHAPTER I.

INTRODUCTION.

DEFINITIONS IN NAVIGATION.

§ 1. Navigation, as a general term, denotes that science by means of which the place of a ship on the sea, and her course to or from any given point, are determined. This, of course, includes the effect of winds, currents, tides, local magnetic attraction, etc., which have to be considered. The necessary information on these heads is to be obtained from the sailing directions, tide tables, compass manuals, etc.; and the term Navigation is usually restricted to the finding a ship's position and direct course.

This may be done in two ways: (1) by the application of the common rules of Plane Trigonometry, the necessary angles and distances being supplied by means of the compass and log line; (2) by means of astronomical observations, treated by the rules of Spherical Trigonometry.

It has been proposed to call the first of these methods geo-navigation (γῆ, the earth), and the second cælo-navigation (*cælum*, the heavens), but the terms Navigation and Nautical Astronomy have become sanctioned by long use, and are therefore retained.

§ 2. The earth is very nearly a sphere, the length of the longest diameter (about the equator) being 7,926 miles; and that of the shortest (that which joins the poles) being 7,899 miles. For all the practical purposes of Navigation the earth is considered as a sphere.

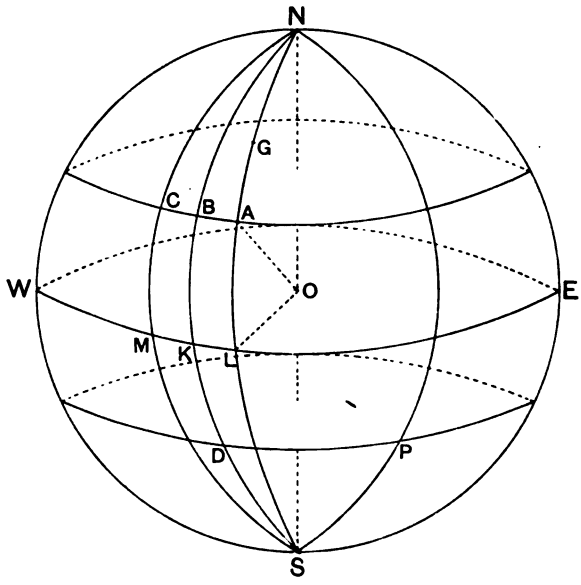


FIG. 1.

Def. 1. *The Axis of the earth is that diameter about which it revolves, with uniform motion, from west to east.*

Def. 2. *The Poles of the earth are the points where its axis meets its surface;* as *N*, *S* in the figure.

Def. 3. *The Earth's Equator is a great circle on its surface equidistant from its poles;* as *EW*.

N.B.—The plane of a *great* circle passes through the centre of a sphere, the plane of a *small* circle does not.

Def. 4. *Meridians are great circles which pass through the poles of the earth.*

Def. 5. *The Latitude of a place is the arc of a meridian intercepted between the equator and the place,* measured from 0° to 90°, N or S. A place north of the equator is said to be in north latitude, a place south of the equator in south latitude.

Thus *AL* is the north latitude of the place *A*, *DK* is the south latitude of the place *D*.

Def. 6. *The Co-latitude is the complement of the latitude;* thus $NA = 90° - AL$ is the co-latitude of *A*.

Def. 7. *Parallels of Latitude are small circles whose planes are parallel to the plane of the equator,* all places on a parallel

having the same latitude. Thus if ABC be a parallel of latitude, $AL = BK = CM$.

Def. 8. *A Nautical Mile is equal to the mean length of a minute of latitude, and is reckoned as 6080 feet.*

Def. 9. *The True Difference of Latitude between two places is the arc of a meridian intercepted between their parallels, expressed in minutes of arc or in nautical miles.*

Thus the true difference of latitude between B and P is the arc BD between their respective parallels.

It is evident that, if the places are on the same side of the equator, their true difference of latitude is obtained by subtracting the less latitude from the greater, while, if they are on opposite sides, it is obtained by adding the two latitudes together. Thus the true difference of latitude between B and P is the sum of the north latitude BK (of B), and the south latitude KD (of P).

The true difference of latitude is marked N or S according as the place arrived at or *in* is to the north or south of the place left or *from*.

Examples.—Find the true difference of latitude between Cape Clear, lat. 51° 26′ N, and Cape Finisterre, lat. 42° 54′ N.

Lat. from, - - - - - -	51° 26′ N
Lat. in, - - - - - - -	42 54 N
	8 32
	60
true diff. lat., - - - -	512 miles S

Find the true difference of latitude between Cape St. Roque, lat. 5° 28′ S. and Cape Verd, lat. 14° 43′ N.

Lat. from, - - - - - -	5° 28′ S
Lat. in, - - - - - - -	14 43 N
	20 11
	60
true diff. lat., - - - -	1211 miles N

A ship sails from 42° 14′ N, 215 miles north, find the latitude in.

Lat. from - - - - -	42° 14′ N
true diff. lat., 215′ N = - - -	3 35 N
Lat. in, - - - - - -	45 49 N

A ship sails from latitude 27° 15′ S, 320 miles north, find the latitude in.

Lat. from,	- - - - -	27° 15′ S
true diff. lat., 320′ N =	- - -	5 20 N
Lat. in,	- - - - -	21 55 S

Find the true difference of latitude in the following cases:—

	Lat. from.	Lat. in.		Lat. from.	Lat. in.
(1)	27° 16′ N	39° 41′ N	(4)	35° 19′ S	18° 45′ S
(2)	47 25 N	37 43 N	(5)	12 27 N	5 16 S
(3)	18 25 S	36 17 S	(6)	3 19 S	22 47 N

Find the latitude in, in the following cases:—

	Lat. from.	true diff. lat.		Lat. from.	true diff. lat.
(7)	14° 26′ N	569′ N	(10)	39° 16′ S	2195′ N
(8)	29 41 N	1679 S	(11)	4 21 N	916 S
(9)	17 47 S	1427 S	(12)	5 27 S	1624 N

Def. 10. *When two places are on the same side of the equator, the mean of their latitudes is called the Middle Latitude.*

Def. 11. *The First or Prime Meridian is that fixed meridian by reference to which the longitude of places on the earth is measured.*

The meridian of Greenwich has been almost universally adopted as the first meridian.

Def. 12. *The Longitude of a Place is the smaller arc of the equator intercepted between the first meridian and the meridian of the place.*

If G be the position of Greenwich in the figure, B, C and D are in west longitude, and P in east longitude.

Longitude may also be defined as the angle at the pole between the first meridian and the meridian of the place, or as the angle at the centre of the earth subtended by the arc of the equator between the meridians.

It is reckoned from 0° to 180° E or W, but is never considered as $> 180°$. Places which are more than 180° in longitude eastward are considered as being in west longitude, and *vice versâ*.

Def. 13. *Difference of Longitude between two places is the smaller arc of the equator intercepted between their meridians.*

If they are both in E or both in W longitude, the difference of longitude is the difference between their longitudes, expressed in miles or minutes of arc; if one is in E longitude and the other in W longitude, the difference of longitude is the sum of their longitudes, taken from 360° if it exceeds 180°.

Examples.—Find the difference of longitude between Ushant and the east point of Madeira.

Long. from,	5°	3′ W
Long. in,	16	39 W
	11	36
	60	
diff. long.,	696′	W

Find the difference of longitude between the Cape of Good Hope and Tristan d'Acunha.

Long. from,	18°	29′ E
Long. in,	12	2 W
	30	31
	60	
diff. long.,	1831′	W

Find the difference of longitude between 120° W and 79° E.

Long. from,	120	W
Long. in,	79	E
	199	E
	360	
	161	W
	60	
diff. long.,	9660′	W

A ship sails from longitude 1° 20′ W and changes her longitude 236 miles to the eastward. What is her longitude *in*?

Long. from,	1°	20′ W
diff. long., 236′ E,	3	56 E
Long. in,	2	36 E

Examples.—Find the difference of longitude in the following cases:—

	Long. from.	Long. in.		Long. from.	Long. in.
(13)	43° 20′ W	57° 18′ W	(17)	5° 47′ W	3° 16′ E
(14)	97 41 W	69 57 W	(18)	4 24 E	9 37 W
(15)	117 22 E	142 19 E	(19)	164 29 W	134 19 E
(16)	55 11 E	39 17 E	(20)	127 32 E	96 47 W

Find the longitude *in*, in the following cases:—

	Long. from.	diff. long.		Long. from.	diff. long.
(21)	27° 18′ W	2461′ W	(25)	5° 29′ E	787′ W
(22)	74 16 W	3547 E	(26)	4 27 W	953 E
(23)	114 29 E	1293 E	(27)	169 25 E	1347 E
(24)	87 23 E	1459 W	(28)	158 47 W	1729 W

Def. 14. *The Rhumb Line or Loxodromic Spiral is a curve which cuts all meridians at the same angle.*

It is generally an equiangular spiral on the surface of the sphere, always approaching the pole, but never actually reaching it; the pole must always bear due north, and therefore cannot be reached on any course except due north.

Def. 15. *The Course Steered is the angle between a meridian and the ship's fore and aft line.*

Def. 16. *The Course Made Good is the angle between the meridian and the rhumb line joining the place left and the place arrived at.*

Def. 17. *The Distance between two places, or the Distance Run by a ship on any course, is the length of the arc of the rhumb line expressed in nautical miles.*

The Departure is the distance, in nautical miles, made good, east or west, by a ship sailing on a rhumb line; or it is the distance, in nautical miles, between two places on the same parallel of latitude. It is marked E or W according as the ship's course has been towards the east or the west.

Def. 18. *If an infinite number of points be taken on the rhumb line joining two places, the meridian through each of these points cuts off an arc of the parallel of latitude through the next point. The sum of all these arcs is called the Departure.*

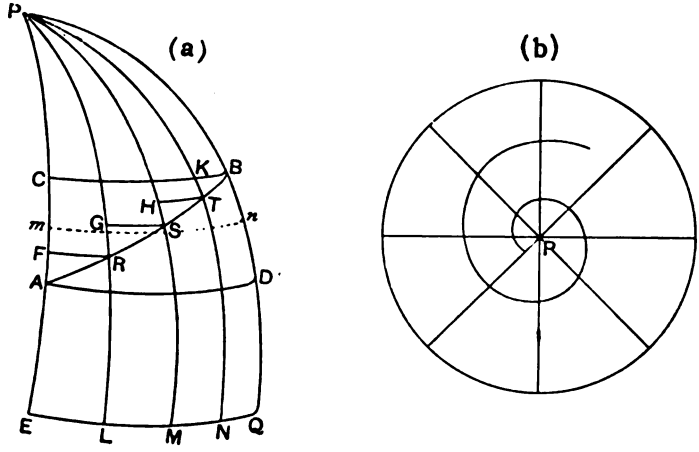


FIG. 2.

P is one of the earth's poles; A is the place left, and B is the place arrived at. If PAE, PBQ be meridians, and AD,

CB be parallels of latitude through *A* and *B*, and *EQ* the equator; *AC* or *DB* is the difference of latitude; and *EQ* the difference of longitude between *A* and *B*.

AB is the rhumb line between *A* and *B*, a curve similar to that shown in (Fig. 2 (b)). *A*, *R*, *S*, *T*, ... are some of an infinite number of points taken on the rhumb line *AB*; *PR*, *PS*, *PT*... are the meridians drawn through these points; the angles *PAR*, *PRS*, *PST*... are all equal, as *AB* is a rhumb line, and these angles are the Course Made Good between *A* and *B*; *FR*, *GS*, *HT*... are arcs of parallels of latitude through *R*, *S*, *T*.... The sum of all these arcs *FR*, *GS*, *HT*... is called the Departure between *A* and *B*.

This sum is clearly less than *AD* and greater than *BC*. Hence there must be some parallel between *AD* and *BC* such as *mn*, which is equal to the departure. The latitude *Em* of the parallel *mn* is called the True Middle Latitude (Def. 35). In all ordinary cases of navigation, this may be considered as the mean of the latitudes of *A* and *B*.

Def. 19. *The Tropics are parallels of latitude, in latitude about* 23° 27½′ *N or S.* The tropic of Cancer is in N latitude; the tropic of Capricorn in S latitude.

Def. 20. *The Arctic and Antarctic Circles are parallels of latitude, in latitude about* 66° 32½′ *N and S respectively.*

CHAPTER II.

COMPASS. CORRECTION OF COURSES. LOG LINE.

THE instruments used in navigation for obtaining courses, bearings, and distances, are the *compass* and the *log line* and *glass*.

Def. 21. *The Points of the Compass are divisions on the rim of the Compass Card, whose angular distance apart is* 11° 15′.

Def. 22. *The Magnetic Meridian is the great circle in whose plane the compass needle lies when unaffected by local attraction.*

Def. 23. *The Compass Meridian is the great circle in whose plane the compass needle lies when affected by local attraction.*

Def. 24. *The Dip of the Needle is the angle between the horizontal plane and the direction of a magnetic needle freely suspended at its centre of gravity.*

§ 3. The way in which the compass card and bowl are suspended and fixed in position can be much better understood by actual inspection; a general description of the compass card is therefore all that is now given.

The compass card is a circular plate or disc of mica or cardboard, divided into four quadrants, each containing eight "points" as they are called, their angular distance apart being therefore 11° 15′. Thus there are thirty-two points in all, which are again divided into quarter points. The circumference of the card is also divided into degrees, measured from 0° to 90° from the north and south points to the east and west points.

These points—north, south, east, and west—called the *Cardinal* points, are at the extremities of two diameters at right angles to each other, the north point being marked by a *fleur de lys*,

the east point being on the right hand when the observer is looking towards the north.

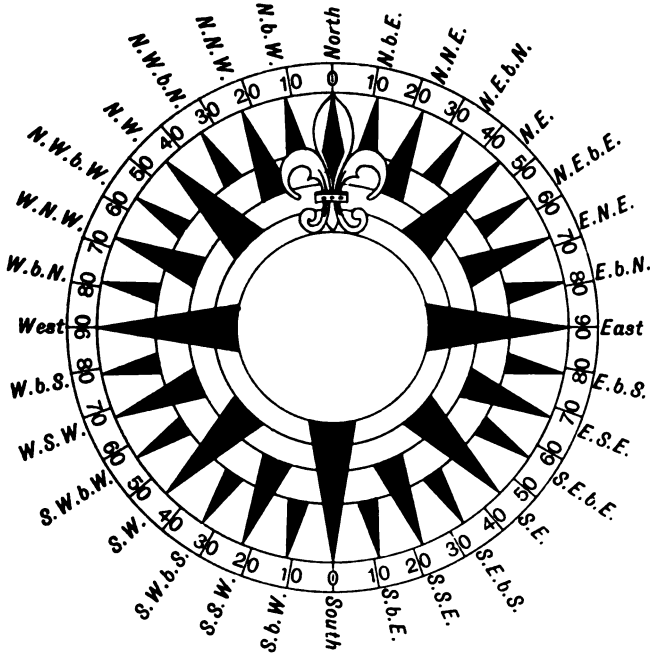


FIG. 3.

The other points are named on the following principle:—

The middle point between N and E is - - - - NE,
formed by putting the letters together.

The middle point between N and NE is - - - NNE,
„ „ E and NE is - - - ENE,
formed in a similar manner.

One point from N towards E is N by E or NbE,
„ E „ N is E by N or EbN,
„ NE „ N is NE by N or NEbN,
„ NE „ E is NE by E or NEbE,

the points in the other quadrants being formed and named in a similar manner; NE, SE, SW, and N.W. being named *Intercardinal* or *Quadrantal* points.

The half and quarter points are named from the adjacent points, with the exception of those, such as ENE and NEbN, which commence and end with the same letter; they are usually expressed as briefly as possible, attention being paid

to the sound produced. Thus between north and east we have—

N ¼E	NNE ¼E	NE ¼E	EbN ¾N
N ½E	NNE ½E	NE ½E	EbN ½N
N ¾E	NNE ¾E	NE ¾E	EbN ¼N
NbE	NEbN	NEbE	EBN
NbE ¼E	NE ¾N	NEbE ¼E	E ¾N
NbE ½E	NE ½N	NEbE ½E	E ½N
NbE ¾E	NE ¼N	NEbE ¾E	E ¼N
NNE	NE	ENE	

It will be observed that such a name as NNE ½N is not used, as it has a much more awkward sound than NbE ½E; the letter attached to the quarter or half point being the same as the final letter of the adjacent point, except in the case of those measured from the cardinal or quadrantal points, where brevity is the object aimed at.

Similar names apply to the other quadrants.

All Nautical Tables contain a table giving the points of the compass, with the number of points or degrees, etc., from the north or south point.

In reading the compass it is to be remembered that the observer is supposed to be situated at the *centre*, and the reading is then so many points or degrees to the right or left of N or S respectively.

Looking on the compass as though it were a map is a fruitful source of error in beginners.

Thus east is to the *right* of north and to the *left* of south; west is to the *left* of north and to the *right* of south.

Examples.

NNE ½E is 2½ points to the right of N.
SSW ½W „ „ „ S.
SSE ½E „ „ left of S.
NNW ½W „ „ „ N.

Examples.—Express the following in points and degrees to the right or left of N:—

(1) N ¾E	(5) NEbE ½E
(2) EbN ½N	(6) NW ¾W
(3) NNW ¾W	(7) NbE ¾E
(4) WbN ¼N	(8) NW ¼N

Express the following in points and degrees to the right or left of S:—

(9) S ½W	(13) E ½S
(10) SbE ¾E	(14) SEbE ¾E
(11) SSW ½W	(15) WbS ½S
(12) W ¾S	(16) EbS ¾S

§ 4. On the under side of the compass card are fixed one or more magnetized needles, the direction of the needles being parallel to the north and south diameter of the card, which is kept in a horizontal position by small brass weights counteracting the "dip" of the needle.

If the north point of the compass always pointed accurately to the north point of the horizon, *i.e.* the point in which the meridian intersects a horizontal plane through the position of the observer, the true course would be at once seen by the indication of the compass.

But this is very seldom the case, as the compass needle, and therefore also the north point on the card, is deflected from the true north point of the horizon by two influences: (1) that which arises from the earth's attraction, and is the same at any place for every ship, and for every direction of the ship's head; (2) that which arises from local attraction, caused by the materials of which a ship is built, the direction of the ship's head while building, the cargo, armament, etc., which differs for every ship, and depends on the direction of the ship's head at any time.

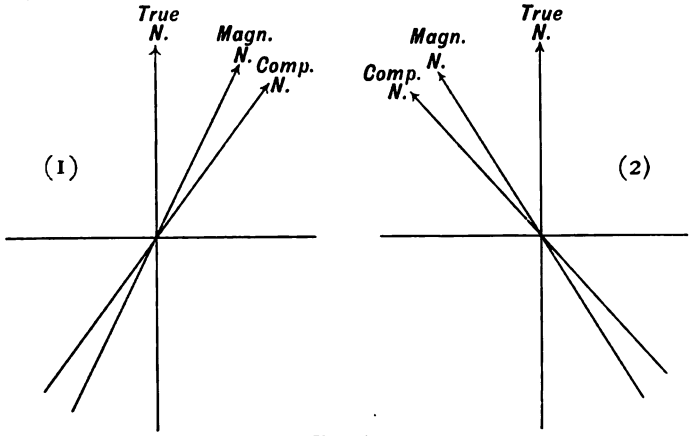


FIG. 4.

The error of the compass caused by the first is named the *Variation.*

Def. 25. *Variation is the angle between the planes of the true and magnetic meridians at any place, or the angle between the directions of the true and magnetic north.*

When, as in fig. 4 (1), the direction of the magnetic north is to the right of the direction of the true north, the variation

is said to be *easterly*; when, as in fig. 4 (2), it is to the left of the direction of the true north, the variation is said to be *westerly*.

The error of the compass caused by local attraction is named the *Deviation*.

Def. 26. *Deviation is the angle between the planes of the magnetic and compass meridians, or the angle between the directions of the magnetic and compass north.*

It is said to be easterly or westerly according as the direction of the *compass north* is drawn to the right or left of the direction of the *magnetic north*.

In order therefore to obtain the *true course steered* from the course as shown by the compass, it is necessary to correct the compass course for variation and deviation. To do this, *easterly* variation or deviation is to be allowed to the *right*, *westerly* to the *left*, of the compass course, for this reason:—

In fig. 4 (1) the compass error is represented as easterly, and it is clear that any course measured from the direction of the compass north will be so much farther to the right or so

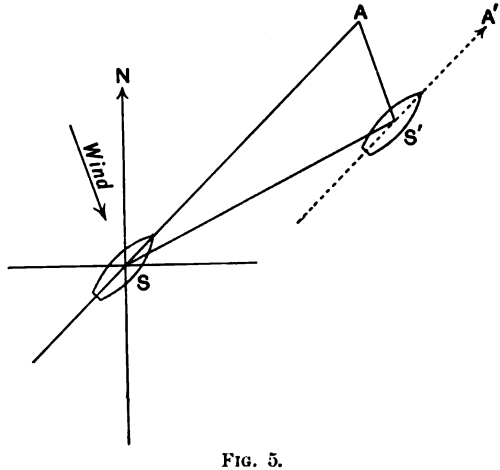


FIG. 5.

much less far to the left, of the true north; while in fig. 4 (2), where the compass error is westerly, any course measured from the direction of the compass north will be so much farther to the left, or less far to the right, of the true north.

The same reasoning applies to courses measured from the compass south, or to the obtaining a magnetic course or bearing from a compass course or bearing.

In order to obtain the *true course made good*, it is necessary further to apply a correction for *leeway*. This is caused by the lateral pressure of the sea or the wind on the hull and sails of a ship, causing her to drift somewhat sideways.

Thus, if *NSA* is the true course steered by the ship, the direction of the wind being shown by the arrow, the course made good will be *NSS'*, the ship's head still pointing in the direction *S'A'* parallel to *SA*. (See fig. 5, p. 12.)

Hence the correction for leeway must be applied *away from* the direction of the wind.

Def. 27. *Leeway is measured by the angle between a ship's fore and aft line and her wake.*

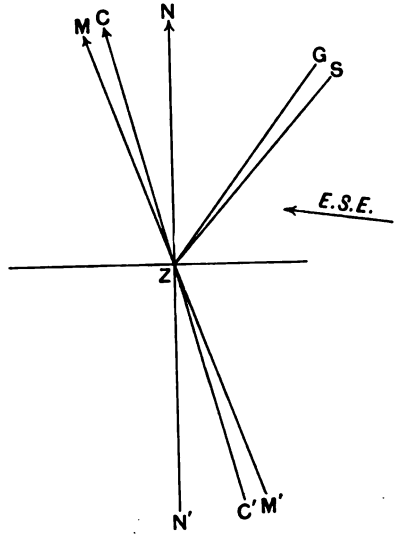


FIG. 6.

Examples.—Obtain the true course made good, having given, compass course NEbE½E, variation 25° 20′ W, deviation 4° 10′ E, leeway ½ point, the wind being ESE.

Compass course,	61°	52′	30″	*r*	of N
Deviation,	4	10	0	*r*	
Magnetic course,	66	2	30	*r*	„
Variation,	25	20	0	*l*	
True course steered,	40	42	30	*r*	„
Leeway,		37	30	*l*	
True course made good,	35	5	0	*r*	„
or	N 35	5	0	E	

ZN, *ZC*, and *ZM* being the directions of the true, compass, and magnetic north—

the angle $NZM = 25° 20'$ is the variation (westerly),
the angle $MZC = 4° 10'$ is the deviation (easterly),
the angle $CZS = 61° 52'$ is the compass course,
the angle $MZS = 66° 2'$ is the magnetic course,
the angle $NZS = 40° 42'$ is the true course steered,
the angle $NZG = 35° 5'$ is the true course made good,
the angle $GZS = 5° 37'$ being the leeway, allowed away from the direction of the wind, as shown by the arrow.

Obtain the true course made good, having given, compass course WbN$\frac{1}{2}$N, variation 35° 20′ W, deviation 2° 10′ W, leeway $\frac{3}{4}$ point, the wind being NNW.

Compass course, - - -	73°	7′	30″	*l* of N
Deviation, - - - -	2	10	0	*l*
	75	17	30	*l* „
Variation, - - - -	35	20	0	*l*
True course steered, - -	110	37	30	*l* „
or - - - - -	69	22	30	*r* of S
Leeway, - - - -	8	26	15	*l*
True course made good, -	60	55	15	*r* „
or - - - - -	S 60	55	0	W

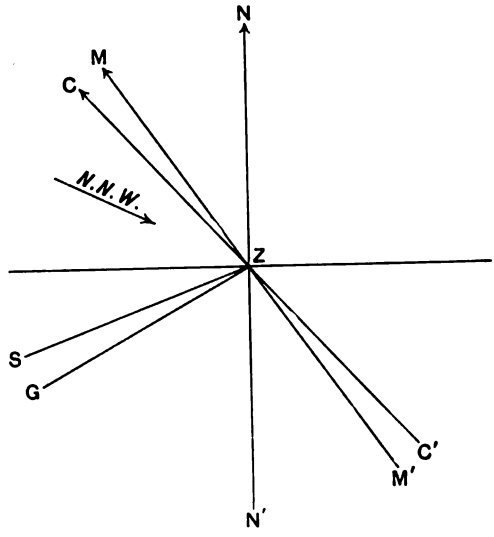


FIG. 7.

$NZM = 35° 20'$	$CZS = 73° 7'$	$N'ZS = 69° 23'$
$MZC = 2 10$	$NZS = 110 37$	$N'ZG = 60 55$

Examples.—Construct figures and obtain the true course made good in the following cases :—

	Compass Course.	Deviation.	Variation.	Leeway.	Wind.
(1)	SWbS	2° 10′ E	18° 15′ E	¼ pt.	WNW
(2)	SbE ¼E	2 20 W	21 40 W	¾ pt.	SW ½W
(3)	S ¼W	1 15 E	16 50 W	½ pt.	ESE
(4)	E ½N	3 10 W	15 0 E	1¼ pt.	SEbS
(5)	SWbW ½W	1 50 E	4 20 W	¼ pt.	S ½E
(6)	NW ¾N	4 15 W	34 30 W	1½ pt.	NEbN

If it be required to find the course to be steered by compass in order to reach a port whose true direction is known, the corrections must be applied in exactly the opposite direction to that given in the preceding section, *i.e. easterly* variation and deviation to the *left*, *westerly* to the *right*, and *leeway towards* the direction of the wind.

Examples.—Find the compass course, having given, true course to be made good N 8° W, leeway ½ point, wind (by compass) EbN, variation 17° 10′ W, deviation 3° 20′ E.

True course to be made good,	8°	0′	0″	*l* of N
Leeway, - - - - - -	5	37	30	*r*
True course to be steered, -	2	22	30	*l* „
Variation, - - - - - -	17	10	0	*r*
Magnetic course, - - -	14	47	30	*r* „
Deviation, - - - - - -	3	20	0	*l*
Compass course, - - -	11	27	30	*r* „
or - - - - -	N 11	27	30	E

NZG = 8° 0′

SZG = 5 38

NZM = 17 10

MZG = 14 47

MZC = 3 20

CZG = Compass course

= 11° 27′

= NbE nearly.

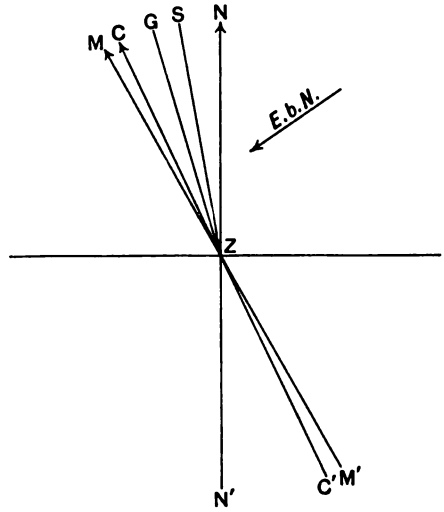


FIG. 8.

Find the compass course, having given, true course to be made good S 65° E, leeway ¾ point, wind (by compass) NbE, variation 25° 30′ E, deviation 2° 10′ E.

True course to be made good,	65°	0′	0″	*l* of S
Leeway, - - - - -	8	26	15	*l*
True course to be steered, -	73	26	15	*l* „
Variation, - - - -	25	30	0	*l*
Magnetic course, - - -	98	56	15	*l* „
Deviation, - - - -	2	10	0	*l*
Compass course, - - -	101	6	15	*l* „
or - - - - -	78	53	45	*r* of N
or - - - - -	N 79 E=EbN nearly.			

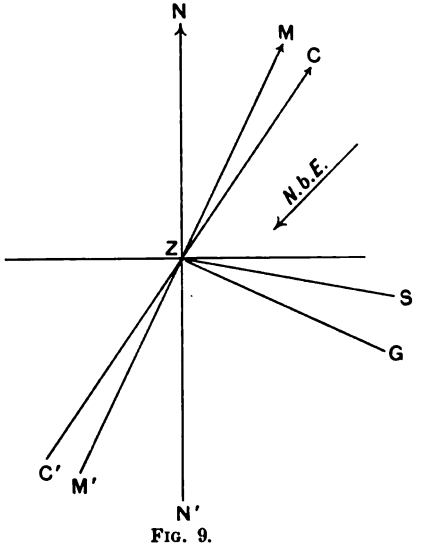


FIG. 9.

∴ $N'ZG = 65°\ 0'$
$SZG = 8\ 26$
$NZM = 25\ 30$
$CZM = 2\ 10$
$MZS = 81\ 4$
$CZS = 78\ 54$

Examples.—Construct figures and find the compass course in the following cases :—

	True course to be made good.	Leeway.	Wind.	Variation.	Deviation.
(7)	N 27° 30′ W	¼ pt.	W	18° 30′ E	3° 15′ W
(8)	S 47 E	½ pt.	SSW	27 40 W	3 20 E
(9)	S 75 W	0	E	33 10 W	1 50 W
(10)	N 69 E	1½ pt.	SE	14 20 E	2 50 E
(11)	N 6 E	¾ pt.	NWbW	27 45 E	1 10 W
(12)	S 17 E	1¼ pt.	SW	22 40 W	3 30 W

NOTE.—If it should happen that the deviations are large and changing rapidly from point to point, which is very seldom the case in the present day when compasses are so carefully adjusted, the deviation should be taken from the table for the nearest point of the magnetic course, so as

to obtain an *approximate* compass course, the deviation being then taken out for this course, and again corrected, if necessary, to any required degree of accuracy.

If, for example, in No. (12), instead of a small deviation, the table had given

Deviation for SSW, - - 12° W,
„ SWbS, - - 15° W,

the calculation would be as follows:—

	S	17°	0′	E
Leeway, -		14	4	
	S	2	56	E
Variation, -		22	40	W
		19	44	W, *i.e.* nearly SSW
Deviation, -		12	0	W
	S	31	44	W, approximate compass course.

Magnetic course, - - - - -	S	19°	44′ W
Corrected deviation, - - - -		14	30 W
	S	34	14 W

A further approximation gives a deviation of nearly 15° 10′ W, and a compass course of S 35° W, *i.e.* 3° different from the first approximation.

§ 5. The variation at any given place being the same for all ships, is calculated and marked on the charts, a special "variation chart" also being constructed.

The deviation has to be obtained for each ship and each position of the ship's head. It is calculated for the "standard compass," the course by any other compass which may be used for steering purposes being adjusted by reference to the "standard."

§ 6. The methods of obtaining the deviation by astronomical observations will be discussed later (§§ 123 *seqq.*). The following may be considered now—technically called "swinging ship":—

(1) Reciprocal bearings.

An azimuth compass is set up on shore, in sight of the ship, in some place which is free from disturbing elements. The ship is then swung slowly round and steadied on each point. At given signals the bearing of the shore compass from the ship and of the ship's compass from the shore are simultaneously observed, with the position of the ship's head at the time.

It is evident that the bearings thus observed would be exactly opposite if there were nothing besides the earth's attraction to affect them. Hence the difference between the bearing of the shore compass from the ship and that of the ship's compass from the shore, *reversed*, must be the deviation for the observed position of the ship's head; *easterly* if the compass bearing is to the *right* of the reversed shore bearing, *westerly* if to the *left*.

In this manner a deviation table may be formed for each position of the ship's head.

Example.

Ship's Head.	Bearing from Ship's Compass.	Bearing from Shore Compass.
N	S 18° 40′ W	N 21° 20′ E
NE	S 29 50 W	N 31 0 E
W	S 47 10 W	N 44 50 E
SW	S 75 0 W	N 72 30 E

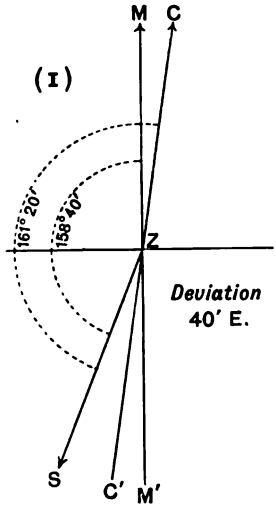


FIG. 10 (1).

Let *MZM′* be the magnetic meridian. Then

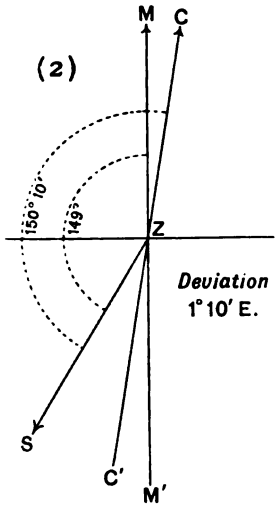
$$MZS = 180° - 21° 20′ = \text{N } 158° 40′ \text{ W}$$

is the correct magnetic bearing of the shore compass from the ship. Hence *SZC* the compass bearing being greater than *MZS*, *ZC* the direction of the N point of the compass will fall to the right (as seen from *Z*) of *ZM*.

NOTE.—It is better for beginners *always* to decide their compass errors by reference to the *north point*, as it avoids confusion, although they can of course be equally well determined by reference to the south point.

The deviation is therefore easterly, and $=21°\ 20' - 18°\ 40' = 2°\ 40'$ E, when the ship's head points north.

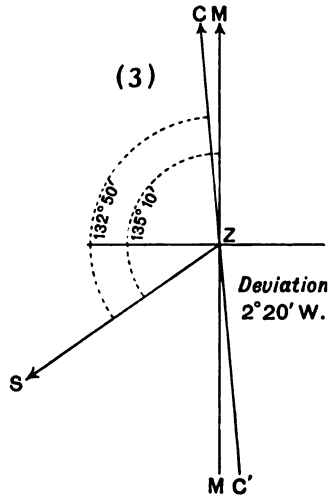
Ship's Head NE



Deviation.
1° 10' E

FIG. 10 (2).

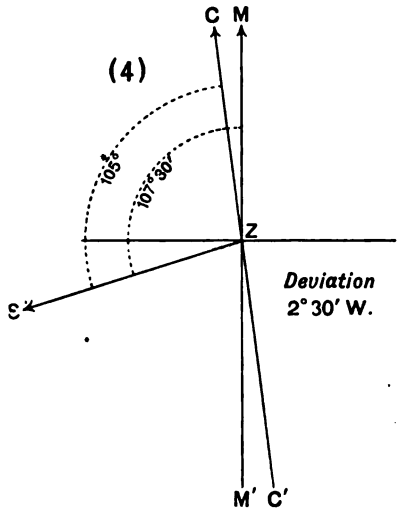
Ship's Head W



Deviation.
2° 20' W

FIG. 10 (3).

Ship's Head SW



Deviation.
2° 30' W

FIG. 10 (4).

Examples.—Find the deviation, when we have given

	Ship's Head.	Bearing from Ship's Compass.	Bearing from Shore Compass.
(1)	N	N 37° 20′ E	S 33° 50′ W
	NbW	N 43 20 E	S 39 40 W
	WNW	N 57 40 E	S 56 10 W
	SE	S 59 10 E	N 57 40 W
	S	S 70 30 E	N 67 50 W

	Ship's Head.	Bearing from Ship's Compass.	Bearing from Shore Compass.
(2)	SW	N 79° 30′ E	S 82° 20′ W
	SSW	N 71 20 E	S 75 30 W
	SbE	N 45 30 E	S 41 10 W
	SE	S 28 50 W	N 25 40 E
	EbN	S 26 40 W	N 30 50 E

§ 7. Second method. By bearing of an object a few miles distant.

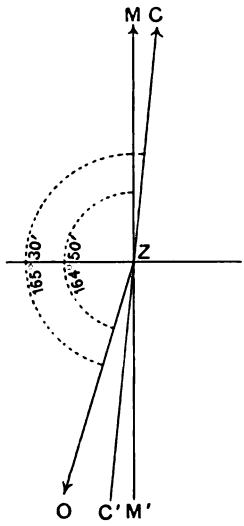


FIG. 11.

The object selected must be at a distance which is sufficiently large compared with the curve described by the standard compass during the operation of "swinging," otherwise its true bearing would differ appreciably for the different positions of the ship's head—about six to eight miles is usually sufficient, or less if the ship is warped round in a small circle (cf. § 128).

The ship is steadied as her head comes to each point of the compass, and the bearing of the distant object noted; the mean of all the bearings is considered as the correct magnetic bearing, and the difference between it and any compass bearing determines the deviation for the corresponding direction of the ship's head.

Example.—Find the correct magnetic bearing of the distant object, and the deviations, having given

Ship's Head by Standard Compass.	Bearing of distant object by Standard Compass.	Ship's Head by Standard Compass.	Bearing of distant object by Standard Compass.
N	S 14° 30′ W	S	S 17° 0′ W
NE	S 11 30 W	SW	S 19 30 W
E	S 9 30 W	W	S 20 30 W
SE	S 12 0 W	NW	S 16 50 W

	14° 30′
	11 30
	9 30
	12 0
	17 0
	19 30
	20 30
	16 50
8)	121 20
	15 10

Correct magnetic bearing, S 15° 10′ W.
Let $OZM = 180° - 15° 10′ = 164° 50′$,
$OZC = 180° - 14° 30′ = 165° 30′$.
Then $CZM = 40′$, and the deviation is east on all points on which the compass bearing is less than the magnetic; and, similarly, it will be west on those points on which the compass bearing is greater than the magnetic.

Therefore the deviations are

Ship's Head.	Deviation.	Ship's Head.	Deviation.
N	0° 40′ E	S	1° 50′ W
NE	3 40 E	SW	4 20 W
E	5 40 E	W	5 20 W
SE	3 10 E	SW	1 40 W

N.B.—In practice it is sufficient to take the mean of the bearings on the eight principal points as the correct magnetic bearing.

§ 8. When the deviations are small, as is usually the case in ships whose compasses are carefully adjusted, the deviations for the intermediate positions of the ship's head may be found by means of a diagram. Take a sheet of ruled foolscap paper, mark the points of the compass on thirty-two consecutive lines. From a vertical line, drawn through the centre of the paper, lay off on the horizontal lines the deviations for the eight principal points on any desired scale, and draw a curve through the points thus obtained. The distance from the vertical line to the curve on the given scale will give the deviation, with considerable accuracy, for any desired direction of the ship's head.

For larger deviations and greater accuracy "Napier's Diagram" may be employed. This is supplied in the "chart box" in the Royal Navy.

Examples.—(1) From the following observations find the correct magnetic bearing of the distant object, and thence the deviations.

Ship's Head by Standard Compass.	Bearings of distant object by Standard Compass.	Ship's Head by Standard Compass.	Bearings of distant object by Standard Compass.
N	S 48° 40′ W	S	S 52° 20′ W
NbE	S 48 10 W	SbW	S 50 40 W
NE	S 47 10 W	SW	S 47 30 W
ENE	S 48 0 W	WbS	S 48 20 W
E	S 48 50 W	W	S 48 30 W
SEbE	S 50 0 W	WNW	S 50 40 W
SE	S 53 20 W	NW	S 52 50 W
SSE	S 52 50 W	NWbN	S 52 20 W

(2) From the following observations find the correct magnetic bearing of the distant object, the deviations for the given positions of the ship's head, and the deviations for the other points by a diagram.

Ship's Head by Standard Compass.	Bearings of distant object by Standard Compass.	Ship's Head by Standard Compass.	Bearings of distant object by Standard Compass.
N	N 88° 30′ W	S	N 87° 40′ W
NE	S 89 50 W	SW	N 86 10 W
E	S 88 10 W	W	N 84 30 W
SE	S 89 30 W	NW	N 86 0 W

THE LOG LINE.

§ 9. The logship or log is a flat piece of wood in the shape of a sector of a circle, the curved part being weighted with lead so as to cause it to float in an upright position below the surface of the water. To this is fastened about 150 fathoms of line, the inner end being secured to the spokes of a reel on which the line is wound, and which, turning freely, suffers the line to run off when the log is thrown overboard, without bringing it home.

About 10 to 20 fathoms from the log is fixed a piece of bunting to mark off "stray line"; the use of the stray line being to allow the log to get clear of the ship, and the line run off the reel at uniform speed, before the time is taken.

From this point to the inner end the line is divided into equal portions called "knots"; and the number of knots which run out in a fixed time, as measured by a sand-glass, shows the speed of the ship, because the length of the knots is determined by the following principle, called the "Principle of the Log Line":—

"The length of a knot on the line bears the same proportion to the length of a nautical mile (6080 feet) as the number of seconds in which the sand-glass runs out bears to the number of seconds in an hour. In the Royal Navy the standard glass is a 28-seconds glass; hence the length of a knot should be $\frac{6080}{3600} \times 28 = 47{\cdot}3$ feet nearly. When the ship's speed is greater than about six knots, a 14-seconds glass is used, and the number of knots that has run out is doubled."

NOTE.—In log-line problems, it is necessary to remember that, though there are *two* glasses, a long and a short, there is but *one* line; the length of the knots being calculated for the *long* glass.

§ 10. The line may have contracted through constant wetting, or have become stretched during hauling in if the speed is great. There may also be errors in the length of the glass: the condition of the sand or the size of the hole through which it runs may be affected by change of temperature or humidity. Hence it is necessary to know how to make allowance for any of these errors.

It is evident that the longer the glass the more line will run out, and the longer the knot the smaller will be the number of knots which run out in a given time; in other words, the speed shown is directly proportional to the length of the glass, and inversely proportional to the length of a knot.

Let t seconds be the correct length of the glass.
t' „ erroneous „ „
d the correct speed.
D the speed shown, the length of the knot being, so far, assumed to be correct.

Then $d : D :: t : t'$ .....(1)

But if l' be the length of the knot used, and l the correct length, D' the speed shown,

$$D : D' :: l' : l,$$

or $$D = \frac{D'l'}{l} \quad \text{.....(2)}$$

If, then, both time and knot are incorrect, substituting in (1) the value of D from (2) we have the general formula

$$d = \frac{l't}{t'l} . D'.$$

Or we may obtain the same formula thus:—

Speed shown when length of knot is l and time $t' = D'$,
„ „ „ 1 „ $t' = D'l'$.
„ „ „ 1 „ $1 = \frac{D' . l'}{t'}$,
„ „ „ l „ $1 = \frac{D'l'}{t'l}$,
„ „ „ l „ $t = \frac{D'l't}{t'l}$.

§ 11. The log line and glass are almost entirely superseded by various patent logs, but as the log is still used in the Royal Navy, it is as well that the principles should be understood.

The difference between a knot and a nautical mile should be noticed.

When a ship's speed is said to be "so many knots," what is meant is that so many knots run off the reel in a given

time, and that therefore the ship is going at the rate of so many nautical miles per hour.

Def. 28. *A knot, therefore,* when thus used, *is a rate of a nautical mile per hour,* while *a nautical mile is a definite distance, the mean length of a minute of latitude.*

Examples.—If the length of the knot is only 45 feet and the glass runs out in $26\frac{1}{2}$ seconds, what is the true distance run by a ship which by the log has run 265 miles?

Correct length of knot for 28-seconds glass $= 47{\cdot}3$ feet;

$$\therefore \text{ distance run} = \frac{265 \times 45 \times 28}{47{\cdot}3 \times 26\frac{1}{2}} = 266{\cdot}4 \text{ miles.}$$

Using a 29-seconds glass, the ship ran 150 miles by the log, the true distance by the chart being 160 miles. Required the error in the log line.

$$\text{Length of knot for 29-seconds glass} = \frac{6080 \times 29}{3600} = 49 \text{ feet.}$$

$$D : D' = l' : l,$$

$$160 : 150 = l' : 49;$$

$$\therefore\ l' = \frac{49 \times 160}{150} = 52{\cdot}3 \text{ feet;}$$

$\therefore$ length of knots is $3{\cdot}3$ feet too long.

Examples.—(1) Taking the nautical mile to be 6080 feet, what is the length of a knot on the log line when a 30-seconds glass is used? What error would be introduced in the estimated run of a ship if the glass were found to have run out in 26 seconds instead of 30?

(2) If a 31-seconds glass and a 48-feet line be used, find the true distance corresponding to a registered distance of 217 miles.

(3) The apparent speed of a ship was $9{\cdot}5$ knots, but it was found that the (short) glass ran out in 13 seconds instead of 14 seconds, and that the length of the knots on the line was 5 inches too long. What is the true speed?

(4) Show that the time a ship will take to steam a given distance in slack water is the harmonic mean of her times with and against the tide (supposed uniform).

A ship ran a measured mile in $3^m\ 20^s$ with the tide, and in $4^m\ 15^s$ against the tide. Using a log line with a 28-seconds glass, her speed was made out to be $16{\cdot}5$ knots. What was the error in the length of a knot on the line? Find also the rate of the tide.

(5) The distance run by log was 304 miles, and the true distance 295 miles. The short glass ran out in $13\frac{1}{2}$ seconds. What was the error in the length of the knots on the log line?

(6) A ship at full-speed trial steamed a measured mile in $3^m\ 27^s$ with the tide, and in $4^m\ 47^s$ against it. A common log, tested (with a 28-seconds glass) during the trial, showed a speed of $14{\cdot}3$ knots. How long would the ship have taken over the mile in slack water, and what was the error in the line?

CHAPTER III.

THE "SAILINGS." CONSTRUCTION AND USE OF TRAVERSE TABLE. DEAD RECKONING. MISCELLANEOUS EXAMPLES IN THE SAILINGS. CONSTRUCTION OF MERCATOR'S CHART.

THE SAILINGS.

THE methods by means of which the position, course, etc., of a ship are determined in "Navigation," are called the "Sailings."

(1) *Plane Sailing*, which includes *Traverse*, *Current*, and *Windward sailing*.

Def. 29. *Plane Sailing is a name sometimes given to those formulæ of navigation which can be established by the use of Plane Trigonometry alone, the formulæ namely which connect the Course, Distance, Departure, and True Difference of Latitude.*

§ 12. Suppose a ship to sail from A to B on a rhumb-line, cutting the meridians in R, S, T, ... When these points are taken indefinitely near to one another, the triangles ARF, RSG, STH, ... become right-angled plane triangles, and the angles FAR, GRS, HST, ... are all equal to one another, being the Course between A and B.

Also $FR+GS+HT$ ... is called the Departure between A and B, and $AF+RG+SH$ ... is the true Difference of Latitude ....

Then in the triangle

FAR, $FR = AR$. sin $FAR = AR$. sin Course.

GRS, $GS = RS$. sin $GRS = RS$. sin Course.

HST, $HT = ST$. sin $HST = ST$. sin Course.

.....etc.

Adding, $FR+GS+HT+\ \ldots\ =(AR+RS+ST\ \ldots)\sin$ Course.

But $FR+GS+HT+\ \ldots$ Departure.

$AR+RS+ST+\ \ldots$ Distance.

Departure = Distance . sin Course.

Similarly, $AF+RG+SH+\ldots\ =(AR+RS+ST+\ \ldots)\cos$ Course.

Or True Diff. Lat. = Distance . cos Course.

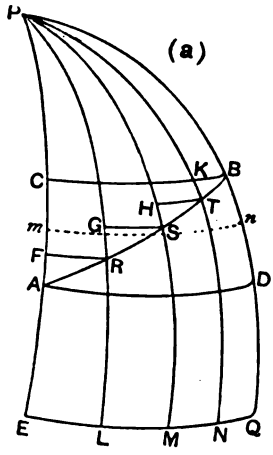


FIG. 12.

Hence, in plane sailing, the distance, true difference of latitude, departure, and course may be represented by the three sides and one acute angle of a plane right-angled triangle.

Def. 30. *When a ship sails on more than one course in succession she is said to describe a "Traverse," and the calculation of the total amount of difference of latitude and departure made good, and the resultant single course and distance, is called "Traverse sailing."*

§ 13. Suppose a ship to sail from A to D (fig. 13) by means of the courses from A to B, B to C, C to D; NS and EW representing the directions of the meridians and parallels of latitude. Dropping perpendiculars from A, B, C, D on NS and EW, it is evident that the difference of latitude AM, and departure AR, are, respectively, the algebraic sum of the several differences of latitude and departures made good on each of the courses.

Hence the difference of latitude and departure must be calculated for each course, and entered in a table, with headings

N, S, E, W respectively. Add up the columns; then the difference of the sums of those marked *N* and *S* will be the total difference of latitude, to be marked with the name of the

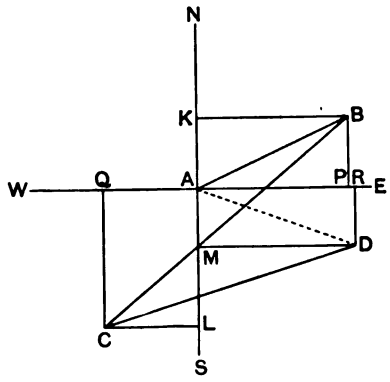


FIG. 13.

greater; the total departure being similarly obtained from the columns marked *E* and *W*.

Then $$\frac{\text{dep.}}{\text{true diff. lat.}} = \tan \text{course},$$

$$\text{and distance} = \text{diff. lat.} \times \sec \text{course}$$

by the ordinary rules of Plane Trigonometry.

§ 14. These calculations are facilitated by means of the "Traverse Table."

Def. 31. *The Traverse Table is a table containing the values of the perpendiculars and bases of right-angled triangles when the hypothenuse and vertical angle are given;* from 1 to 296 and 1° to 89° respectively.

In a right-angled triangle

$$\text{perpendicular} = \text{hypothenuse} \times \cos \text{vertical angle},$$

$$\text{base} = \text{hypothenuse} \times \sin \text{vertical angle}.$$

If, therefore, for hypothenuse, vertical angle, perpendicular, and base, we read distance, course, true difference of latitude, and departure, we see that the solution of right-angled triangles may be made available for the purposes of navigation in the form of the "Traverse Table"; the distance being placed at the top, the course, in points or degrees, at the side, and the corresponding difference of latitude and departure in the columns headed *diff. lat.* and *dep.*

As the cosine of an angle = the sine of its complement, the diff. lat. and dep. are calculated as far as 4 points or 45°. If the course is greater, the *diff. lat.* is taken from the departure column for the complement of the course and *vice versâ*. To avoid mistakes the courses above 45° are placed on the *right-hand side* of the table, and the corresponding *diff. lat.* and *departure* marked at *the bottom.*

With courses less than 45°, therefore, look for the headings at the *top* of the table; with courses greater than 45°, look for the headings at the *bottom* of the table.

For distances greater than 296, take out the *diff. lat.* and *dep.* for any part of the given distance which comes within the limits of the table, and increase them in the required proportion.

Examples.—(1) A ship sails NEbN 115 miles from latitude 32° 15′ N. Find the latitude in and the departure.

In this case, $AB = 115'$,

$CAB = 33°\ 45'$.

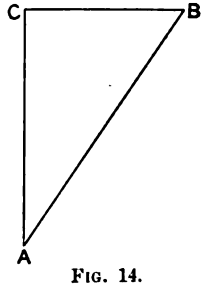


FIG. 14.

$AC = 115 \cos 33°\ 45'$,

2·060698

9·919846

1·980544

$AC = 95{\cdot}62$.

$BC = 115 \sin 33°\ 45'$,

2·060698

9·744739

1·805437

$BC = 63{\cdot}89$.

Lat. from,	32°	15′	0″	N
diff. lat.,	1	35	37	N
Lat. in,	33	50	37	N

dep., 64 miles.

By Traverse Table, with 115 at the top and 3 points at the side, we have diff. lat. = 95·6, dep. = 63·9.

(2) A ship sails from latitude 27° 15′ S to latitude 24° 39′ S, and makes 79·5 miles of easterly departure. Find the course and distance made good.

Latitude from,			27° 15′	S
Latitude in,			24 39	S
diff. lat.,			2 36	N = 156′ N.

$$\tan \text{course} = \frac{79{\cdot}5}{156},$$

1·900367

2·193125

9·707242

Course = N 27° 0′ 15″ E.

Distance = 156 sec course,

2·193125

10·050135 Distance = 175·1 miles.

2·243260

The nearest *diff. lat.* and *dep.* in the Traverse Table are 155·9 and 79·4. The corresponding course and distance are 27° and 175 miles.

(3) A ship sails from latitude 50° 30′ N as follows :—

SW ½S	40′
SSE	55′
NbW	15′
WbN	70′

find the latitude in and departure.

	diff. lat. N.	diff. lat. S.	dep. E.	dep. W.
S 3½ pts. W 40′, - -		30·9		25·4
S 2 „ E 55′, - -		50·8	21·0	
N 1 „ W 15′, - -	14·7			2·9
N 7 „ W 70′, - -	13·7			68·7
	28·4	81·7	21·0	97·0
		28·4		21·0
	diff. lat.,	53·3 S	dep.,	76·0 W

Lat. from, -	50°	30′	0″	N
diff. lat., - -	0	53	18	S
Lat. in, -	49	36	42	N

departure, 76 miles W.

Examples.—Find the latitude in and the departure in the following cases :—

	Lat. from.	Course.	Distance.
(1)	54° 10′ N	EbS ½S	273 miles.
(2)	49 23 N	SW ½W	229 „
(3)	2 15 S	N ½E	146 „

Find the course and distance made good in the following cases : —

	diff. lat.	dep.
(4)	110·4′ N	58·7′ E
(5)	94·8 S	145·9 W
(6)	112·9 N	155·3 E

(7) A ship sails on the following courses ; find the course and distance made good :—

N ½E, 75′ ; E ¾S, 66′ ; SWbW ½W, 115′ ; SbE ¾E, 98′ ; WNW, 47′.

NOTE.—A few examples will be given on the direct principle of each of the sailings, and at the end of the chapter a number of miscellaneous examples.

§ 15. When a ship experiences a current or tide of known strength and direction, the distance and course on which she would have been carried by the current is to be considered as a separate course and distance, and included in the calculations by Traverse Table, attention being paid to the fact that a current is named after the point of the compass *towards* which it is flowing, unlike a wind, which is named after the point *from* which it is blowing.

Def. 32. *The point of the compass towards which a current flows is called its "set," the distance per hour its "rate," and the whole distance in a given time its "drift."*

When it is required to find the course to be steered through a known tide or current in order to reach a given port, and in similar problems, it is generally necessary to make use of those rules of Plane Trigonometry which apply; but results which are *very nearly* correct may usually be obtained by means of the Traverse Table.

Examples.—(1) A ship ran NEbN 24 miles in 3 hours, through a current setting WbS 2 miles per hour. Required the course and distance made good.

	N.	S.	E.	W.
NEbN 24', - - -	20		12·8	
WbS 6', - - -		1·2		5·9
	20	1·2	12·8	5·9
	1·2		5·9	

diff. lat., 18·8 N dep., 6·9 E

Course and distance made good—
N 20° E. 20 miles.

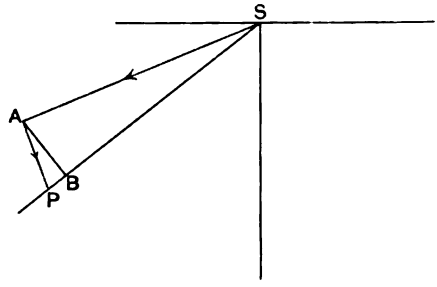


FIG. 15.

(2) What course must a ship, whose speed is 7 knots, steer through a current setting SSE 2 miles per hour, to reach a port bearing S 52° W ?

First, by Trigonometry.

Let S (fig. 15) be the position of the ship, P that of the port. It is evident that the ship will have to steer in the direction SA, where SA is proportional to 7 miles, and AP, in a SSE direction, is proportional to 2 miles.

$$\therefore \quad \frac{\sin ASP}{\sin APS} = \frac{AP}{AS} = \frac{2x}{7x} = \frac{2}{7}.$$

$$\therefore \quad \sin ASP = \tfrac{2}{7} \sin APS = \tfrac{2}{7} \sin 74^\circ 30'.$$

$$\begin{array}{r} \cdot 301030 \\ 9 \cdot 983910 \\ \hline 10 \cdot 284940 \\ \cdot 845098 \\ \hline 9 \cdot 439842 \end{array}$$

$\therefore$ $ASP = 15^\circ\ 59'$, and the course to be steered $= 52^\circ + 16^\circ =$ S 68° W.

If the distance SP is given, SA may be easily found (by rule of sines). This, divided by the speed of the ship, will give the time taken to reach the port.

Second, by Traverse Table; to find the course.

Taking AP as distance and the angle $APS = 74^\circ\ 30'$ as course, AB will represent the departure $1 \cdot 9x$ (AB being drawn at right angles to SP). Then with $SA = 7x$ as distance, and $AB = 1 \cdot 9x$ as departure, the corresponding course $= 16^\circ$, and the course to be steered $=$ S 68° W, as before.

The Traverse Table also enables us to find the amount made good per hour in direction of the port; thus, SP in the figure $= SB + BP = 7 \cdot 23$, since SB is the diff. lat. corresponding to distance 7 and departure 1·9, and $= 6 \cdot 7$; and PB similarly $= \cdot 53$.

(3) A vessel sails by her log NW $\frac{3}{4}$N 156 miles in 24 hours. Her position by observation is 125 miles NWbW of her former position. Required the set, drift, and rate of the current.

	diff. lat.	dep.
NW $\frac{3}{4}$N 156′ gives - -	125·3 N	92·9 W
NWbW 125′ „ - -	69·4 N	103·9 W
$\therefore$ current has caused - -	55·9 S	11·0 W

$\therefore$ set is S 11° W; drift, 57 miles; rate, $\frac{57}{24} = 2\frac{3}{8}$ miles per hour.

Examples.—(1) A ship sails (7·5 knots) for a port 15 miles SW of her. A current sets SEbE 4 knots. What course must she steer, and how long will she take to reach the port? The wind is SSE, causing $\frac{1}{2}$ point of leeway.

(2) A ship steams NNE 12 miles per hour for 5 hours through a three-knot current setting WNW. How long will it take her to return to her starting point, and what course must she steer?

(3) A ship steams SbE 16 knots directly away from a rock, through a current setting WSW. After half-an-hour's steaming the rock bears NbE. Find the rate of the current.

(4) What course must a ship, steaming 10 knots, steer to reach a port SW of her, through a current setting SSE 2 knots?

(5) If a ship is moving through the water 10·4 knots, how must she steer across a current setting SEbS 3 knots in order to make good an EbN course, and what rate does she make good?

§ 16. When a ship has to sail to a port against a wind which is more or less foul, she has to beat to windward. Her course will depend on the number of points from the wind within which she can sail on either tack, on the principle that she is to near her port as much as possible from instant to instant, etc.

In theoretical questions no time or distance is supposed to be lost while in stays, so that the distance run is the same whether she makes one or several tacks, and she is supposed to be able to reach her port when her head will point directly to it; allowing for leeway.

Current and windward sailing questions may be combined.

Example.—A boat can sail 6 knots within $4\frac{1}{2}$ points of the wind, with half a point leeway. She has to reach a ship 7 miles due East and dead to windward. A tide is running ESE 1·5 knots. How long will she take to reach the ship?

Starting on the starboard tack, the boat's course will be in direction AC, where $CAB=5$ points, the current causing her to arrive at D, where she must tack, so that, heading in the direction DE, where $EDG=3$ points, the current will cause her to arrive at B.

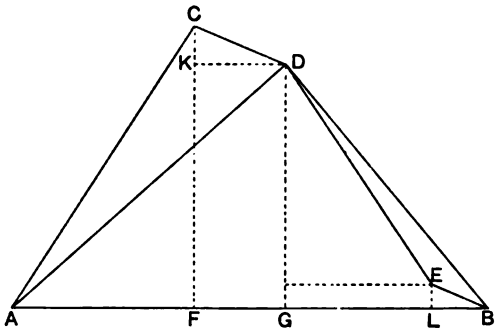


FIG. 16.

The distance AB is made up of AF, FG, GL, LB.

If t be the whole time; t' the time that has elapsed when she tacks

$$AF=6t' \cos 5 \text{ pts.}, \qquad FG=1{\cdot}5t' \cos 2 \text{ pts.},$$
$$GL=6(t-t')\cos 5 \text{ pts.}, \qquad LB=1{\cdot}5(t-t')\cos 2 \text{ pts.}$$
$$\therefore \quad AB=6t \cos 5 \text{ pts.}+1{\cdot}5t \cos 2 \text{ pts.}$$
$$7=t(3{\cdot}33+1{\cdot}39).$$
$$\therefore \quad t=\frac{7}{4{\cdot}72}=1\tfrac{1}{2} \text{ hours nearly.}$$

Two tacks only have been considered for the sake of simplicity.

NOTE.—Problems of this description may be solved with considerable accuracy by projection, if the work is carefully done.

The value of AB might have been obtained at once by resolving the whole distance due to the wind and the whole distance due to the current, in the direction of AB.

Examples.—(1) In the foregoing example, find the distance of D from B.

(2) The wind was due East. A ship sailing 10 knots within 5 points of the wind, through a current setting ESE, took 16·7 hours to reach a port 142 miles dead to windward of her starting point. Find the rate of the current.

(3) A ship using a 28-seconds glass and sailing within 6 points of the wind, at an apparent rate of 6 knots, reached a buoy 14 miles to windward of her on her second tack, in 6·5 hours. Find the error in the length of a knot on the line.

(4) A ship wishes to weather a point of land bearing from her WbN, distant 9 miles. The wind is WbS, and the ship lies with $5\frac{1}{2}$ points of the wind with $\frac{3}{4}$ point leeway. How far must she sail on one tack so as just to weather the point on the next tack?

(2) *Parallel Sailing.*

§ 17. Plane sailing only takes account of difference of latitude and the distance run east or west. It does not depend on the ship's position on the earth's surface, and tells us nothing as to how much the ship has changed her *longitude*.

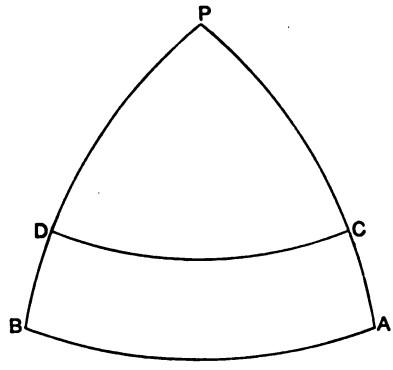


FIG. 17.

As soon as the idea of longitude is introduced we must take account of the fact that the earth is practically a sphere. The meridians through consecutive degrees of longitude are 60′ apart at the equator and meet at the pole. Hence the distance on a parallel of latitude between two meridians must vary with the latitude, and, conversely, the difference of longitude corresponding to a given number of miles of departure must also vary with the latitude.

Parallel sailing is the first of the sailings to introduce the idea of difference of longitude.

Def. 33. *Parallel sailing, as the name denotes, is the method by means of which the position, course, etc., of a ship is determined when the ship sails on a parallel of latitude due east or west.*

By a well-known property in Spherical Trigonometry, when DC is the arc of a small circle on a sphere parallel to BA the arc of a great circle,

$$\frac{DC}{BA} = \cos CA \quad \text{(see fig. 17)},$$

or distance = difference of longitude × cos latitude, if BA = the arc of the equator between the meridians through C and D.

This formula solves all questions in parallel sailing.

Examples.—A ship sails from longitude 64° 30′ W to longitude 47° 19′ W on the parallel of latitude 40°. Required the course and distance made good.

Long. from,	64° 30′ W	
Long. in,	47 19 W	distance = 1031 × cos 40°.
	17 11 E	3·013259
	60	9·884254
diff. long.,	1031′ E	2·897513
Course,	East.	distance = 789·8 miles.

A ship changes her longitude 5° in sailing 275 miles due west. What is the latitude of the parallel sailed on?

	2·439333
cos lat. = $\frac{275}{300}$.	2·477121
	9·962212

Latitude of parallel, 23° 33′ 15″.

Examples.—Find the course and distance from A to B, having given

	Latitude.	Longitude.		Latitude.	Longitude.
(1) A	50° 20′ N	47° 25′ W	(3) A	12° 17′ S	34° 29′ E
B		68 13 W	B		79 16 E
(2) A	26 27 N	49 50 W	(4) A	5 19 N	83 17 E
B		15 52 W	B		65 27 E

Find the longitude in, having given

	Course.	Distance.	Lat. from.	Long from.
(5)	East	675′	37° 18′ N	59° 27′ W
(6)	East	981	34 57 S	39 47 E
(7)	West	1519	13 12 N	115 35 W
(8)	West	597	14 19 S	3 48 E
(9)	East	398	41 25 S	175 27 E

(3) *Middle-latitude sailing.*

§ 18. When a ship sails on a meridian she changes her latitude only; when on the equator or a parallel of latitude, her longitude only; in any other case she must change both latitude and longitude.

Def. 34. *Middle-latitude sailing is the method by which a relation is found between the departure and the difference of longitude when a ship does not sail on a parallel;* the course distance, etc., being obtained with the aid of the formulæ of plane sailing.

When a ship sails from A to B (see fig. 12, § 12), it is clear that the departure $F'R+GS+\ldots$ is less than AD the distance between the meridians in the latitude of A, and greater than the distance BC in the latitude of B. There must therefore be some latitude between A and B at which the distance between the meridians is equal to the departure. It is *assumed* that this is the mean of the latitudes between A and B, and it is called the middle latitude.

If mn thus represent the departure, we have, as in parallel sailing,

$$mn = EQ \cos \text{lat. of } m;$$

$$\therefore \text{ dep.} = \text{diff. long.} \times \cos \text{ mid. lat.} \quad \ldots\ldots\ldots\ldots\ldots\text{(i.)}$$

or $$\text{diff. long.} = \text{dep.} \times \sec \text{ mid. lat.} \quad \ldots\ldots\ldots\ldots\ldots\text{(ii.)}$$

By plane sailing $\dfrac{\text{dep.}}{\text{diff. lat.}} = \tan \text{course};$

$$\therefore \tan \text{course} = \frac{\text{diff. long.} \times \cos \text{ mid. lat.}}{\text{diff. lat.}}, \quad \ldots\ldots\ldots\text{(iii.)}$$

also $$\text{distance} = \text{diff. lat.} \times \sec \text{ course}, \quad \ldots\ldots\ldots\ldots\ldots\text{(iv.)}$$

These formulæ will solve all problems in middle-latitude sailing.

§ 19. Since dep. = diff. long. × cos mid. lat., the diff. long. can be found from the Traverse Table. With *mid. lat.* as course, look out the *dep.* in the *diff. lat.* column, and the corresponding *distance* will be the *diff. long.* required, or the departure can similarly be found from the *diff. long.*

Examples (use of Traverse Table).

Lat. from 37° 25′ N, diff. lat. 70′ N, dep. 97′. Required the diff. long.

	37°	25′ N
	1	10 N
Lat. in,	38	35 N
	37	25 N
2)	76	0
Mid. lat.,	38	0

In the Traverse Table the nearest quantity in the *diff. lat.* column is 96·9′, with *course* 38°. The *distance* 123′ is therefore the *diff. long.* required.

Lat. from 49° 20′ N, diff. lat. 160′ S, diff. long. 167′ E. Required the departure.

	49°	20′ N
	2	40 S
Lat. in,	46	40
	49	20
2)	96	0
Mid. lat.,	48	0

In the Traverse Table, with 167′ as *distance* and 48° as *course*, the quantity in the *diff. lat.* column is 111·7, which is therefore the *departure* required.

The course and distance made good may then be found by looking in the Table for *diff. lat.* 160, *dep.* 111·7. The nearest quantities are *diff. lat.* 159·7, *dep.* 111·8, the corresponding *course* and *distance* being, therefore, S 35° E, 195′.

§ 20. The assumption made is not quite correct.

Def. 35. *The True Middle Latitude is that latitude in which the distance between the meridians is equal to the departure.*

This is always somewhat nearer to the pole than the mean of the latitudes. The correction may be found in Table 6, *Inman's Tables*, new edition.

The results, therefore, obtained by middle-latitude sailing are not *theoretically* correct—

(1) in high latitudes; because the cosines and secants of large angles change rapidly; and a small error in the mid. lat. produces a larger error in the dep. or diff. long., or course;

(2) when the difference of latitude is great or course small; because the greater the difference of latitude, the farther is the *true* middle latitude from the mean of the latitudes;

(3) when the two places are on opposite sides of the equator; the principle being manifestly inapplicable.

In this case it is usually sufficiently accurate in an ordinary day's run to consider the departure as equal to the difference of longitude.

No error of any practical importance is caused by the use of middle-latitude sailing in the ordinary conditions of a day's run. For long distances the more accurate method of Mercator's sailing, which is a rigorous method, is usually employed.

Examples.—(1) Find the course and distance from *A* to *B*, having given—

	Latitude.	Longitude.
A,	50° 15′ N	27° 19′ W
B,	47 30 N	31 14 W

Lat. A,	50°	15′	N		50°	15′	0″	N	Long. A,	27°	19′ W
Lat. B,	47	30	N		47	30	0	N	Long. B,	31	14 W
	2	45		2)	97	45	0			3	55
	60			Mid. lat.,	48	52	30			60	
diff. lat.,	165′	S							diff. long.,	235′	W

cos mid. lat.,	9·818030	sec course,	10·136788
diff. long.,	2·371068	diff. lat.,	2·217484
	12·189098		2·354272
diff. lat.,	2·217484		
tan course,	9·971614	distance = 226·1′.	

Course, S 43° 7′ 45″ W.

(2) Find the latitude and longitude in, having given—

Lat. from, 27° 15′ S; long. from, 93° 21′ E; course, S 61° W; distance, 325′ miles.

cos course,	9·685571	tan course,	10·256248
distance,	2·511883	diff. lat.,	2·197454
	2·197454	sec mid. lat.,	10·056376
			2·510078
diff. lat.	= 157·6′	diff. long.	= 323·7′ W
	= 2° 38′ S		= 5° 24′ W
Lat. from,	27 15 S	Long. from,	93 21 E
Lat. in,	29 53 S	Long. in,	87 57 E
	27 15		
2)	57 8		
Mid. lat.,	28 34		

Examples.—Find the course and distance from A to B, having given—

	Latitude.	Longitude.		Latitude.	Longitude.
(1) A,	39° 27′ N	47° 18′ W	(3) A,	12° 41′ N	52° 32′ E
B,	36 15 N	53 13 W	B,	18 37 N	45 21 E
(2) A,	35 26 S	52 29 E	(4) A,	18 47 S	27 57 W
B,	41 3 S	58 19 E	B,	15 21 S	32 3 W

Find the latitude and longitude in, having given—

	Latitude from.	Longitude from.	Course.	Distance.
(5)	29° 18′ N	57° 17′ W	S 49° W	381′
(6)	45 27 S	79 18 E	N 71 E	326
(7)	36 29 N	70 14 W	N 58 E	517
(8)	17 50 N	50 9 E	S 43 W	409

§21. *Mercator's Sailing.*

Def. 36. *Mercator's sailing is the method of finding a ship's position, course, etc., by means of principles derived from the construction of a Mercator's chart.*

This obtains its name from Gerard Mercator, a Fleming (A.D. 1569), who appears to have been the first to construct a chart in which the length of the degrees of latitude increased as they receded from the equator. His method was improved upon by Mr. E. Wright, of Caius College, Cambridge (A.D. 1599).

§ 22. Suppose a globe or sphere, with meridians, parallels of latitude, rhumb lines, etc., drawn on it, to be circumscribed by a hollow cylinder, which touches it all round the equator, and suppose that this globe is continually expanded until each point of it, in succession, touches the cylinder, the expansion at that point then ceasing, while that of the portion of the sphere not in contact with the cylinder still goes on. The meridians will become straight lines, and the parallels circles on the surface of the cylinder, the former in the direction of the axes, the latter parallel to the base.

If now the cylinder be unrolled and spread out into a plane, the surface thus produced will represent a Mercator's chart, which gives a true representation of the form of each small portion of the earth's surface; but varies greatly as to the scale on which these portions are represented in different latitudes. The polar regions, in particular, are very much distorted.

§ 23. From the above description it appears that the distance between the meridians, on a Mercator's chart, is everywhere made equal to the difference of longitude, the meridians becoming straight lines at right angles to the equator.

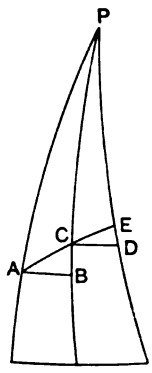


FIG. 18.

Each parallel of latitude, therefore, must have been increased in the ratio of sec lat.: 1, by the principle of parallel sailing; the rhumb line, which cuts all the meridians at the same angle, becomes a straight line; and the arcs of the meridians must, therefore, to preserve the relative proportions of the small figures, such as *ABC*, *CDE* (fig. 18), be also increased in the same proportion as the parallels of latitude, *i.e.* sec lat.: 1. Hence the degrees of latitude on a Mercator's chart increase as they recede from the equator, as the secant of the latitude increases; and all the parallels of latitude, and every part of them, are larger than they are on the globe, in the proportion sec lat.: 1.

Hence also, though the latitudes and longitudes and bearings of places are accurately represented on a Mercator's chart, the distances are distorted in various proportions.

Def. 37. *The Meridional Parts of a certain latitude give the length, expressed in nautical miles, of the distance on a Mercator's chart from the equator to the parallel of that latitude.*

Def. 38. *The Meridional Difference of Latitude between two places is the length, expressed in nautical miles, of the line on a Mercator's chart, which represents the difference of latitude.*

To calculate the Meridional Parts for any latitude.

§ 24. Let PU and PV represent two meridians on the surface of the earth; UV the equator, AB, CD, etc., arcs of parallels of latitude. Let these arcs be expanded in fig. 19 (2) into the distances ab, cd, etc., on a Mercator's chart, uv being equal to UV the difference of longitude.

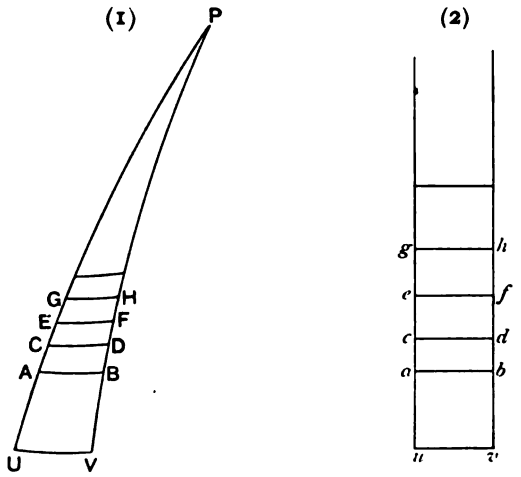


FIG. 19.

Then the differences of latitude BD, DF, etc., must be expanded similarly into bd, df, etc., where

$$bd = BD \sec \text{lat. } B,$$
$$df = DF \sec \text{lat. } D,$$
$$fh = FH \sec \text{lat. } F,$$
$$\text{etc.} = \quad \text{etc.}$$

Now suppose that $BD = DF = FH \ldots$, and that each $= 1'$ of latitude, the latitude of B being l. Then

$bd + df + fh =$ meridional difference of latitude between B and H

$= \sec \text{lat. } B + \sec \text{lat. } D + \ldots$

$= \sec l^\circ + \sec (l^\circ + 1') + \sec (l^\circ + 2') + \ldots .$

Now suppose B to be a place on the equator, then

$bd + df + fh =$ meridional parts for $3'$

$= \sec 0 + \sec 1' + \sec 2'.$

In the same way it may be shown that meridional parts for l°

$= \sec 0 + \sec 1' + \sec 2' + \ldots + \sec (l^\circ - 1').$

A closer approximation may be obtained by taking the distances BD, DF, etc., each $= 1''$, but the above is sufficiently accurate for all practical purposes. For accurate calculation of meridional parts a formula obtained by means of the Integral Calculus is required; and the fact that the earth is not a perfect sphere must be taken into account.

When the meridional parts for any latitude have been calculated, those for any other latitude may be found by multiplying the true difference of latitude by the secant of the *true* middle latitude, which can be obtained by means of Table 6, *Inman's Tables*, new edition (cf. § 26), and adding the result to the given meridional parts.

To obtain the meridional difference of latitude between two places take from the Table the meridional parts for each latitude, and then take the difference or the sum, according as the latitudes are of the same or different names.

To show that $\tan \text{course} = \dfrac{\text{diff. long.}}{\text{mer. diff. lat.}}$

§ 25. Let the figure (fig. 20) represent a portion of a Mercator's chart, A and B the positions of two places on it. Completing the right-angled triangle ABC, BC will represent the difference of longitude and AC the meridional difference of latitude.

Suppose that $BC =$ departure $\times m$, then by the principle of the construction AC must $=$ diff. lat. $\times m$.

Now $\tan \text{course} = \frac{\text{dep.}}{\text{diff. lat.}} = \frac{\text{dep.} \times m.}{\text{diff. lat.} \times m} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}$.

$\therefore$ CAB represents the course from A to B.

If AD be the true diff. lat. between the two places, and DE be drawn parallel to CB, AE will represent the true distance between them, because the *relative* proportions of distances, etc., on the globe are preserved on the chart.

$$\therefore \quad AE = AD \sec EAD,$$

or $$\text{distance} = \text{true diff. lat.} \times \sec \text{course}.$$

These two formulæ solve all problems in Mercator's sailing.

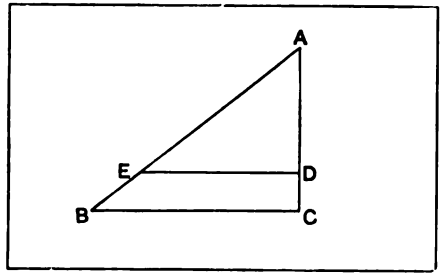


FIG. 20.

Considering the two expressions

$$\tan \text{course} = \frac{\text{dep.}}{\text{diff. lat.}}, \quad \tan \text{course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}},$$

we obtain $\text{diff. long.} = \text{dep.} \frac{\text{mer. diff. lat.}}{\text{diff. lat.}}$,

by means of which the diff. long. may be obtained from the departure, if desired, without the use of the middle latitude.

§ 26. Also, since

$$\text{dep.} = \text{diff. long.} \times \cos \text{true mid. lat.},$$

$$\therefore \quad \cos \text{true mid. lat.} = \frac{\text{diff. lat.}}{\text{mer. diff. lat.}},$$

which gives the latitude in which the distance between the meridians *is* equal to the departure made good between two places.

If the course is nearly 90°, and there is any probability of error in it, the diff. long. obtained from the expression

$$\text{diff. long.} = \text{mer. diff. lat.} \times \tan \text{course}$$

may be incorrect, as the tangents of angles near 90° change very rapidly.

In this case it may be preferable to use the middle-latitude method, which is used, in practice, for obtaining the diff. long. from the departure in the ordinary daily reckoning.

Examples.—(1) Find the course and distance from *A* to *B*, having given—

	Latitude.	Longitude.
A,	50° 15′ N	27° 19′ W
B,	47 30 N	31 14 W

		Mer. parts.		
Lat. from,	50° 15′ N	3497·87′	Long. from, 27° 19′ W	
Lat. in,	47 30 N	3246·91	Long. in, 31 14 W	
	2 45	250·96		3 55 W
	60	Mer. diff. lat.		60
diff. lat.,	165′ S		diff. long.,	235′ W

diff. long.,	2·371068	sec course,	10·136728
mer. diff. lat.,	2·399605	diff. lat.,	2·217484
tan course,	9·971463		2·354212
Course, S 43° 7′ 15″ W		distance,	226·5′

the course differing by about 30″ and the distance by less than half a mile from the results obtained by middle-latitude method.

(2) Find the latitude and longitude in, having given—
lat. from 27° 15′ S, long. from 93° 21′ E, course S 61° W, distance 325 miles.

			Mer. parts.
cos course,	9·685571	27° 15′	1700·37
distance,	2·511883	29 53	1880·30
	2·197454		179·93
diff. lat.	=157·6	mer. diff. lat.,	2·255104
	=2° 38′ S	tan course,	10·256248
			2·511352
Lat. from,	27 15		
Lat. in,	29 53 S	diff. long.	=324·6
			=5° 25′ W
		Long. from,	93 21 E
		Long. in,	87 56 E

which differs from the middle-latitude method result by 1 mile of longitude.

Examples.

(In several of the following examples the data are the same as in those given on middle-latitude sailing, as the general agreement of the two results in ordinary cases is thus made plain. For *long* distances Mercator's sailing should be used, unless the course is nearly 90°.)

Find the course and distance from *A* to *B*, having given—

		Latitude.	Longitude.
(1)	*A*,	39° 27′ N	47° 18′ W
	B,	36 15 N	53 13 W
(2)	*A*,	35 26 S	52 29 E
	B,	41 3 S	58 19 E
(3)	*A*,	12 41 N	52 32 E
	B,	18 37 N	45 21 E
(4)	*A*,	32 30 S	158 12 E
	B,	21 10 N	159 22 W

Find the latitude and longitude in, having given—

	Latitude from.	Longitude from.	Course.	Distance.
(5)	29° 18′ N	57° 17′ W	S 49° W	381′
(6)	45 27 S	79 18 E	N 71 E	326
(7)	36 29 N	70 14 W	N 58 E	517
(8)	51 20 S	60 15 W	N 73 E	3400

The last example worked by middle-latitude method gives 13° 55′ E as the resulting longitude, but that method should not be used for such long distances.

To obtain a ship's position by "Dead Reckoning."

§ 27. This is obtained by a combination of the various "sailings," traverse and middle-latitude sailings being those principally employed. When the position is that obtained at noon from the courses and distances sailed in the previous twenty-four hours, the work is sometimes called the "day's work," though the whole "day's work" includes the finding the position by observation, current, course and distance made good, etc., as will be explained later.

The method of obtaining the dead reckoning will be best understood by means of a fully-worked example, which is supposed to represent data taken from a ship's log.

For practice in turning points into degrees the courses are given in points. In actual navigation courses are most usually expressed in degrees.

Example. A ship sailed from Lat. 54° 3′ N, Long. 10° 25′ W, as by the following log account. Find the latitude and longitude in next day at noon.

Hours.	Knots.	Tenths.	Standard Compass Course.	Deviation.	Wind.	Leeway.	Remarks.
						Points.	
1	7	2	NNW ½W	3° E	NE ½N	½	
2	6	8					
3	6	5					
4	6	8					
5	7	2					
6	7	5					
7	5	8	NbW ¾W	2° 30′ E	NE	¾	
8	5	5					
9	4	8					
10	4	5					
11	5	2					
12	4	2					Midnight.
1	5	5					
2	5	5					
3	4	8	SEbE ¼E	1° W	NEbE	1	
4	4	8					
5	4	2					
6	4	2					
7	4	8	ESE	2° W	NEbN	½	Variation allowed, 27° W
8	4	8					
9	5	2					
10	5	2					
11	7	8	EbN	2° 30′ W	NbW	0	
12	8	2					

I.—*To correct the courses.*

	(1)	(2)	(3)
Course,	NNW ½W	NbW ¾W	SEbE ¼E
	28° 7′ 30″ *l* N	19° 41′ 15″ *l* N	59° 3′ 45″ *l* S
Deviation,	3 0 0 *r*	2 30 0 *r*	1 0 0 *l*
Magnetic course,	25 7 30 *l* N	17 11 15 *l* N	60 3 45 *l* S
Variation,	27 0 0 *l*	27 0 0 *l*	27 0 0 *l*
True course steered,	52 7 30 *l* N	44 11 15 *l* N	87 3 45 *l* S
Leeway,	5 37 30 *l*	8 26 15 *l*	11 15 0 *r*
	57 45 0 *l* N	52 37 30 *l* N	75 48 45 *l* S
True course made good,	N 58° W	N 53° W	S 76° E

	(4)	(5)
Course,	ESE 67° 30′ 0″ *l* S	EbN 78° 45′ *r* N
Deviation,	2 0 0 *l*	2 30 *l*
Magnetic course,	69 30 0 *l* S	76 15 *r* N
Variation,	27 0 0 *l*	27 0 *l*
True course steered,	96 30 0 *l* S	49 15 *r* N
Leeway,	5 37 30 *r*	0
	90 52 30 *l* S	49 15 *r* N
	180	
	89 7 30 *r* N	
True course made good,	N 89° E.	N 49° E.

In practice it is not necessary to work to seconds. If the minutes in the course are less than 30, they may be neglected; if over 30, take the next higher whole degree as the course. If great accuracy is desired, take the course to the nearest half degree (seldom necessary).

II.—*To find the distances.*

Add up the knots and tenths run on each course, then

III.—Form a Table as follows, with the aid of the Traverse Table:—

Course.	Distance.	diff. lat.		departure.	
		N.	S.	E.	W.
N 58° W	42′	22·3	—	—	35·6
N 53 W	41	24·7	—	—	32·7
S 76 E	18	—	4·4	17·5	—
N 89 E	20	0·3	—	20·0	—
N 49 E	16	10·5	—	12·1	—
		57·8	4·4	49·6	68·3
		4·4			49·6
Diff. lat.,		53·4 N			18·7 W dep.

It is advisable, before filling up the diff. lat. and departure columns, to draw a dash (—) in those places which will not be required. In this way the mistake of entering in the wrong column can be avoided.

IV.—The diff. lat. and departure having been thus obtained, to the latitude from, apply the diff. lat. and obtain the latitude in; and thence the middle latitude. Then, as

diff. long. = dep. × sec mid. lat.,

obtain the diff. long. by calculation or the Traverse Table (if the distances are large and the course not an exact degree, the Traverse Table must be used carefully).

Thus

Lat. from,	54°	3′	0″ N	dep.,	1·271842	
diff. lat..	0	53	24 N	sec mid. lat.,	10·236002	
Lat. in.,	54	56	24 N		1·507844	
	54	3	0			
				diff. long.,	32·2′	W
2)	108	59	24			
Mid. lat.,	54	29	42	Long. from,	10° 25′ 0″	W
				diff. long.,	0 32 12	W
				Long. in,	10 57 12	W

By Traverse Table, with 18·7 as diff. lat. and 54° 30′ as course, the nearest distance is 32′, which is practically the same as the result by calculation.

Results—Lat. in, 54° 56′ 24″ N; long. in, 10° 57′ 12″ W.

§ 28. It is sometimes stated that, when the latitude is greater than 50°, the diff. long. should be calculated for each course separately. The foregoing example, in which the courses are almost all to the northward, treated in this manner produces a diff. long. 31·3, which is less than one mile of longitude, and about half a mile in actual distance, from that already obtained, and would probably be still less had more of the courses been to the southward. Unless there is any *very* great difference of latitude, in a high latitude, the probable error resulting from omitting to work each course separately will be less than those arising from slight errors in steering, compass error, etc., and may safely be neglected, as far as an ordinary day's run is concerned, in all those parts of the earth in which navigation usually takes place. For long distances and the determination of the course the methods of Mercator's sailing are more suitable.

§ 29. Sometimes, in examples for practice, the ship's bearing and distance is given from a point of land whose latitude and longitude are known. In this case the bearing *reversed* (named the *departure course*) and distance are to be treated like the rest of the courses, the deviation applied being that due to the direction of the ship's head.

In actual navigation the position of the ship is fixed, and the courses, etc., reckoned from that position.

Examples.—(1) A ship in latitude 55° 52′ N, and longitude 131° 4′ W, sailed as by the following log account. Work up the reckoning from noon to noon.

Hours.	Knots.	Tenths.	Course.	Wind.	Leeway.	Deviation.	Remarks.
					Points.		
1	10	5	SWbW	S	½	7° 30′ W	P.M.
2	10	5					
3	10	8					
4	11	2					
5	11	0					
6	10	0	W ½S	SSW	½	8° 30′ W	
7	10	0					
8	10	5					
9	10	5					
10	11	8					
11	12	2					
12	12	5	WbN ¼N	SW	¾	8° W	Midnight.
1	13	0					A.M.
2	13	0					
3	13	5					
4	13	0					
5	12	5	S ½W	WSW	½	3° 30′ W	Variation allowed, 27° E.
6	11	8					
7	11	2					
8	10	5					
9	10	0					
10	10	0	S ¼E	SW	1	2° 30′ W	
11	9	0					
12	9	0					Noon.

(2) A ship in latitude 27° 43′ N, and longitude 18° 10′ W, sailed as by the following log account. Find her latitude and longitude in, next day at noon.

Hours.	Knots.	Tenths.	Course.	Wind.	Leeway.	Deviation.	Remarks.
					Points.		
1	9	0	NbW ½W	NEbE	½	0° 30′ W	P.M.
2	9	5					
3	9	5					
4	10	0					
5	10	0					
6	11	2	NNW ¼W	NE	¾	2° W	
7	11	0					
8	10	8					
9	10	5					
10	10	5					
11	9	5	EbS ½S	NEbN	½	6° 30′ E	
12	9	5					Midnight.
1	9	0					A.M.
2	9	8					
3	10	2					
4	10	5					
5	10	5					
6	11	0	NbW	NE	0	1° E	Variation allowed, 21° W.
7	11	5					
8	11	5					
9	10	5	EbN	NNE	¼	8° 30′ E	
10	10	8					
11	10	2					
12	9	5					Noon.

(3) A ship in latitude 23° 30′ S, longitude 35° 50′ E, afterwards sailed as by the following log account. Find the latitude and longitude in, next day at noon.

Hours.	Knots.	Tenths.	Course.	Wind.	Leeway.	Deviation.	Remarks.
					Points.		
1	5	4	SWbS	WbN	1	6° 10′ W	P.M.
2	5	9					
3	4	7					
4	4	9					
5	6	1					Variation of compass, 2½ pts. W.
6	6	3					
7	6	0					
8	5	1					
9	5	0	EbS	NNE	2	7° 10′ E	
10	7	2					
11	8	7					
12	7	3					Midnight.
1	5	0	SE	ENE	½	2° E	A.M.
2	5	7					
3	5	9					
4	6	4	WSW	E	0	8° W	
5	6	0					
6	5	1					A current set the ship the last four hours 2 knots an hour N 60° E magnetic.
7	3	9					
8	4	7					
9	5	2					
10	5	9					
11	6	8					
12	8	4					Noon.

(4) A ship in latitude 33° 12′ S, and longitude 37° 40′ W, afterwards sailed as by the following log account. Find the latitude and longitude in, next day at noon.

Hours.	Knots.	Tenths.	Course.	Wind.	Leeway.	Deviation.	Remarks.
					Points.		
1	7	2	EbS	NE	1	8° E	P.M.
2	7	0					
3	7	8					
4	7	5					
5	7	0	SE	ENE	¾	11° E	
6	6	5					
7	6	5					Variation of compass, 21° W.
8	6	0					
9	7	0	SEbS	E	1¼	9° E	
10	6	8					
11	6	5					
12	7	5					Midnight.
1	8	2					A.M.
2	8	0					
3	9	0	NW ½N	NNE	¾	6° W	
4	8	8					
5	8	5					
6	9	5					
7	9	2					
8	9	5	SWbS	N	0	9° W	
9	9	5					
10	9	5					
11	9	5					
12	10	0					Noon.

(5) A ship in latitude 37° 28′ S, and longitude 57° 38′ W, afterwards sailed as by the following log account. Find the latitude and longitude in, next day at noon.

Hours.	Knots.	Tenths.	Course.	Wind.	Leeway.	Deviation.	Remarks.
					Points.		
1	8	7	ESE	S	¾	1° 0′ E	P.M.
2	9	2					
3	9	5					
4	10	6					
5	10	6	SEbS	SWbS	¾	2° E	
6	11	2					Variation of compass, 1½ pts. E.
7	11	2					
8	9	0					
9	10	0					
10	10	5					
11	10	5					
12	11	5	N ¾E	E	¼	3½° E	Midnight.
1	11	5					A.M.
2	10	0					
3	10	0	SW ½W	SSE	½	7° W	
4	9	2					
5	9	4					
6	9	4					
7	9	8	NWbW	S	—	5½° W	
8	9	2					
9	9	0					
10	10	5	EbS ½S	NEbN	¼	7° E	
11	10	5					
12	10	0					Noon.

(6) A ship in latitude 50° 10′ S, and longitude 58° 37′ W, sailed as by the following log account. Find the latitude and longitude next day at noon.

Hours.	Knots.	Tenths.	Course.	Wind.	Leeway.	Deviation.	Remarks.
					Points.		
1	9	0	NEbE	SE	¼	5° 29′ W	P.M.
2	9	0					
3	9	5					
4	9	5					
5	10	0					
6	11	2	E ¼S	SSE	¾	6° 18′ W	Variation of compass, 27° 10′ W.
7	10	4					
8	10	6					
9	10	8					
10	11	0					
11	11	0					
12	10	6	ENE	SE	½	6° 10′ W	Midnight.
1	10	8					A.M.
2	10	5					
3	10	7					
4	11	0					
5	11	4					
6	11	0	NEbE ½E	SE	½	3° 15′ W	
7	11	5					
8	11	5					
9	11	0					
10	10	4	EbS ¼S	S	¾	1° 18′ E	
11	10	6					
12	9	0					Noon.

Formulæ made use of in the "Sailings," collected for reference.

Plane sailing:

$$\text{diff. lat.} = \text{dist.} \times \cos \text{course}, \quad \ldots\ldots(1)$$

$$\text{dep.} = \text{dist.} \times \sin \text{course}, \quad \ldots\ldots(2)$$

$$\tan \text{course} = \frac{\text{dep.}}{\text{diff. lat.}}, \quad \ldots\ldots(3)$$

$$\text{dist.} = \text{diff. lat.} \times \sec \text{course}, \quad \ldots\ldots(4)$$

Parallel sailing:

$$\text{dist.} = \text{diff. long.} \times \cos \text{lat.}$$

Middle-latitude sailing:

$$\text{dep.} = \text{diff. long.} \times \cos \text{mid. lat.}, \quad \ldots\ldots(1)$$

$$\text{diff. long.} = \text{dep.} \times \sec \text{mid. lat.}, \quad \ldots\ldots(2)$$

$$\tan \text{course} = \frac{\text{diff. long.} \times \cos \text{mid. lat.}}{\text{diff. lat.}}, \quad \ldots\ldots(3)$$

$$\text{dist.} = \text{diff. lat.} \times \sec \text{course}, \quad \ldots\ldots(4)$$

Mercator's sailing:

$$\tan \text{course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}, \quad \ldots\ldots(1)$$

$$\text{dist.} = \text{true diff. lat.} \times \sec \text{course}, \quad \ldots\ldots(2)$$

$$\text{diff. long.} = \frac{\text{dep.} \times \text{mer. diff. lat.}}{\text{true diff. lat.}}, \quad \ldots\ldots(3)$$

$$\cos \textit{true} \text{ mid. lat.} = \frac{\text{diff. lat.}}{\text{mer. diff. lat.}}, \quad \ldots\ldots(4)$$

MISCELLANEOUS EXAMPLES ON THE "SAILINGS."

Find the compass course and distance from *A* to *B*, having given—

(1)	*A*, Lat., 22° 16′ S	*B*, Lat., 22° 16′ S	Variation, 10° E
	Long., 166 55 E	Long., 171 14 W	Deviation, 3 E
(2)	*A*, Lat., 30 25 S	*B*, Lat., 30 25 S	Variation, 27 W
	Long., 101 11 E	Long., 43 20 E	Deviation, 4 E
(3)	*A*, Lat., 35 20 N	*B*, Lat., 47 10 N	Variation, 12 E
	Long., 165 17 W	Long., 171 20 E	Deviation, 5 E
(4)	*A*, Lat., 32 30 N	*B*, Lat., 40 27 N	Variation, 8 W
	Long., 64 55 W	Long., 74 1 W	Deviation, 2 E

Leeway (wind NNE), ½ pt.

(5) In sailing along a parallel of latitude, I find that the distance actually made good is 47 miles, while I have changed my longitude one degree. On what parallel am I sailing?

(6) A ship, in latitude 40° 25′, sailing on a parallel, changes her longitude 4° 15′. Find the distance sailed.

(7) A certain place on the earth's surface is revolving 250 miles per hour faster than a place whose latitude is 20° farther North. Find its latitude.

(8) Two ships sail from the same place. The first sails NW x miles, then SW x miles; the second sails SW x miles, then NW x miles. Will they reach the same position, and, if not, why not?

(9) A vessel sailed from Lat. 62° 6′ N, Long. 7° 37′ W, 196 miles due N, 196 miles due W, 196 miles due S, and 196 miles due E. How far was she then from her starting point?

(10) A ship sails from Lat. 33° 56′ S, Long. 18° 28′ W, on a NEbE course. In what latitude will she cross the Prime Meridian, and, in what longitude, the Equator?

(11) A ship in Lat. 50° 12′ N, Long. 4° 16′ W, sails SW ¼W, until her diff. long. is 620 miles. Required her latitude in, and the distance sailed.

(12) Two ships are chasing to windward, the wind being SE ½E. At the end of a given time the bearing of B from A is E ½S, distance 1560 yards. Find, by the Traverse Table, how much B has weathered on A.

(13) A ship sails from Lat. 57° 40′ N, Long. 42° 20′ W, between South and East till she arrives in Long. 40° 12′ W, and her mer. diff. lat. is 496 miles. Find the course and distance made good, and the latitude in.

(14) The meridional parts for 20° 40′ being 1267·8, calculate those for 20° 46′.

(15) The meridional parts for 62° 18′ being 4813·51, find those for 62° 26′.

(16) I am bound for a port in Lat. 12° 3′ N, its longitude being 5° 13′ to the Eastward, my latitude being 18° 20′ N. Next day I am in Lat. 15° 40′ N, and have made departure 115 miles E. Required my course and distance to the port.

(17) Two ships, sailing at the same rate, start together from Lat. 15° 50′ S, Long. 5° 40′ W for a port in Long. 14° W. One sails due West till she reaches the required longitude, and then due North to the port. The other sails due North till she reaches the latitude of the port, then due West, and is 15 miles from the port when the first arrives. What is the latitude of the port?

(18) Two ships leave a port in North latitude sailing at the same speed N $x°$ W, N $2x°$ W respectively. After a certain interval the diff. lat. was 104′ in one case, and 17′ in the other. Find the distance sailed.

(19) A ship sails on a certain course between South and West for a distance of 109 miles, and finds that she is in Lat. 1° 13′ higher than that of the place from which she sailed. On what course has she sailed, and how much westing has she made?

(20) A ship sails from a place in Lat. 60° N for 360′ on a course N 20° E. Find the error in the longitude of the place arrived at obtained by using middle-latitude sailing.

(21) Two ships, in Lat. l_1, are 385 miles apart. They both sail due North until they reach Lat. l_2, when their distance apart is 420 miles. Find l_1 and l_2, having given that $l_1 - l_2 = 12° 30′$.

(22) A ship sails WNW from Lat. 59° 40′ N, Long. 32° W, and at the end of 24 hours finds the difference of longitude to be double of the departure. Find the latitude and longitude in, and the average speed of the ship.

(23) A ship, using a 28-seconds glass, steams SEbE 150 miles (true), and finds that she has been overlogged, the T.D. lat. being 8·4′ too much, and the D. long. 17·6′ too much. Determine the error in the length of the knot on the line, and the middle latitude.

(24) A boat that can lie within 5 points of the wind wishes to reach a ship 8 miles due North of her, the wind being at NW. Required her course and distance on each tack.

(25) A cutter is 7 miles from a ship at anchor, which bears from her SSW. The wind is SbE, and the cutter lies within 6 points of it. How far must she sail on the port tack, so as to reach the ship on the other tack?

(26) A ship sails NEbE½E 10 miles from a headland in 1½ hours; she then finds that the headland bears SW distant 15 miles. Find the direction and strength of the current.

(27) A port bears from a ship ENE 102 miles. After proceeding on that course for 8 hours at 10½ knots, the port bore NNE 24 miles. Find the set and drift of the current.

(28) A ship steaming 11 knots wishes to reach a port bearing WSW 75 miles. How must she steer so as to allow for a current setting S¼E 2·5 knots, and how long will she be in completing the distance?

(29) A ship steaming from Gibraltar (36° 7′ N, 5° 21′ W) at the rate of 15·6 knots by the log, passes Gozo (36° 7′ N, 14° 28′ E) in 2 days 14 hours. Find the error in the length of the knots on the log line used.

(30) The wind is NbW, and a ship, sailing 10 knots within 5 points of the wind through a current setting WNW, took 4 hours to reach a port 36 miles dead to windward of her starting point. Find the rate of the current.

(31) A boat can sail 6 knots within 5 points of the wind. She has to reach a ship 10′ NE and dead to windward. It takes her 4 hours to reach the ship, starting on the starboard tack. If the rate of the tide is 2 knots, find its direction.

To construct a Mercator's chart.

§30. Draw a straight line near the *lower* margin of the paper if the chart is to represent north latitude; near the *upper* margin if it is to represent south latitude; or at a suitable position towards the centre if both north and south latitudes are to be represented; or it may be found convenient to take, as the line of reference, a parallel *about* the middle of the paper. Divide this line into equal parts to represent longitude according to the scale on which the chart is to be constructed,

and the number of degrees of longitude required. At each extremity erect perpendiculars. Take out the meridional parts for each convenient degree of latitude for the limits between which the chart is to be drawn, and take the difference between each successive pair, thus obtaining the meridional differences of latitude.

As the given scale represents 60 miles of longitude, multiply the scale by the number of miles in the meridional differences and divide by 60; the results will be the lengths, on the given scale, between the chosen degrees of latitude. Lay off these lengths successively on the perpendicular lines, and through the points thus obtained (but not through half degrees, unless the scale is very large) draw straight lines parallel to the original line, to represent parallels of latitude. Draw also straight lines at convenient intervals parallel to the perpendiculars to represent meridians.

The frame of the chart is thus completed. Its accuracy should be tested by seeing whether the two diagonals of the rectangle thus formed are equal, and whether the intervals representing longitude are of the same length at each end of the chart.

The intervals representing latitude and longitude should then be divided conveniently, the principal divisions numbered, all lines inked in, a neat margin added, and pencil lines rubbed out, when the chart is ready for use, such positions, land, etc., as may be required, being inserted.

§31. (1) *To fix a position on the chart.*

Place the edge of a parallel ruler along the parallel of latitude nearest to the given latitude: move it until it passes through this latitude on the graduated edge of the chart. Measure the distance of the given longitude from the nearest meridian, and lay off this along the edge of the ruler. The position thus obtained is the position required, which should be marked by a small cross.

The latitude and longitude of any position may be taken from the chart by the reverse of this method.

(2) *To lay off courses or bearings.*

On charts for actual use compasses are drawn; those for use near the land, on the magnetic meridian, marked to quarter points, and, in recent charts, to degrees.

On those made for practice it is sufficient to draw one or more straight lines, representing the magnetic meridian and passing through the intersection of a meridian and a parallel.

Correct the given compass course or bearing for the deviation due to the *direction of the ship's head*, so as to obtain the *magnetic* course or bearing. Lay this off from the magnetic meridian; a line parallel to it through the given position will be the required course or bearing.

The intersection of the bearings of two or more points thus laid off fixes the position of the ship; as she must be somewhere on each line, and, therefore, at their intersection.

The converse of this method will obtain the course between any two positions, or the bearing of any point, etc.

(3) *To find the distance between two points.*

The distance is found (nearly) by transferring the interval between the two positions to the graduated meridian, as nearly as possible opposite to the positions, *i.e.* as much below the more southern latitude as above the more northern: this space turned into minutes is the distance required. If the two places have the same latitude, half the distance should be measured on the graduated meridian on each side of the parallel of latitude; the total space measured, expressed in minutes, is (nearly) the distance required. If the places have the same longitude, the algebraic difference of their latitudes will be the distance between them.

(4) *To lay off a given distance*, take it, similarly, from the graduated meridian and apply it to the line representing the course or bearing.

The whole subject may be illustrated by the following example, worked in full.

Example.—Construct a Mercator's chart on a scale of 1·5 inches to a degree of longitude, extending from 53° 30′ N to 55° 30′ N, and from 8° W to 11° W. Place on it the following positions (fig. 21):

Clare Island Light, -	53° 50′ N	9° 59′ W
Achill Head, - -	53 58 N	10 15 W
Carrigan Head, - -	54 37 N	8 41 W
Rathlin O'Birne Island,	54 40 N	8 50 W
Horne Head, - -	55 14 N	8 0 W
Tory Island Light, -	55 17 N	8 15 W

NOTE.—For construction of charts on the Gnomonic projection, for use in high latitudes, see Great Circle Sailing.

A ship's position was fixed by Cross Bearings, the variation allowed being 25° 20′ W.

A, {Clare Island Light, SEbE / Achill Head, NNE}, Deviation, 2° 50′ W.

Afterwards sailed as follows:

Standard Compass Course.	Deviation.	Distance.	Cross Bearings.
NNW ½W	2° 50 W	31′	
E	5 40 E	75	B, {Rathlin O'Birne Island, N ½W / Carrigan Head, E ¼S}
N ¾W	3 0 W	60	
ESE	5 30 E	60	C, {Horne Head, S ¾W / Tory Island Light, SWbW ¾W}

Lay down the various courses and fix the positions A, B, C.

(1) To construct the Frame.

		Meridional Diff. Lat.
Mer. Parts for 53° 30′,	3813·90	
		50·74
„ 54 0,	3864·64	
		103·33
„ 55 0,	3967·97	
		52·63
„ 55 30,	4020·60	

	Inch.	Inch.	½ inch.
$50·74 \times \frac{1·5}{60} = \frac{50·74}{40}$		$= 1·2685$	$= 2·5370$
$103·33 \times \frac{1·5}{60} = \frac{103·33}{40}$		$= 2·58325$	$= 5·1665$
$52·63 \times \frac{1·5}{60} = \frac{52·63}{40}$		$= 1·31575$	$= 2·6315$
Total length of chart		$= 5·16750$	

4020·60 mer. parts for 55·30
3813·90 „ 53·30

206·70 = mer. diff. lat.

$206·7 \times \frac{1·5}{60} = \frac{206·7}{40}$

$= 5·1675$

This result, agreeing with the one obtained by considering the separate mer. diff. lats., shows that the work is correct. This method should be always used as a check.

Draw a line near the lower margin of the paper at such a distance as to bring the completed chart symmetrically in the centre ; divide it and erect perpendiculars as directed ; lay off the distances, taken from a diagonal scale, which represent the mer. diff. lat. ; draw parallels through latitudes 54° and 55°, and through 55° 30′, as it is the Northern limit of the chart ; also meridians through longitudes 9° and 10°. Divide each degree of latitude and longitude into 6 parts to represent 10′. As the variation is assumed

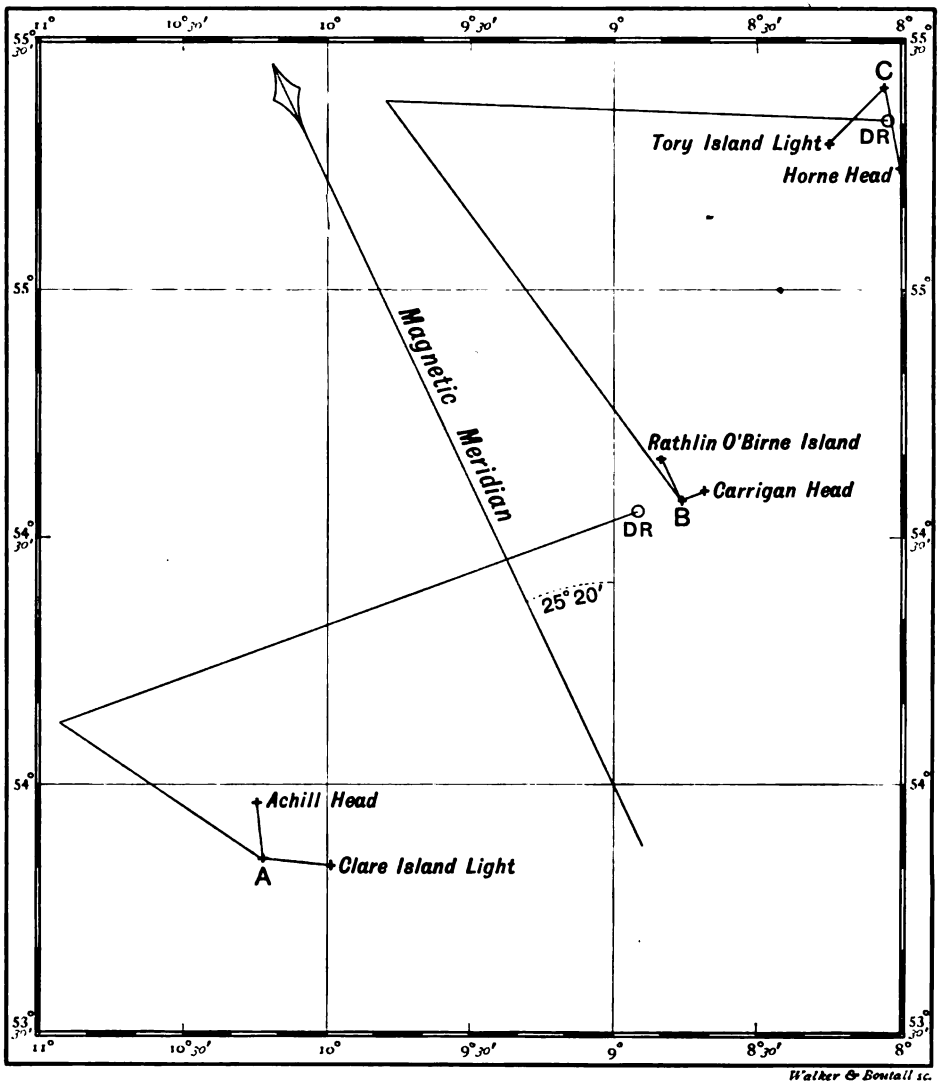

11°
10°30′
10°
9°30′
9°
8°30′
8°
55°30′
55°
54°30′
54°
53°30′
53°
C
DR
Tory Island Light
Horne Head
Magnetic Meridian
Rathlin O'Birne Island
Carrigan Head
B
DR
25° 20′
Achill Head
Clare Island Light
A
Walker & Boutall sc.

FIG. 21.

to be the same over the whole chart, draw a line making an angle of 25° 20′ with a meridian to represent the magnetic meridian, from which to lay off the courses and bearings. (If desired, the courses, etc., may be corrected for variation as well as deviation, and the results laid off from the true meridian.)

(2) Fix the given positions.

(3) Correct and lay off the bearings and courses—

Clare Island Light.	Achill Head.	1st Course.
S 56° 15′ E	N 22° 30′ E	N 28° 8′ W
2 50 W	2 50 W	2 50 W
59 5	19 40	30 58
S 59° E	N 20° E	N 31° W

2nd Course.	Rathlin O'Birne Island.	Carrigan Head.
S 90° 0′ E	N 5° 38′ W	S 87° 11′ E
5 40 E	5 40 E	5 40 E
84 20	0	81 31
S 84½° E	North	S 81½° E

3rd Course.	4th Course.	Horne Head.	Tory Island Light.
N 8° 27′ W	S 67° 30′ E	S 8° 27′ W	S 64° 41′ W
3 0 W	5 30 E	5 30 E	5 30 E
11 27	62 0	13 57	70 11
N 11½° W	S 62° E	S 14° W	S 70° W

N.B.—In laying off courses, when a position has been fixed by cross-bearings, the next course should be laid off from the position thus obtained, and *not* from the extremity of the line representing the previous course, as this latter position is only an approximate one.

	A	*B*	*C*
Results—Lat.,	53° 50′ N	54° 35′ N	55° 24′ N
Long.,	10 13 W	8 45 W	8 3 W

CHAPTER IV.

METHODS OF FIXING A SHIP'S POSITION ON A CHART.

To lay off on a chart the bearing of any object.

§ 32. Correct the given compass bearing for the deviation due to the *direction of the ship's head.* Measure this corrected bearing, which will be the magnetic bearing, on the nearest compass drawn on the chart (such compasses being constructed on the magnetic meridian), by placing the parallel rulers so that the edge passes through the centre of the compass and the requisite degree on the circumference. Then move the ruler till the edge passes through the object, when the line drawn along the edge will represent the bearing required. If the compass on the chart is not marked to degrees, correct the magnetic bearing for the variation, and measure off this true bearing from the true meridian. A line parallel to this through the object will represent the bearing required.

(1) *Cross Bearings.*

§ 33. If the bearing be taken of a headland, lighthouse, or other well-defined object, and this bearing laid off on a chart, it is manifest that the position of the ship must be somewhere on this line.

Similarly, if the bearing of another object, taken immediately after, be laid off, the position of the ship must be somewhere on this line. And, as the only point common to two lines is the point where they meet, the position of the ship on the chart will be at the point of intersection of the two lines of bearing.

The objects should be chosen so that the lines do not intersect at a very acute angle, as the point of intersection, in such a case, is somewhat doubtful. In practice the bearing of a third point is always taken as a check.

(2) *By bearing and sextant angle.*

§ 34. If two well-defined objects cannot be seen at the same time from the standard compass, the position may be fixed by taking the bearing of one object, and the sextant angle between it and another object.

Lay off the bearing of the first object, and from any point of it lay off the sextant angle. A line through the second object, parallel to the line thus obtained, will cut the bearing of the first object in a point which is the position of the ship.

(3) *Doubling the angle on the bow.*

§ 35. The angle between the direction of the ship's head and the bearing of an object is called "the angle on the bow." Let this angle be observed and the time noted; and, when the angle on the bow is doubled, let the time be again noted. Then the distance of the ship from the object is equal to the distance run by the ship in the interval. Lay off the second corrected bearing and on the line measure the distance run, the point obtained will be the position of the ship.

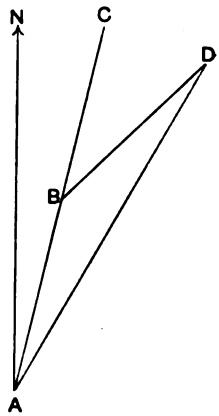


FIG. 22.

Proof.—Suppose that the ship is proceeding in the direction ABC (fig. 22), and, when she is at A, let BAD be the "angle on the bow," *i.e.* the difference between the ship's course and the bearing of the object D. Let B be the position of the ship when the angle on the bow is doubled, *i.e.* when $CBD = 2\,BAD$. But $CBD = BAD + BDA$. Therefore BDA must $= BAD$, and the side $BD =$ the side BA, *i.e.* the distance of D from the ship = the distance run.

Example (fig. 23).—A ship steering S 36° E by compass observes that a lightship bears S 8° E; after running 2 miles the angle on the bow is observed to be doubled. Required the magnetic bearing of the lightship at the second observation; the deviation being 2° W;

$BAS = 38°$
$LAS = 10$
$BAL = 28$ = angle on the bow,
$\therefore\ CBL = 56$
and $CBS' = 38$
$\therefore\ LBS' = 18$

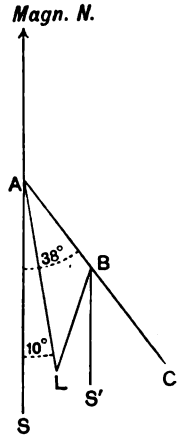


FIG. 23.

i.e. magnetic bearing of lightship at second observation is S 18° W.

Laying off N 18° E 2′ from the lightship the position of the ship is obtained. The compass bearing will be S 20° W. In this manner the required compass bearing, when the angle has been doubled, may be calculated, and all that has to be done is to note the time when the object has this calculated compass bearing.

Or, since $ALB = BAL$, the position of the ship may be obtained by drawing from L a line LB, making with AL an angle equal to the angle on the bow, and on this line measuring the distance run.

(4) *Four-point bearing.*

§ 36. This is a particular case of the previous method.

The object is observed when its bearing is four points or 45° from the direction of the ship's head, and again when on the beam: the distance from the object at the second observation being clearly the same as the distance run in the interval.

These observations are easily made by means of a brass plate, with radiating lines, let into the bridge, the compass therefore not being required.

(5) *Two bearings of the same object, and the distance run in the interval.*

§ 37. Lay off, from the object, the second bearing *reversed*, and the course of the ship; on the latter lay off the distance run in the interval, and from the point thus obtained draw a line parallel to the first bearing. The point in which this meets the second bearing will be the position of the ship at the second observation.

A Table based on trigonometrical calculation of these positions is inserted as Table 15 in the new edition of *Inman's Tables.*

In these cases, in which there is an interval between the observations, the results can only be approximate, as the ship's course will be affected by tides, etc. But, if the interval is not large, the results will be sufficiently accurate for the purposes of navigation.

(6) *Angles observed with the sextant only.*

§ 38. When the view from the standard compass is obstructed, the position of the ship may be obtained by sextant angles. Three objects are selected, and the angle subtended between the central object and each of the others measured with a sextant.

The position of the ship is determined on the following principle:—

If the positions of the central and right-hand objects be joined, and, on the joining line, a segment of a circle described containing the angle subtended between them, the position of

the observer must be somewhere on the arc of that segment; and similarly, he must be somewhere on the arc of the segment described on the line joining the positions of the central and left-hand object. Hence his position must be at the point where the segments intersect. From Euclid's construction for describing segments of circles containing given angles, illustrated by the accompanying figures (fig. 24), it follows that when the given angle is less than a right angle, its complement should be

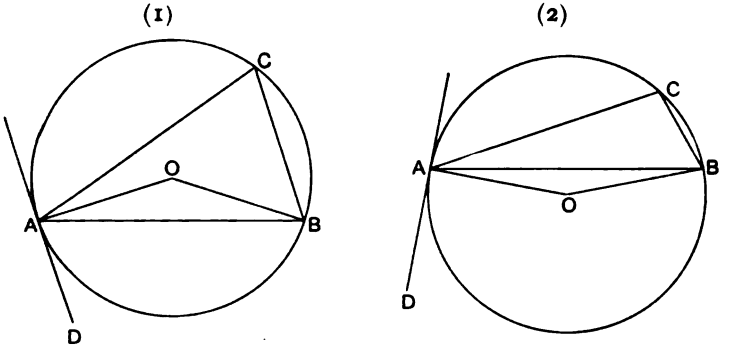


FIG. 24.

laid off from each end of the line *towards* the position of the observer; the lines meeting at the centre of the required segment; but when the given angle is greater than a right angle, the difference between it and 90° must be laid off from each end of the line *away from* the position of the observer.

Thus the given angle being BAD, to which ACB in the segment is equal, OAD being a right angle in each figure, in fig. 24(1), OAB and OBA each $= 90° - BAD$, in fig. 24(2), OAB and OBA each $= BAD - 90°$.

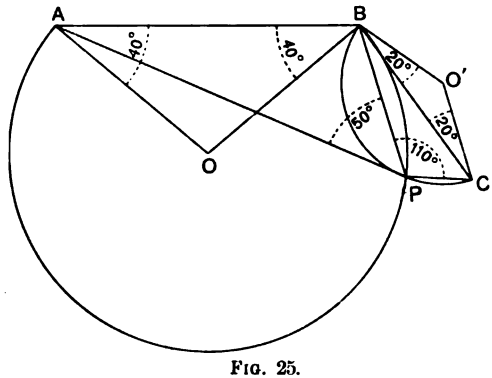


FIG. 25.

Example.—Let A, B, C (fig. 25) be the objects and suppose the angle subtended between A and $B = 50°$, and between B and $C = 110°$, or as it is usually written A 50° B 110° C.

Angles $= 40°$ being laid off from each end of AB, and $= 20°$ from each end of BC on the opposite side, the centres of the segments are found to be O and O', and the position required is at P, where the segments intersect.

The positions of the three objects must not lie nearly on the circumference of either circle, because in this case the two circles would nearly coincide. The objects selected should, if possible, be nearly in a line, or with the central one nearer than the other two.

§ 39. In practice, the position is determined by means of the "Station Pointer." This consists of a graduated circle, with three arms radiating from the centre (fig. 26). The middle one, OA, is fixed, OB and OC are moveable, and can, by means of a clamping screw and vernier, be adjusted to any required angle. The arms having been thus adjusted, the arm OA is placed over the central object and the instrument moved about until the other arms pass over the other objects; the position required is then immediately underneath the centre O. Or, the angles may be laid off on a piece of tracing paper, and the position will be at the point where the lines meet, when they pass over the positions of the selected objects.

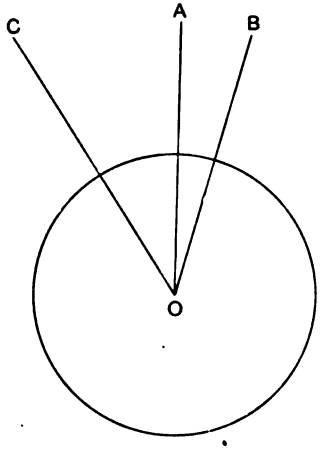


FIG. 26.

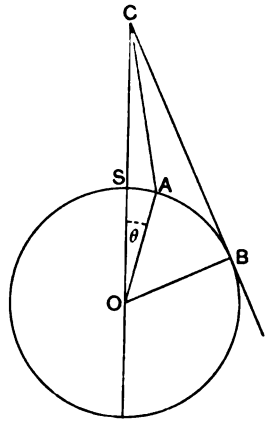


FIG. 27.

The following problems may perhaps be profitably inserted here, though not actually pertaining to the subject of the chapter.

§ 40. *Ryder's Horizon Method for finding distance of a target at sea* (see fig. 27).

Let C be the position of the observer at a height h above the water, A the target, CB the tangent to the horizon, θ the angle subtended at the centre O by the distance SA.

The angle OCA is known, as it is the difference between the observed angle ACB and the complement of the dip; call it ϕ. In triangle OCA

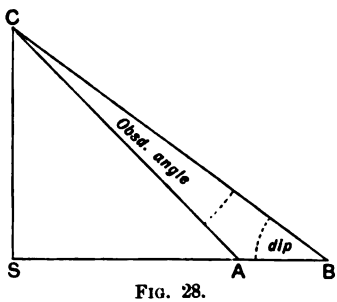


Fig. 28.

$$\frac{\sin CAO}{\sin OCA} = \frac{OC}{OA} = \frac{r+h}{r},$$

$$\therefore \sin \overline{\theta+\phi} = \frac{r+h}{r} \sin \phi.$$

Hence θ may be found.

Very little practical error is caused by finding SA from the expression

$$SA = h \,.\, \cot (\text{observed angle} + \text{dip}) \text{ (fig. 28).}$$

§ 41. *To find the distance of the ship from the base of a mountain peak, etc., of known height.*

Let P be the summit and A the base of the mountain, etc., observed from C (fig. 29).

Let a be the observed altitude (corrected for index error and dip, $\frac{1}{12}$ of the estimated distance being also subtracted to allow for the effects of terrestrial refraction), θ the angle subtended at the centre of the earth by the distance CA, d the true dip due to the height $PA = h$. Then

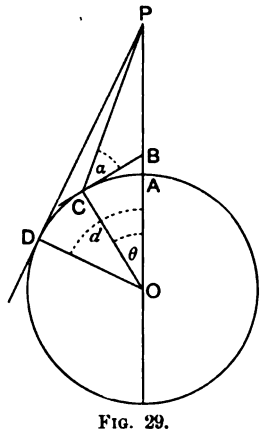


Fig. 29.

$$PCO = 90° + a,$$

$$CPO = 180° - (90° + a + \theta) = 90° - \overline{a+\theta}.$$

In triangle CPO

$$\frac{\sin CPO}{\sin PCO} = \frac{CO}{PO} = \frac{r}{r+h} = \cos d,$$

$$\therefore \frac{\cos \overline{a+\theta}}{\cos a} = \cos d,$$

$$\cos a + \theta = \cos a \,.\, \cos d, \quad \ldots\ldots\ldots\

the term $4\sin^2\frac{a}{2}\sin^2\frac{d}{2}$ being neglected as it is of the 4th power.

$$\therefore \sin^2\frac{a+\theta}{2}=\sin^2\frac{a}{2}+\sin^2\frac{d}{2},$$

writing $\overline{a+\theta}\sin 1'$ for $\sin a+\theta$, etc.,

$$(a+\theta)^2=a^2+d^2.$$

Hence, as a general rule, add the square of the altitude (in minutes) to the square of the dip (in minutes) due to the height of the mountain, and from the square root of the result subtract the altitude. The result will be the distance required in miles.

§ 42. *To find the distance of the ship from a mountain of unknown height, by means of two altitudes, and the distance run* (in the direction *of the mountain*) *in the interval between the observations.*

By the previous article

$$\cos\overline{a+\theta}=\cos a\,.\cos d,$$

where θ is the distance. Similarly, if β be the altitude when $\theta+\phi$ is the distance, ϕ being the angle subtended by the distance run in the interval (the distance being obtained, if the run has not been directly towards the mountain, etc., as in the "correction for run," § 121).

$$\cos\overline{\beta+\theta+\phi}=\cos\beta\,.\cos d,$$

$$\therefore \frac{\cos\overline{a+\theta}}{\cos a}=\frac{\cos\overline{\beta+\theta+\phi}}{\cos\beta},$$

$$\therefore \frac{\cos a-\cos\overline{a+\theta}}{\cos\beta-\cos\overline{\beta+\theta+\phi}}=\frac{\cos a}{\cos\beta}.$$

But $\frac{\cos a}{\cos\beta}=1$ *very* nearly, in all probable cases of the problem, a and β being always small angles.

$$\therefore \sin\frac{2a+\theta}{2}\sin\frac{\theta}{2}=\sin\frac{2\beta+\theta+\phi}{2}\sin\frac{\theta+\phi}{2};$$

or, since all the angles are small, writing $\overline{2a+\theta}\sin 1'$, etc., for $\sin\overline{2a+\theta}$, etc.,

$$(2a+\theta)\theta=(2\beta+\theta+\phi)(\theta+\phi),$$

$$\therefore \theta(a-\beta-\phi)=\phi\left(\beta+\frac{\phi}{2}\right),$$

$$\therefore \theta=\frac{\phi\left(\beta+\frac{\phi}{2}\right)}{a-\beta+\phi}.$$

Hence, multiply the distance run towards the object by the sum of the first altitude (in minutes) and half the distance, and divide the product by the difference between the second altitude (in minutes) and the sum of the first altitude (in minutes) and the distance run.

The quotient will be the distance, in miles, at the time of the second observation.

§§ 41 and 42 are the investigation of the practical rules given in Raper's *Practice of Navigation*, 10th Edit., §§ 355, 359, 363.

§ 43. *Danger Angle.*

If it is desired to pass outside a danger which does not show above water, this result may be obtained by calculating the "danger angle," on the following principles:—

Let *A* and *B* be two well-defined objects on the shore, *R* the position of the submerged rock, shoal, etc. If the angle *ARB* be measured by a protractor, the same angle will be subtended between *A* and *B* at any point on the segment of

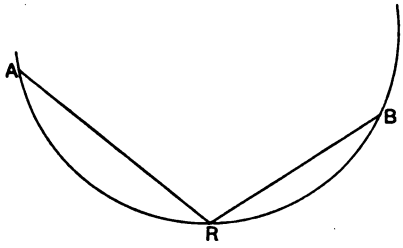


FIG. 30.

a circle passing through *A*, *R*, *B*, angles in the same segment of a circle being equal to each other. If, therefore, an angle *less* than *ARB* be placed on a sextant, so long as this angle is subtended between *A* and *B*, the ship must be *outside* the segment *ARB*, and therefore clear of the danger. *ARB* is called the "danger angle."

If it is desired to pass at any given distance outside *R*, a circle must be described with *R* as centre and radius equal to the given distance.

A circle must then be described through *A*, *B*, touching this circle, and the angle subtended between *A* and *B* at the point of contact will be the "danger angle."

If there be a point whose height is *accurately* known, a vertical danger angle may be calculated

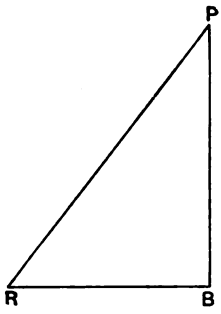


FIG. 31.

$$\tan PRB = \frac{h}{RB},$$

and so long as the observed altitude of P is less than PRB, the ship must be outside R.

N.B.--The chart used in the following examples is 1824*b*, West Coast of Ireland, which provides a great number of points suitable for observation.

No examples have been given on the use of the Station Pointer or on the Danger Angle, as they belong to the subject of Pilotage rather than to "Navigation" as defined at the commencement of this book, though it was thought advisable to state the principles on which they depend.

Examples.—N.B.—All bearings are by compass.

Cross Bearings.

(1) Daunt's Rock Lightship,	S 87° W	Deviation, 2° E.
Roche Point,	N 22 W	
Poor Head,	N 29 E	
(2) Sheep Head,	N 78½ E	Deviation, 3½° W.
Three Castle Head,	S 50½ E	
Mizen Head,	S 26½ E	
(3) Great Skellig Light,	N 2 E	Deviation, 3° W.
Hog Island Centre,	N 81 E	
Bolus Head,	N 51 E	

Bearing and Sextant Angle.

(4) Old Head of Kinsale,	N 43½° E	Deviation, 1½° E.
Sextant angle between Old Head of Kinsale and Seven Heads,	73° 50′	
(5) Kerry Head,	S 28° E	Deviation, 2½° W.
Sextant angle to Loop Head,	59½°	
(6) Clare Island Light,	S 61° E	Deviation, 2° E.
Sextant angle to Achill Head,	78°	

Doubling the angle on the bow.

(7) A ship steaming S 72° E 16 knots, observes the Fastnet Light 24° on the bow. A quarter of an hour after the angle on the bow was doubled. Deviation, 4° W.

(8) A ship steaming N 84° E 15 knots, observes the Old Head of Kinsale Light 16° on the bow. After 20 minutes the angle on the bow was doubled. Deviation, 3° E.

(9) A ship steaming N 9° W 10 knots, observes the Great Skellig Light 24° on the bow. Half an hour after the angle on the bow was doubled. Deviation, 2° W.

Four-point bearing.

(10) A ship steaming N 8° E 12 knots, observes Black Rock Light 4 points on the bow. A quarter of an hour afterwards the Light was abeam. Deviation, 5° E.

(11) A ship steaming N 82° W 15 knots, observes the Fastnet Light 4 points on the bow. Twenty minutes afterwards the Light was abeam. Deviation, 5° E.

(12) A ship steaming N 50° E 12 knots, observes Daunt's Rock Lightship 4 points on the bow. Ten minutes afterwards the Lightship was abeam. Deviation, 4° E.

Two bearings of an object, and the distance run in the interval.

(13) A ship steaming S 3° E, observes the Great Skellig Light bearing S 41° E; after she has run 5 miles the bearing is N 57° E. Deviation, 7° W.

(14) A ship steaming N 46° E, observes Achill Head bearing S 85° E; after she has run 4 miles the bearing is S 18° E. Deviation, 4° E.

(15) A ship steaming N 60° W, observes Tory Island Light bearing S 88° W; after she has run 3 miles the bearing is S 55° W. Deviation, 4° W.

CHAPTER V.

GREAT CIRCLE SAILING. COMPOSITE SAILING.

Great Circle Sailing.

§ 44. The introduction of steam power having rendered steamships practically independent of the direction of the wind, they are able to steer towards their port by the most direct route.

This most direct route is the smaller arc of the great circle passing through the position of the ship and that of the port.

The larger the diameter of the circle drawn through any two points, the more nearly will the arc coincide with the chord joining them. And, as a great circle is the circle of largest diameter that can be drawn on a sphere, its arc will be the shortest distance on any circle joining the two points.

On a Mercator's chart the rhumb line is represented by a straight line, and the great circle by a curve; so that, at first sight, it may appear that the distance on the rhumb line is less than that on the great circle. But the rhumb line appears as a straight line because of the distortion of the chart; and, if a series of points on it be taken, and their positions marked on a globe, it will be found that the length of a piece of string which passes through these points will be longer than that which measures the arc of the great circle.

If, then, a ship could be kept on a great circle, her head would be always pointing directly towards her port, and all her distance would be made good. This, however, would necessitate a continual alteration of the course (unless, as is not very likely, the ship's track was due E or W on the equator, or due N or S on a meridian), unlike the rhumb line course, which remains the same, although the ship's head does not actually point towards the port until it is in sight.

In practice, therefore, points are taken on the great circle, at a convenient distance apart, and the ship kept on the rhumb

line from point to point. In this manner the benefits of great circle sailing are obtained, without frequent change of course. As the smaller arc of the great circle is the shortest distance, it is evident that its curve must always be turned towards the elevated pole.

§ 45. **Def. 39.** *The first course sailed on in Great Circle sailing is called the Initial course, and the last one the Final course.*

In fig. 32 the angle PAB is the initial course, and $P'BC$ (1), or PBC (2), the final course. The angle PBA *is not* the final course.

Def. 40. *The Vertex of a Great Circle is the point at which the highest latitude is reached, or the point at which the curve most nearly approaches the pole, and is at right angles to the meridian.*

The *smaller* arc of the great circle being the one always referred to in great circle sailing, it is not necessary to speak of more than one vertex, although there are of course two to every great circle.

If both the angles at the base of the triangle formed by the great circle and the meridians of the two places are less than 90°, the vertex will lie between the two points; if one of these two angles is greater than 90°, the vertex will lie on the arc produced, on that side on which the angle is greater than 90°.

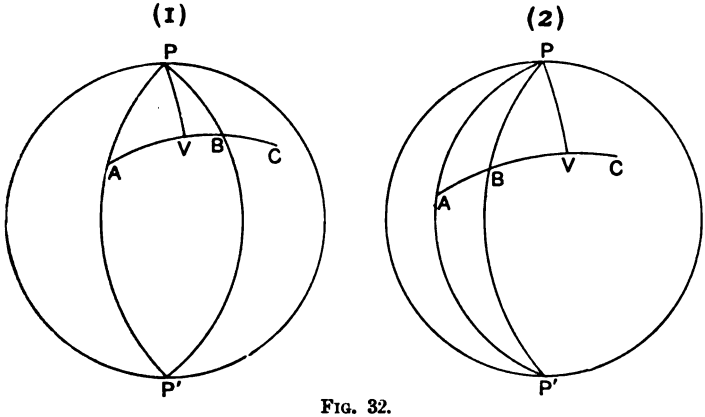


FIG. 32.

In (1) the angles PAB and PBA are each less than 90°; $\therefore$ V lies between A and B, the angles at V being right angles.

In (2) the angle PBA is greater than a right angle; V lies on AB produced on the side nearer to B.

§ 46. Since all great circles on a sphere bisect each other, the great circle and the equator bisect each other.

Hence EVQ and ECQ each equal 180°. (Fig. 33.)

Also in the right-angled triangles PVE, PVQ, PE and PQ are equal, being quadrants, and PV is common; $\therefore EV = QV$.

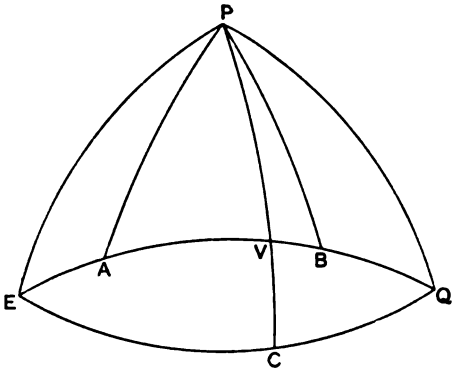


FIG. 33.

$\therefore QV = 90°$, or the vertex of a great circle is 90° from the point in which it would cut the equator.

PQV, the course on crossing the equator, is equal to the arc PV, since QV and QP each equal 90°; *i.e.* the complement of the course on a great circle when crossing the equator is equal to the latitude of the vertex.

§ 47. Problems in great circle sailing may be solved by the ordinary rules of Spherical Trigonometry according to the data in each case. As this would involve a considerable amount of calculation, various tables and diagrams have been devised in order to shorten the labour. One of the simplest of these is that proposed by Mr. Godfray of St. John's College, Cambridge, by means of a chart on the Gnomonic Projection.

In this projection a tangent plane is supposed to touch the earth at the pole, the eye of the observer being supposed to be at the centre of the earth. The meridians will then appear on the tangent plane, as straight lines radiating from the pole; all great circles will be represented by straight lines (because the planes of the meridians and all great circles pass through the centre of the earth); and the parallels of latitude will appear as concentric circles whose radii are proportional to the cotangent of the latitude.

Let *acb* be a portion of the tangent plane, *P* the pole, *O* the centre. (Fig. 34.) Then Pa or $Pb = PO \tan POB = PO \cot \text{lat } B$.

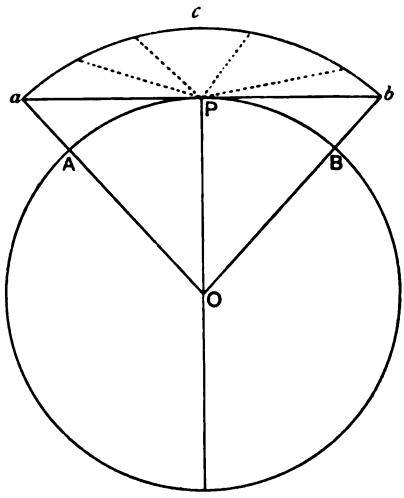


FIG. 34.

Hence a chart on this projection may be very easily drawn to any desired scale: the accompanying figure (35) being constructed between the limits 75° and 35° of latitude, the radius for lat. 45° being 1·25 inches, and the meridians being drawn for each 10° of longitude.

The great circle being drawn as a straight line joining the two points, it can be seen at once if its course is interrupted by land, or would take the ship into too high a latitude.

The latitude and longitude of the vertex can be seen by inspection, or the latitude may be obtained accurately by measuring its distance from the pole, and from the given scale obtaining the cotangent. (A Table of natural cotangents is given in Inman's Tables, new edition.)

The longitude may be obtained accurately by measuring with a protractor the angle between the nearest meridian and that of the vertex; the latitudes and longitudes of as many points as may be desired can be transferred to the Mercator's chart, and the courses and distances between them found.

Godfray's chart is accompanied by a diagram which gives the different courses, and the distance to be run on each, so as to keep within an eighth of a point of the great circle course.

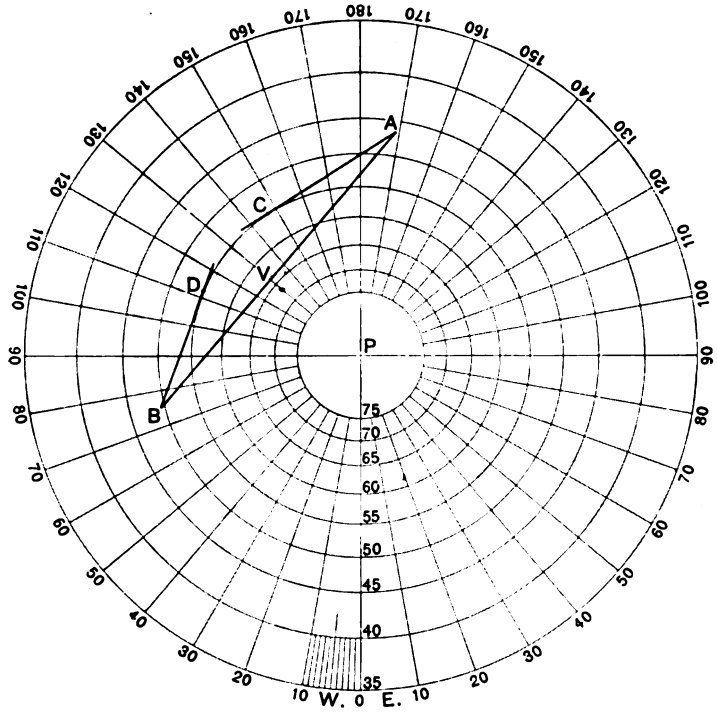


FIG. 35.

On the figure is laid down the great circle track from 46° S, 171° E, to 49° S, 75° 30′ W; the latitude and longitude of the vertex appearing to be in about 63° S, 131° W, calculation giving 63° 30′ S, 130° 30′ W.

The gnomonic chart gives a very distorted representation of low latitudes, but, in such parts of the earth, great circle sailing has not so much advantage over Mercator's, and practical navigation very much depends on Trade Winds, Monsoons, etc. Great Circle sailing is of most advantage in high latitudes, when the difference of latitude is not very great compared with the difference of longitude. It is not so useful when the difference of latitude is great compared with the difference of longitude.

§ 48. If two ships sail from point to point, one on the great circle, the other on the rhumb line, their tracks at first

separate, and at last meet again. There must, therefore, be some intermediate points at which the courses must be parallel; and as at these points their difference of latitude is greatest, the point on the great circle is called "the point of maximum separation in latitude."

§ 49. The principles of great circle sailing may prove useful in the case of a sailing ship meeting a head wind, as they will show which is the more favourable tack. The ship will make more distance good towards her port on that course which is nearer to the great circle track; and, if she is on a long voyage and does not need to tack often, she may really be gaining a great deal, although a comparison of her course with the rhumb line course will not show it.

NOTE.—For a convenient method of finding the initial course by inspection from the "Azimuth Tables," see remarks at the end of Mr. H. B. Goodwin's pamphlet, *The Ex-Meridian considered as a problem in Dynamics*, or the more complete explanation in the *Nautical Magazine* for July, 1895. The principle is that the formulæ for computing the courses in great circle sailing are the same as those for computing time azimuths, the two latitudes and the difference of longitude in the former case corresponding to the latitude, declination, and hour angle in the latter. If neither latitude is less than 23°, a point whose latitude is 23° can easily be found on the great circle, and the courses to this point from any position on the great circle will be the courses required; the practical use of great circle sailing being, not to find, as in theoretical examples, the initial and final courses only, but to find the necessary courses from time to time from any selected points.

Composite Sailing.

§ 50. As the great circle track would sometimes take the ship into too high a latitude, into a region of ice or bad weather, or as land might come in the way, a method of sailing, named "Composite Sailing," has been devised, so as to combine the advantages of great circle sailing with that of being able to determine the highest latitude it will be convenient to reach.

This latitude being decided on, the course is then on a great circle which passes through the starting point and touches the parallel of the highest latitude, then along the parallel, and, finally, along another (*not* the same) great circle touching the parallel and passing through the point to be reached. Thus a ship on a composite course from *A* to *B*

(fig. 36), CD being the parallel of highest latitude, would sail on the great circle AC touching CD at C; along the parallel to D, the point in which the great circle through B and touching CD meets the parallel; and, finally, along DB to B.

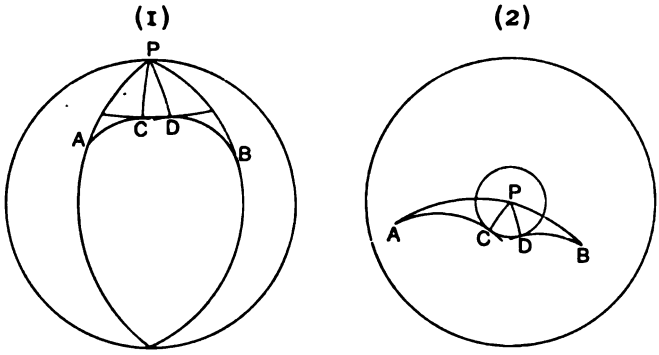


FIG. 36.

In this case the angles PCA, PDB are right angles, as the great circles touch the parallel at C and D, and are, therefore, at right angles to the meridians at those points. CD is, of course, the arc of a small circle parallel to the equator. Composite sailing problems are, therefore, solved by means of the solution of right-angled spherical triangles, and parallel sailing.

§ 51. The chart on the gnomonic projection may be used in this case also; the great circles being represented by straight lines touching the parallel of highest latitude.

If, in the example already taken (fig. 35), it had not been desirable to go to a higher latitude than 55° S, the points C and D will be found to be in about longitudes 146° W and 112° W; calculation giving 145° 36′ W, and 111° 35′ W.

Examples.

(1) A ship sails on a great circle from Cape Hatteras (Lat. 35° 15′ N, Long. 75° 30′ W) to the Lizard (Lat. 49° 58′ N, Long. 5° 12′ W); find the initial course and the distance sailed.

(2) A ship sails on a great circle from Lat. 45° 47′ S, Long. 170° 45′ E to Lat. 12° 4′ S, Long. 77° 14′ W. Find the distance sailed and the initial course.

(3) In sailing on a great circle from Lat. 40° S, Long. 16° E, to Lat. 48° S, Long. 147° E, find the initial and final courses and the highest southern latitude reached.

(4) A ship sailing on a great circle leaves a place in Lat. 20° S, Long. 57° E on a SEbE course. What will be her latitude, longitude, and course after she has sailed 2000 miles?

(5) Two ships sail from A to B; one on a great circle, the other on a rhumb line. Find the longitude of the vertex of the great circle and the latitude of the first ship when her course, as laid off on a Mercator's chart, is parallel to that of the second.

Lat. A,	42° 3′ N	Long. A,	70° 6′ W
Lat. B,	51 25 N	Long. B,	9 29 W

(6) In examples (1) and (2) find the distance saved by sailing on the great circle instead of on the rhumb line.

(7) A ship leaves a place A in Lat. 34° 29′ S, Long. 18° 23′ E, for another place B to the westward, on a composite track, and she is not to go to a higher latitude than 45° S. Find the course at A, the longitude of the place where she will reach the parallel of maximum latitude, and the distance of the place from A.

(8) A ship sails on a composite track from Cape Cod (Lat. 42° 3′ N, Long. 70° 6′ W) to Cape Clear (Lat. 51° 25′ N, Long. 9° 29′ W), not going North of Lat. 52° N. What distance does she sail along the parallel of 52° N?

(9) A ship sails on a composite track from Lat. 30° S, Long. 32° E, to Lat. 43° S, Long. 148° E. The initial course is S 48° E, the final course N 61° 30′ E. What was the highest latitude reached, and the distance sailed on the parallel?

CHAPTER VI.

NAUTICAL ASTRONOMY DEFINITIONS. CONSTRUCTION OF FIGURES.

Astronomical Terms and Definitions.

§ 52. **Def. 41.** *The Celestial Concave is an imaginary spherical surface of infinite radius, having the eye of any spectator for its centre, and which may be considered as a ground on which the sun, stars, etc., are seen projected as in a vast picture.*

The heavenly bodies are, of course, at very different distances from the observer in the depths of space, and it is an optical illusion by which he imagines himself to be at the centre of the universe.

For if A be the position of the observer on the surface of the earth (represented, for the sake of clearness, by the small circle pAp_1), m the position of a heavenly body, it will appear to the observer as if it were situated at M, on the celestial concave PQP_1Q_1.

Had the observer been situated at the centre of the earth C, the body m would appear at M_1 on the celestial concave.

Def. 42. *The Apparent Place of a Heavenly Body is the place on the celestial concave in which its centre is seen by an observer.*

Def. 43. *The True Place of a Heavenly Body is the point in which a line, joining the centre of the earth to the centre of the body, meets the celestial concave.*

Def. 44. *The Axis of the Heavens is an imaginary line coincident with the earth's axis produced.*

The celestial concave *appears* to revolve about this line from *east* to *west* on account of the earth's revolution on *its* axis from *west* to *east*.

Def. 45. *The Poles of the Heavens are those points on the celestial concave towards which the earth's axis is directed;* P and P_1 in the figure.

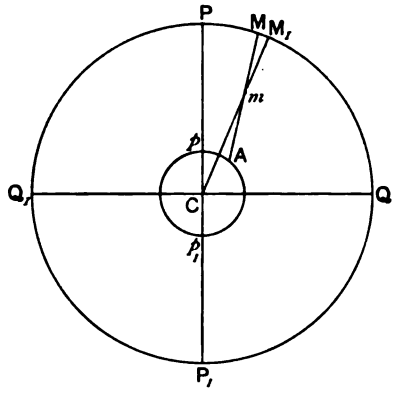


FIG. 37.

Def. 46. *The Celestial Equator or Equinoctial is the great circle of the celestial concave, marked out by the indefinite extension of the plane of the terrestrial equator.*

The poles of the heavens are, therefore, the poles of the celestial equator.

Def. 47. *The Earth's Orbit is the plane and nearly circular curve described by the earth in its annual motion round the sun.*

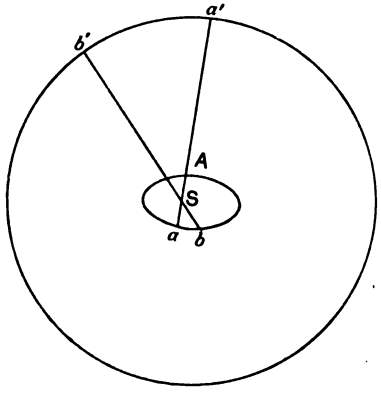


FIG. 38.

Def. 48. *The Ecliptic is the apparent path of the sun on the celestial concave. It is the great circle in which the plane of the earth's orbit cuts the celestial concave.*

The earth in its annual motion round the sun causes the sun to *appear* to describe a great circle on the celestial concave. Thus let Aab (fig. 38) represent the earth's orbit, S the sun. An observer, when the earth is at a, will see the sun apparently on the celestial concave at a', and, when the earth arrives at b, will see the sun apparently at b'. So, when the earth has again arrived at a, the sun will appear to have described a great circle, *the ecliptic*, on the celestial concave.

Def. 49. *The Obliquity of the Ecliptic is the angle at which the plane of the ecliptic is inclined to the plane of the celestial equator.*

It has been found by observation to be about 23° 27′.

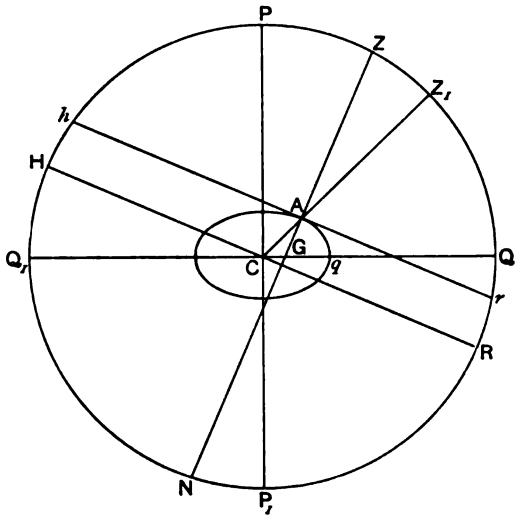


FIG. 39.

The earth is not exactly a sphere, but an oblate spheroid, cf. § 2. Its ellipticity is much exaggerated in fig. 39.

Def. 50. *The True Latitude of a Place is the angle between a perpendicular to the earth's surface at that place, and the plane of the equator; AGq* in the figure.

Def. 51. *The Reduced or Central Latitude of a Place is the angle between the earth's radius at that place, and the plane of the equator; ACq.*

The true and reduced latitudes are the same at the equator and at the poles, the angle CAG or *angle of the vertical* having its greatest value about *lat.* 45°.

For the ordinary purposes of Navigation and Nautical Astronomy, these two latitudes are assumed to be the same, the earth being considered to be a sphere, but in some problems the difference is of importance.

Def. 52. *The Sensible Horizon is a plane touching the earth at the position of an observer and extended to meet the celestial concave; hr* in the figure.

Def. 53. *The Rational Horizon is a plane passing through the centre of the earth parallel to the sensible horizon, and extended to meet the celestial concave;* as *HR.*

At the infinite distance of the celestial concave an object of the size of the earth would be imperceptible. Hence the great circles in which the sensible and rational horizons meet the celestial concave are coincident.

Def. 54. *The Celestial Horizon or* the *Horizon is the great circle in which the planes of the sensible and rational horizons meet the celestial concave.*

Def. 55. *The Sea Horizon is that small circle of the celestial concave which bounds the vision of the observer. It is the apparent meeting of sea and sky.*

Def. 56. *The Dip of the Sea Horizon is the angle between a horizontal plane through the position of an observer and a line drawn from that position to the sea horizon.*

Def. 57. *The Zenith and Nadir of an observer are the two points on the celestial concave, vertically over his head and vertically under his feet; Z* and *N* in the figure, the points in which *GA*, produced both ways, meets the celestial concave. These points are poles of the horizon.

Def. 58. *The Reduced Zenith is the point in which the radius of the earth, which passes through the place of the observer, meets the celestial concave;* as Z_1.

Def. 59. *The Celestial Meridian of an observer is the great circle marked out on the celestial concave by the prolongation of the plane of his terrestrial meridian.*

It passes through the poles of the heavens and the zenith; as PZP_1.

The arc *ZQ* on the celestial concave measures the angle *AGq*; hence the arc of a celestial meridian, between the zenith and the equator, measures the latitude of a place.

The arc Z_1Q measures the angle ACq; hence the reduced latitude of a place is always less than the true latitude.

Def. 60. *Circles of Altitude or Vertical Circles are great circles on the celestial concave passing through the zenith and nadir; as ZXH.* (Fig. 39 A.)

The celestial meridian is a circle of altitude which passes through the poles of the heavens.

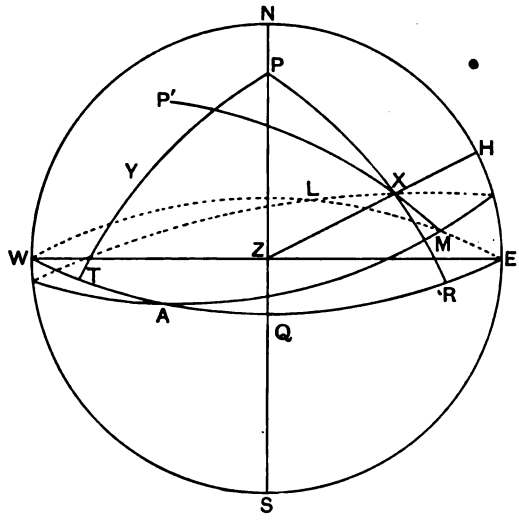


FIG. 39 A.

Def. 61. *The North and South Points of the Horizon are those points where the celestial meridian cuts it; as N* and *S.*

Def. 62. *The Prime Vertical is a vertical circle perpendicular to the celestial meridian;* as *WZE.*

Def. 63. *The East and West Points of the Horizon are those points where the prime vertical cuts it;* as *E* and *W.*

Def. 64. *Circles of Declination are great circles passing through the poles of the heavens;* as *PXR.*

Def. 65. *Parallels of Declination are small circles whose planes are parallel to the plane of the celestial equator.*

Def. 66. *The Declination of a Heavenly Body is its angular distance from the celestial equator, measured on a circle of declination;* as *XR.*

Declination is measured from 0° to 90°, N or S.

Def. 67. *The Polar Distance of a Heavenly Body is its angular distance from the pole, measured on a circle of declination;* as *PX*.

It is equal to 90° ∓ the declination, according as the latitude of the place of observation and the declination of the body are of the *same* or of *different* name.

Def. 68. *Circles of Celestial Latitude are great circles passing through the poles of the ecliptic;* as *PXM*.

Def. 69. *Parallels of Celestial Latitude are small circles whose planes are parallel to the plane of the ecliptic.*

Def. 70. *The Latitude of a Heavenly Body is the arc of a circle of latitude intercepted between the ecliptic and the place of the body;* as *XM*.

It is measured from 0° to 90°, N or S, from the ecliptic.

Def. 71. *The ecliptic AL and the equinoctial WAE intersect in two points. That in which the Sun passes from the South to the North of the equinoctial is called the First Point of Aries or the Vernal Equinox. That in which the Sun passes from the North to the South of the equinoctial is called the First Point of Libra, or the Autumnal Equinox;* as *A* and *L*.

Def. 72. *The Right Ascension of a Heavenly Body is the arc of the equinoctial intercepted between the First Point of Aries and the circle of declination passing through the place of the body, measured from the First Point of Aries from* 0 *hrs. to* 24 *hrs. in a direction* opposite *to that of the motion of the hands of a watch;* as *AR*.

Def. 73. *The Longitude of a Heavenly Body is the arc of the ecliptic intercepted between the First Point of Aries and the circle of latitude passing through the place of the body, measured from the First Point of Aries from* 0° *to* 360°, *in a direction* opposite *to that of the motion of the hands of a watch;* as *AM*.

Hence the position of a heavenly body is fixed, when its right ascension and declination, or its latitude and longitude, are known.

Def. 74. *A Polar Angle is the angle at the pole between the circles of declination of two heavenly bodies, or between two positions of the circle of declination of the same body.*

Def. 75. *The Hour Angle of a Heavenly Body is the angle at the pole included between the hour circle (or circle of declination) passing through the place of the body and the celestial meridian of the place of observation, measured from the meridian from* 0 *hrs. to* 24 *hrs. westwards;*

Or, *it is the arc of the equinoctial expressed in time intercepted between the celestial meridian and the body's hour-circle (or circle of declination) measured from the meridian from* 0 *hrs. to* 24 *hrs. westwards.*

Thus the angle ZPY or the arc QT measures the hour angle of the body Y; but the hour angle of the body X is measured by 24 hours − the angle ZPX, or 24 hours − the arc QR.

Hour angles are thus measured in conformity with the apparent diurnal motion.

Def. 76. *The Altitude of a Heavenly Body is its apparent angular elevation above the horizon, measured on a circle of altitude;* as XH.

Def. 77. *The Zenith Distance of a Heavenly Body is the complement of its altitude, or the arc of a circle of altitude intercepted between the zenith and the place of the body;* as ZX.

Def. 78. *The Azimuth of a Heavenly Body is the angular distance, measured on the horizon, between the North (or South) point and the circle of altitude through the place of the body;* as NH;

Or, *it is the angle comprised between two vertical planes, one passing through the elevated pole, and the other through the place of the body;* as PZX.

The Azimuth is measured from 0° to 180°, eastwards or westwards, from the North or South Point.

Def. 79. *The Amplitude of a Heavenly Body is the angular distance, measured on the horizon, between the East point and the place of the body, when rising, or between the West point and the place of the body when setting;* as EH, if H represent the place of a body when rising;

Or, *it is the angle comprised between two vertical planes, one passing through the East or the West point, and the other through the place of the body;* as EZH.

Def. 80. *The Solstices or Solstitial Points are the two points on the ecliptic which are most distant from the equinoctial.*

Def. 81. *The Six O'clock Hour-Circle is that circle of declination which passes through the East and West points.*

Hence, when the latitude of the place and the declination of the sun are of the same name, the sun will rise before and set after six o'clock; and, when they are of different names, will rise after and set before six o'clock.

Def. 82. *The Equinoctial Colure is the circle of declination passing through the equinoctial points.*

Def. 83. *The Solstitial Colure is the circle of declination passing through the solstitial points.*

Def. 84. *The Zodiac is the belt of the heavens included between* 8° *N and* 8° *S of the ecliptic.*

Def. 84a. *The Signs of the Zodiac are twelve equal divisions, each containing* 30° *of longitude, into which the Zodiac is divided.*

Def. 85. *Twilight is the phenomenon caused, before sunrise and after sunset, by reflection of the rays of sunlight from the vapours and minute solid particles which float in the atmosphere, while the sun is between the horizon and a small circle parallel to and about* 18° *below it.*

Def. 86. *The Twilight Parallel is a small circle parallel to the horizon and about* 18° *below it.*

Def. 87. *The Duration of Twilight is the interval that elapses between sunset and the time when the sun is on the twilight parallel, or the corresponding interval at sunrise.*

Def. 88. *Parallax is the apparent angular shifting of a body arising from a change in the point of view.*

Def. 89. *The Correction for Parallax in altitude is the angle subtended at a heavenly body by a radius of the earth passing through the place of the observer*—as *AMC* in fig. 37. Cf. Defs. 42, 43.

Def. 90. *The Transit or Meridian Passage of a Heavenly Body is its passage over the celestial meridian; the upper transit is the one nearer to the zenith, the lower is the one further from the zenith.*

Def. 91. *The Geographical Position of a Heavenly Body is that spot on the surface of the earth, which has the centre of the heavenly body in its zenith.*

Construction of Figures for Nautical Astronomy Problems.

These figures are constructed on the Stereographic Projection, either on (1) the plane of the horizon, (2) the plane of the equator, (3) the plane of the meridian, the last being very seldom used.

In the stereographic projection (or the projection of a solid on a plane) as applied to a sphere, the eye is supposed to be placed at some point on the surface, and a plane drawn through the centre at right angles to the diameter through the position of the eye. The apparent position, on this plane, of points or circles on the opposite hemisphere, as obtained by means of lines drawn from the assumed position of the eye, constitute the stereographic projection on the plane.

Circles on the sphere whose planes pass through the position of the eye will appear as straight lines, others as circles; and the angle between two circles when projected will be the same as the angle between them on the sphere.

In (1), cf. fig. 40, which is most useful when it is required to represent declinations, altitudes, daily motions, etc., the eye

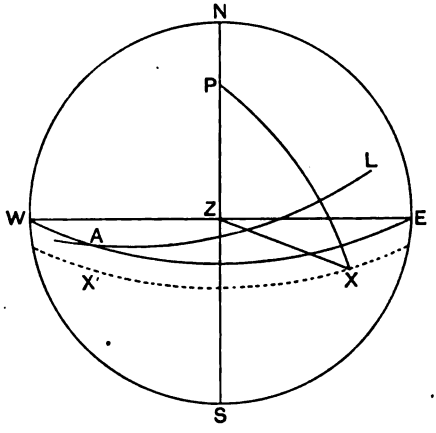


FIG. 40.

is supposed to be placed at the Nadir. The zenith, therefore, will appear to be the centre, the meridian and circles of altitude straight lines through Z; the equator, ecliptic, circles and parallels of declination, being represented by the circles EW, AL, PX, XX'. P is the pole, N, S, E, W the north, south, east, and west points respectively.

In (2), cf. fig. 41, which is most useful when time alone is considered, the eye is supposed to be placed at the end of the axis of the heavens opposite to the elevated pole. The pole,

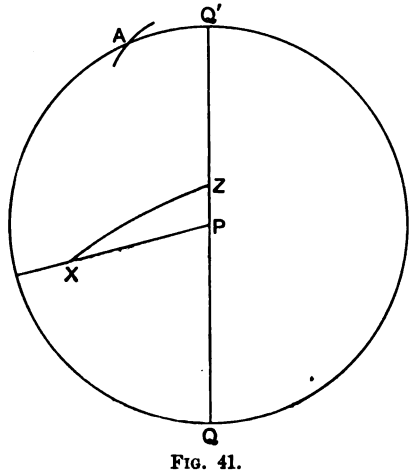


FIG. 41.

therefore, will appear to be the centre, the meridian and circles of declination straight lines through the pole; circles of altitude, if required, appearing as circles.

In (3), cf. fig. 42, the eye would be supposed to be placed at the east or west point. The west or east point would appear

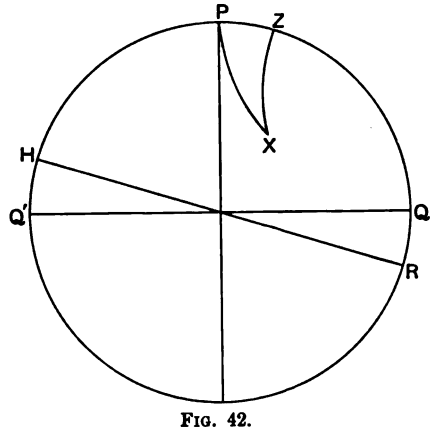


FIG. 42.

at the centre; the equator and horizon as straight lines; circles of altitude and declination as circles.

A little practice will enable the student to construct these figures with sufficient accuracy, *and no problem should be attempted without the assistance of a figure.*

CHAPTER VII.

SEXTANT. VERNIER. CORRECTIONS IN ALTITUDE. ARTIFICIAL HORIZON.

The Sextant.

§ 53. The sextant is an instrument by means of which angles can be measured in any plane. The principle of its construction may be understood by reference to the accompanying figure.

AB is an arc of a circle, of rather more than 60°; whence the name sextant. *I* and *H* are two glass reflectors whose

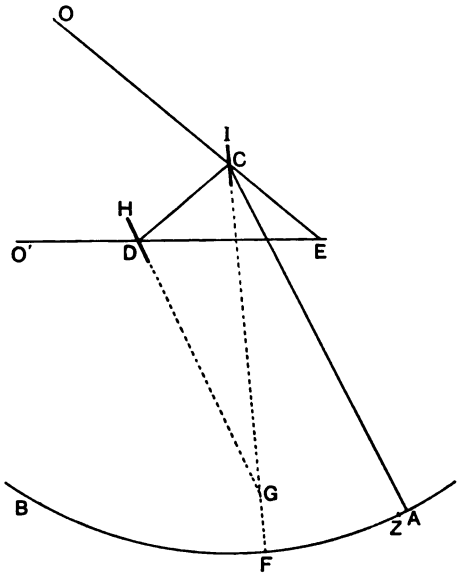


FIG. 43.

planes are perpendicular to the plane of *AB*, the whole of *I* (called the index glass) and the lower half of *H* (called the

horizon glass) being silvered at the back. I is moveable by means of a radius CF (called the index bar) about the centre C of the arc AB; H is fixed at D (so that its plane is parallel to that of I, when the radius CF passes through the zero graduation of the arc), in such a position that a ray of light reflected from the moveable reflector shall be reflected from H to the eye of the observer.

Suppose now that it is required to measure the angular distance between two objects O and O'. The moveable reflector having been turned through such an angle that the reflected image of O coincides with the direct image of O', seen through the unsilvered portion of H, CED measures the angular distance required, and DGC or FCA measures the angle between the planes of the reflectors. But since the angles of incidence and reflection at the surface of a plane mirror are equal (by a well-known law of optics), the angles OCI and DCF, which are the complements of these angles, are also equal; and the angle GCE is equal to the opposite vertical angle OCI. Hence the three angles OCI, DCG, GCE are all equal; and, similarly, the three angles HDO', HDC, EDG. Let each of the former angles $= \alpha$, each of the latter $= \beta$, and $CED = \theta$.

The exterior angle $O'DC$ = the interior angles CED, DCE,

$$i.e.\quad 2\beta = 2\alpha + \theta.$$

$$\therefore\ \theta = 2(\beta - \alpha) \qquad \ldots\ldots(1)$$

Similarly $HDC = DGC + DCG.$

$$\therefore\ \beta = \alpha + DGC.$$

$$\therefore\ DGC = \beta - \alpha.$$

$$\therefore \text{ from (1) } \theta = 2DGC,$$

or the angle at the eye of the observer is twice the angle between the planes of the mirrors.

But $DGC = FCA$, the angle through which the moveable mirror has been turned, which is measured by the arc AF.

Hence the angle at E is twice the arc AF. If, therefore, the arc AB is graduated in such a manner that each half degree is marked as a degree, the number of degrees, etc., read off on the arc AF will measure the angle OEO'.

Sextants are usually cut to 10′ of arc, and graduated for a short distance to the right of the zero, in order to allow small measurements "off the arc."

Adjustments of the Sextant.

§ 54. These are four in number, viz.:

I. *The index glass I must be perpendicular to the plane of the instrument.*

To determine if this is the case, place the radius *CF* about the middle of the arc and look obliquely into the index glass. If the plane of *I* is perpendicular to the plane of the instrument, the arc and its reflected image will form a continuous line. But if the reflected image appears above or below the arc, *I* is out of adjustment and must be adjusted by screws at the back. This is rarely necessary in good instruments, but in any case where it is necessary, the instrument should be sent to a maker.

II. *The horizon glass H must be perpendicular to the plane of the instrument.*

To determine if this is the case, look through the telescope and the horizon glass at the sun, or other well-defined object, as a bright star. Move the index bar so that the image shall pass over the object. If it does so exactly, the adjustment is correct; if not, the adjustment is to be made by means of a screw, sometimes placed under the glass, sometimes behind, sometimes at the side. (In the sextants supplied to cadets on board H.M.S. "Britannia," it is the upper of the two screws at the back of the horizon glass.) This error is usually called "Side Error."

III. *The line of collimation, or optical axis of the telescope, must be parallel to the plane of the instrument.*

To determine if this is the case, fix the inverting telescope in the collar and turn the eye-piece round until the two wires are parallel to the plane of the instrument. Select two heavenly bodies, as the sun and moon, not less than 90° apart, and make a contact between the limbs at the wire which is nearer to the sextant. Then moving the sextant slightly, bring the two bodies to the other wire. If the limbs are still in contact, the adjustment is correct; if they overlap, the farther end of the telescope is inclined away from the plane of the sextant, and the adjustment must be made by slackening the screw in the collar nearest to the instrument, and tightening the other; and *vice versa* if the limbs have separated. The adjustment is

not often required at sea, as the error does not sensibly affect the ordinary observations, and when once made is not very liable to alteration.

For the calculation of the effect of an error in the line of collimation on an observed angle, see § 192.

IV. *The horizon glass should be parallel to the index glass when the index is at zero.*

Sextants are frequently out of adjustment in this respect, the error being named "Index Error," but it is not usually removed unless it is larger than about 3′. Its amount is found and applied to the observed angle.

The adjustment may be made, if required, by means of a screw attached to the horizon glass (the lower of the two in "Britannia" sextants).

Let A be the position of the index when the two mirrors are parallel, instead of its coinciding with Z, as it should do; and let F be any other position of the index. Then it

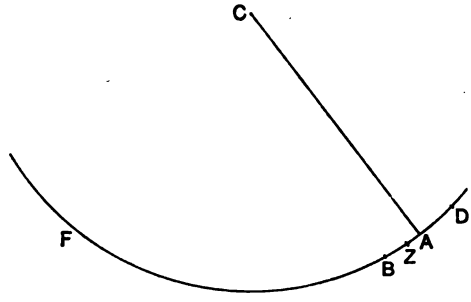


FIG. 44.

is clear that the angle ZCF read off would be too small; while the opposite would be the case if A were to the left of Z.

§ 55. The required correction is usually obtained by measuring a small angle, as the sun's diameter, on and off the arc, *i.e.* to the left and right of Z.

Let B be the point on the arc through which CA passes when there is a contact between the direct and reflected images of a limb of the sun "on the arc," D the corresponding point when the contact is made "off the arc." Then, since the direct and reflected suns must coincide when CA passes through A, the arc AB = the arc AD.

Let $ZB=d$, the measure of the diameter on the arc,
$ZD=d'$, " " " off "
$ZA=c$, the correction required,

$$AB=d+c, \quad AD=d'-c,$$
$$\therefore \; d+c=d'-c,$$
$$\therefore \; 2c=d-d',$$
$$c=\frac{d-d'}{2},$$

or the correction equals half the difference of the readings on and off the arc.

It is evident from the figure that the correction is additive when the greater reading is off the arc, and subtractive when the greater reading is on the arc.

As the arc BD must measure twice the sun's diameter, the sum of the readings on and off the arc, divided by four, ought to correspond with the value of the sun's semi-diameter, given in the *Nautical Almanac* for the day of the observation. This provides a check on the correctness of the observations.

If the sun has a low altitude the horizontal semi-diameter should be measured instead of the vertical, as in such a case the difference in the effect of refraction on the upper and lower limbs may render invalid the semi-diameter obtained as a check.

The index error may also be obtained by observation of the horizon. The direct and reflected images of the horizon being made to form a continuous line, the index should point to zero. If it does not, the reading on the sextant is the index error, additive if off the arc, subtractive if on the arc. Or the direct and reflected images of a star may be brought into contact, when the reading on the sextant will be the index error.

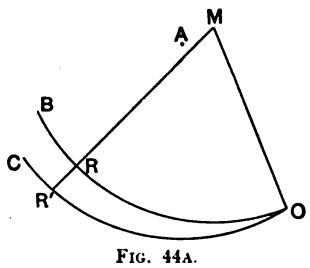


FIG. 44A.

Sextants are liable to errors of centering. Let M be the centre of rotation of the index, OB a circle of centre M. Let A be the centre of the arc OC of the sextant. Then the angle OMR should be read off on ORB, but is read off on $OR'C$.

This error, which may amount to several minutes, may be ascertained by comparing the observed distances of stars, reduced

to the distance as referred to the centre of the earth, with those obtained by calculation from their elements as recorded in the *Nautical Almanac*, or from the meridian altitudes of stars observed in artificial horizon where the latitude of the place is *exactly* known.

Sextants can be examined and certificated at the Kew Observatory.

Reading off on the Sextant.

§ 56. The arc of a sextant is divided into degrees, which are further sub-divided into 20′, 15′, or, in the best instruments, 10′ divisions.

At the end of the moveable radius is fixed the index plate, at the right-hand side of which is a spear-head shaped mark called the index. If the index points exactly to a division on the arc, *e.g.* the third division to the left of 40°, the reading would be at once obtained as 40° 30′. But if, as is more probable, the index points between two divisions, *e.g.* between the third and fourth to the left of 40°, the reading would be *about* 40° 35′. In order to obtain a more accurate reading, a scale called a Vernier is cut on the index plate, the principle of which may be thus explained:

The scale is made of such a length that $n-1$ divisions on the arc coincide with n divisions on the vernier.

Let a = value of a division on the arc,

v = " " " vernier,

$$\therefore \quad nv=(n-1)a,$$

$$\therefore \quad v=\frac{n-1}{n}a,$$

$$\therefore \quad a-v=a-\frac{n-1}{n}a$$

$$=\frac{a}{n}.$$

Hence the difference between an arc division and a vernier division is $\frac{1}{n^{\text{th}}}$ of an arc division.

This difference is called the "Least Reading" or the "Degree of Accuracy."

A similar result is found more generally if $pn-1$ divisions on the arc are taken to coincide with n on the vernier.

We then have

$$nv=(pn-1)a,$$

$$\therefore \quad v=\frac{pn-1}{n}a,$$

$$\therefore \quad pa-v=pa-\frac{pn-1}{n}a$$

$$=\frac{a}{n}.$$

As this shows that the value of p does not affect the accuracy of the reading, by taking $p=2$ or $3\ldots$, we obtain the advantage of being more exactly able to distinguish the divisions which coincide.

In the sextant the value of p is usually taken as 2, so that 60 divisions on the vernier correspond with 119 on the arc.

The value of n is usually 60, while $a=10'$, therefore the degree of accuracy $=\frac{10'}{60}=10''$.

§ 57. Thus by means of the vernier more accurate readings may be obtained, for if the first division on the vernier coincides with a division on the arc, the index must be $\frac{a}{n}$ beyond the division on the arc next before it. If the second division on the vernier coincides with a division on the arc, the index must be $2\frac{a}{n}$ beyond, and generally, if the m^{th} division on the vernier coincides with a division on the arc, the index must be $\frac{m.a}{n}$ beyond. Hence if the 26th division on the vernier coincides with a division on the arc in the example already taken, the correct reading is $40°\ 30'+10''\times 26$, *i.e.* $40°\ 34'\ 20''$.

To simplify the reading, each sixth division on the vernier is longer than the rest, and marked 1, 2, 3 .... These represent the minutes.

In the above case the division which coincides is seen to be the second past the line marked 4, which gives $4'\ 20''$ as before.

A microscope is attached to the index bar to facilitate the accurate reading of the Vernier.

If readings are made "off the arc" the vernier must be read from the 10 minute line and not from the index, or it may be read in the usual manner and the result taken from 10′.

The principle of the vernier is applicable to any scale, straight or curved, and $pn+1$ divisions may, if desired, be taken as well as $pn-1$, but this is not desirable in the case of the sextant, as the vernier would always have to be read backward.

Corrections to be Applied to the Observed Altitude of a Heavenly Body.

§ 58. The altitude, as observed with a sextant, is called the observed altitude, and is affected by several errors: index error, dip, refraction, parallax; and in the case of the sun and moon, the semi-diameter has to be applied. Then the true altitude of the centre is obtained. The semi-diameter of planets need not be taken into account in observations with the sextant, as the telescopes are not sufficiently powerful to show them except as bright points.

Index error.—The calculation of this has just been explained.

Dip.—This arises from the fact that the observer, at O, h feet above the sea level, observes the angle SOH', instead of SOH

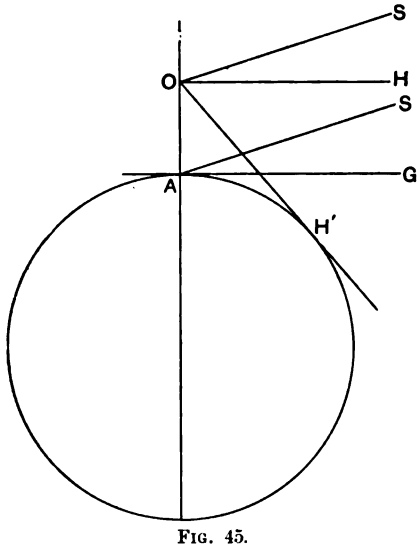


FIG. 45.

or SAG. The value of the correction is $\cdot 984\sqrt{h}$ and is *subtractive* (see § 158).

Refraction.—This arises from the fact that rays of light passing through the earth's atmosphere are deflected more and more from their straight path as the atmosphere becomes more dense. Hence rays from a heavenly body S will travel by a

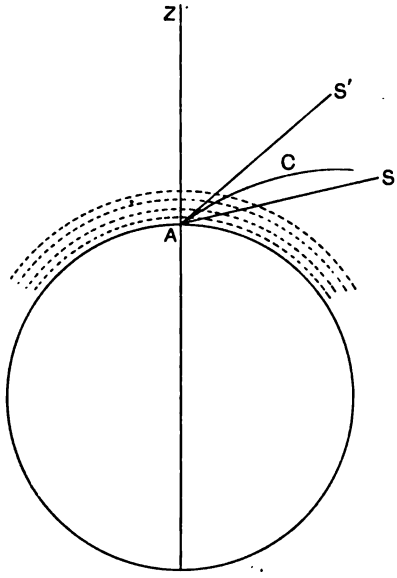


FIG. 46.

curve SCA, and the body will appear to an observer at A to be in the direction AS', where AS' is the tangent to the path of the ray at A.

It is evident that bodies appear to be raised by refraction, and that the correction is therefore subtractive (see § 160).

Parallax.—This arises from the fact that observations are made on the *surface* of the earth and not at the *centre*. The correction for parallax reduces observations to what they would have been if taken at the centre of the earth.

If X (fig. 47) be the position of a heavenly body observed at A, ZAX is the zenith distance, while ZCX is the zenith distance as referred to C. The difference between these is the angle AXC, called the *parallax in altitude*. When the body is in the horizon, as at H, the angle AHC is called the *horizontal parallax*, which is recorded in the *Nautical Almanac*. The parallax in altitude is found from the expression Parallax in altitude = horizontal parallax × cos altitude (corrected for refraction) (see § 169).

It is evident from the figure that the effect of parallax is to cause bodies to appear below their true place, and the correction is therefore additive.

The corrections for dip, refraction, and parallax have been calculated and formed into tables, so that they can be taken out by inspection. In the cases of the sun and moon, the corrections for refraction and parallax have been combined, and tables formed, "Corrections of sun's apparent altitude," and "Correction of moon's apparent altitude"; the former being subtractive, as refraction is greater than parallax in the case of the sun; the latter being additive, as parallax is greater than refraction in the case of the moon.

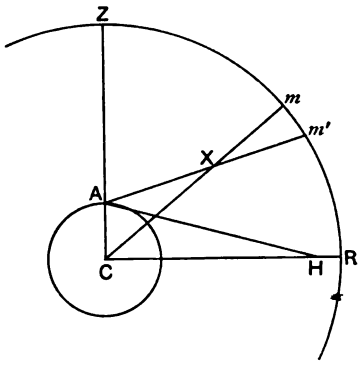


FIG. 47.

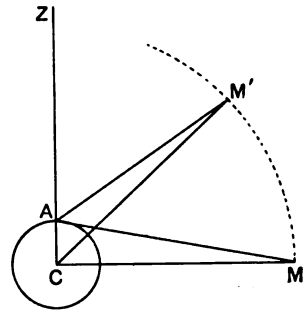


FIG. 48.

Semi-diameter.—If the lower limb is observed, the semi-diameter must be added to obtain the altitude of the centre; if the upper limb is observed, it must be subtracted.

In the case of the moon a correction must first be applied, called the "Augmentation of the moon's horizontal semi-diameter," for the following reason:

The moon's semi-diameter, tabulated in the *Nautical Almanac*, is the horizontal semi-diameter, or the semi-diameter when the moon is on the horizon.

Let M (fig. 48) be the moon on the horizon, M' another position above the horizon, A the place of observation, C the earth's centre. When the moon is at M, CM and AM are practically equal. But AM' is less than CM', and the difference between CM' and AM' increases the nearer M' approaches the zenith.

The moon will, therefore, appear larger as the altitude increases, and it will, therefore, be necessary to increase the

tabulated semi-diameter before it is applied to the observed altitude, in order to obtain the altitude of the centre.

A table has been calculated for this correction (see § 171).

The correction for refraction and parallax is to be applied *before* the semi-diameter in the case of the sun, as the refraction is calculated for the point observed, and the parallax is small and does not alter.

In the case of the moon the semi-diameter is applied first, as the correction is calculated for the moon's centre.

The *observed altitude* is that read off on the sextant. When this has been corrected for index error, dip, and semi-diameter, the result is called the *apparent altitude* of the centre, and the application to this of the correction in altitude produces the *true altitude* of the centre.

Examples.

(1) The obs. alt. of the Sun's Lower Limb was 47° 15′ 50″. The index error was 2′ 50″−, the height of the eye 21 feet, and the semi-diameter 16′ 15″. Required the true altitude of the centre.

47°	15′	50″	obs. alt.
	2	50	− index error.
47	13	0	
	4	31	− dip for 21 feet.
47	8	29	
		48	− Refraction-parallax.
47	7	41	
	16	15	+ semi-diameter.
47	23	56	true altitude of centre.

(2) February 5th in longitude 47° E, about 5 P.M. mean time, the obs. alt. of the Moon's Lower Limb was 49° 18′ 20″. The index error was 2′ 10″+, and the height of the eye was 24 feet. Required the true altitude of the centre.

	H.	M.		Moon's Semi-Diameter.	Horizontal Parallax.
Feb. 5th,	5	0	Noon,	16′ 8·5″	59′ 8·3″
Long.,	3	8	Midnight,	16 16·1	59 36·0
Feb. 5th, G.M.T.,	1	52	Change in 12 hours,	7·6	27·7
			Change in nearly 2 hours,	1·2	4·5
				16 8·7	59 12·8
			Augmentation,	12·8	
				16 21·5	

49°	18′	20″	obs. alt.
	2	10 +	index error.
49	20	30	
	4	49 −	dip for 24 feet.
49	15	41	
	16	21 +	semi-diameter.
49	32	2	
	37	28 }	correction in altitude { for 59′.
		9 }	{ for 13″.
50	9	39	true altitude of centre.

(3) The observed altitude of Venus' centre was 37° 43′ 10″. The index error was 1′ 50″ −, the height of the eye 22 feet, and horizontal parallax 31″. Required the true altitude.

37°	43′	10″	obs. alt.
	1	50 −	index error.
37	41	20	
	4	37 −	dip for 22 feet.
37	36	43	
	1	15	refraction.
37	35	28	
		25 +	parallax.
37	35	53	true altitude of centre.

The above is an extreme case of parallax of a planet. As a rule, the correction for parallax is inappreciable in Nautical Astronomy observations, and, with the semi-diameter, may be neglected.

(4) The observed altitude of Spica (α Virginis) was 67° 42′ 50″. The index error was 3′ 10″ +, the height of the eye 19 feet. Required the true altitude.

67°	42′	50″	obs. alt.
	3	10 +	index error.
67	46	10	
	4	17 −	dip for 19 feet.
67	41	53	
		23 −	refraction.
67	41	30	true altitude.

The Artificial Horizon.

§ 59. The artificial horizon is an instrument by means of which altitudes can be observed when there is no sea horizon, or when great accuracy is desired.

It consists of a trough which can be filled with mercury, and covered with a roof to keep the surface of the mercury free from dust, etc. The roof is made of two pieces of plate-glass fixed into a frame. The upper and lower surface of each plate of glass should be perfectly parallel to each other. In this case the direction of a ray of light, after passing through a plate of glass, is parallel to its former direction. Hence the angular measurement is unaltered. It is, however, advisable in important cases to reverse the roof after a few observations have been taken, so as to eliminate possible errors.

To show that the altitude observed by means of the artificial horizon is double of the true altitude.

Let MM' be the surface of the mercury, which is a horizontal plane, E the observer, ES the direction of the heavenly body, which is parallel to AS on account of the distance of S compared with EA. The altitude of the body is SEN or SAM'; the

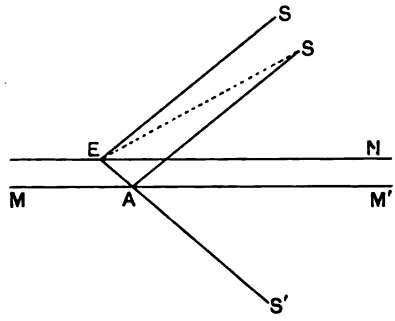


FIG. 49.

angle measured is the angle SAS', as the body will appear, in the mercury, to be in the direction EAS'; and, by the optical law already referred to in the case of the sextant, the angles SAM', $S'AM'$, being equal to the complements of the angles of incidence and reflection, are equal to one another.

Hence the angle observed is twice the angle SAM'; that is, twice the angle SEN, the body's altitude.

When a heavenly body's altitude, therefore, is observed in an artificial horizon, the angle obtained is to be corrected for index error and then divided by 2. This is the observed altitude, to which are to be applied semi-diameter (if necessary), refraction, and parallax.

The correction for "dip" is not applied, having nothing whatever to do with the problem.

By this method errors, whether in the observer or in the sextant, are divided by 2. Hence it is more likely to give accurate results than the horizon method, even if the horizon is clear and suitable. It is, therefore, used when observations are taken to obtain the error and rate of a chronometer.

CHAPTER VIII.

TIME.

Definitions relating to Time, collected for convenience of reference.

Def. 92. *A Day is the interval between two successive transits (both upper or both lower) of some point in the heavens, or some celestial body, over the same celestial meridian.*

N.B.— It is, of course, the meridian of the earth which rotates (diurnal motion), and it is the earth which revolves about the sun (annual motion); but, for convenience, the earth is supposed to remain at rest, and the sun[1] etc., to move.

Def. 93. *A Year is the interval between two successive arrivals of the sun, in its orbit, at some point in the heavens.*

Def. 94. *A Sidereal Day is the interval between two successive transits of the first point of Aries over the same celestial meridian, or it is the time occupied by one complete rotation of the earth upon its axis.*

Def. 95. *A Sidereal Clock is an instrument constructed to show 24 hours in the interval between two successive transits of the first point of Aries over the same celestial meridian.*

Def. 96. *A Sidereal Year is the interval between the sun's leaving a fixed point in the heavens and returning to it again.*

Def. 97. *A Solar Day is the interval between two successive transits of the sun over the same celestial meridian.*

Def. 98. *A Solar Year is the interval between the sun's leaving the first point of Aries and returning to it again.*

Def. 99. *A Mean Solar Year is the mean length of a large number of solar years.*

Def. 100. *The Mean Sun is an imaginary body which is supposed to move along the equinoctial with the average angular velocity of the real sun.*

Def. 101. *A Mean Solar Day is the interval between two successive transits of the mean sun over the same celestial meridian.*

Def. 102. *Apparent Time is the hour angle of the real sun.*

Def. 103. *Mean Time is the hour angle of the mean sun.*

Def. 104. *Sidereal Time is the hour angle of the first point of Aries.*

Def. 105. *Apparent Noon is the instant of the upper transit of the real sun.*

Def. 106. *Mean Noon is the instant of the upper transit of the mean sun; Mean Midnight, of its lower transit.*

Def. 107. *The Astronomical Day is the interval between two successive mean noons, measured from 0 hours to 24 hours.*

Def. 108. *The Civil Day is the interval between two successive mean midnights, and is divided into two intervals, each from 0 hours to 12 hours.*

Def. 109. *The Equation of Time is the difference between mean and apparent time at any instant; or it is the polar angle of the real and mean suns; or it is the arc of the equinoctial (expressed in time) between the circles of declination of the real and mean suns.*

Def. 110. *The Greenwich Date is the Greenwich astronomical time of any observation.*

Time.

§ 60. In order to obtain a distinct notion of that to which we give the name of time or duration, we must refer, as standards of measurement, to the intervals which elapse between the successive occurrences of certain well-defined celestial phenomena.

Those which most naturally occur to us as most universally suitable, are, the rotation of the earth on its axis, and its revolution in its orbit.

The former provides the measure of a "day," the latter of a "year"; the interval between two successive transits of some point in the heavens, or some heavenly body, over the same celestial meridian being called a "day"; the interval between two successive arrivals of the sun, in its orbit, at some point in the heavens being called a "year."

The interval between the sun's leaving a fixed point in the heavens and returning to it again is called a "sidereal year."

The interval between two successive transits of the first point of Aries (which is practically a fixed point as regards daily motion) over the same meridian is called a "sidereal day."

§ 61. These standards are invariable, and are therefore of great use in astronomy; but they are not adapted to the ordinary purposes of life, for which the sun provides the standard of measurement: the interval between two successive transits of the sun over the same celestial meridian being called a "solar day"; the interval between two successive passages of the earth in its orbit (or the sun, if considered to move) through the first point of Aries being called a "solar year."

The length of the solar year is not, however, quite invariable, owing to various irregularities in the motion of the sun and of the first point of Aries. Observations of the sun's longitude, extending over long periods, have given as the mean length of the solar year, and called a "mean solar or tropical year," a period of 365·242242 days.

§ 62. The length of the mean solar year differs from that of the sidereal year, because the first point of Aries is not fixed, the equinoctial points moving back along the equinoctial to meet the sun 50·2″ each year.

This movement is called the "Precession of the Equinoxes," and is caused by the attraction of the sun and moon on the protuberant parts of the earth, the earth being a spheroid and not a perfect sphere.

A result is that the longitudes of the fixed stars increase 50″ a year. The signs of the zodiac have moved back about 30° since the time of Hipparchus, and the first point of Aries, though retaining the name, is not in the constellation of Aries at all.

The equinoctial points make a complete revolution in about 26,000 years, the poles of the heavens describing circles of about 47° diameter round the poles of the ecliptic.

On account of this motion of the first point of Aries, the sun will arrive at it again before he has described a complete circle of the heavens, and his arrival at that point will precede his arrival at some fixed point by the time he takes to describe 50·2″, *i.e.* about 20 m. 23 s.

Hence a mean solar year contains 365 d. 5 h. 48 m. 47 s.
A sidereal year contains 365 d. 6 h. 9 m. 10 s.

Length of Solar Day not constant.

§ 63. If the sun's motion in Right Ascension were uniform, all solar days would be of the same length. But this is not the case. The sun does not move uniformly in its orbit, which is an ellipse and not a circle; and even if it did move uniformly, the corresponding motion in *RA* would not be uniform, as the ecliptic is inclined to the equinoctial.

In order, therefore, to obtain a uniform measure of time depending on the sun, an imaginary body, called the "Mean Sun," is supposed to move along the equinoctial, with the average angular velocity of the real sun. The days measured by this sun will be equal, their length being the average of the length of all the apparent solar days throughout the year; and a clock which goes uniformly may be regulated to the time shown by this mean sun, such a clock showing "mean time."

It is necessary to fix a starting point for the mean sun, so that mean and apparent time may never differ by a large interval. Hence a body, which may be termed an imaginary sun, is supposed to move in the *ecliptic* with the average angular velocity of the true sun, and to start with the true sun at perigee (*i.e.* when the sun is nearest to the earth). Then the mean sun is supposed to start from the first point of Aries at the same time as this imaginary sun. Hence the Right Ascension of the mean sun is equal to the mean longitude of the real sun, which is the longitude of the supposed imaginary sun.

§ 64. Mean noon is the instant when the mean sun is on the meridian, and *mean time* is reckoned by the westerly

hour angle of the mean sun, measured from 0 hours to 24 hours.

These 24 hours of mean time constitute the *astronomical mean day*, which commences at mean noon.

The civil day, however, commences at midnight and ends at the next midnight, being divided into two periods of 12 hours each.

Hence astronomical and civil time are only expressed by the same number of hours in the afternoon of each day; thus

Jan. 10th at 3 P.M. civil time is Jan. 10th 3 hours astronomical time.

Jan. 10th at 3 A.M. civil time is Jan. 9th 15 hours astronomical time,

15 hours having elapsed since the previous astronomical noon on Jan. 9th.

From this we obtain the practical rule: "Civil time P.M. on any day is represented in astronomical time by the same number of hours on that day; but in order to represent civil time A.M. in astronomical time, 12 hours are added to the civil time and the date is put one day back."

§ 65. The equation of time is the difference between apparent and mean time at any instant, or is the angle at the pole or the arc of the equinoctial, expressed in time, between the circles of declination of the real and mean suns.

The equation of time arises from the fact that the ecliptic is inclined to the equinoctial, and that the sun moves in its orbit with varying velocity.

The two causes may be examined separately, and the algebraic sum of the effect will be equal to that due to their combined action.

(1) Neglecting the fact that the sun's orbit is an ellipse.

§ 66. Suppose the sun to describe the orbit $AS'L$ with uniform angular velocity, while the mean sun describes the equinoctial ASL with the same velocity. Suppose them to start together from A. Then when the true sun is at B the mean sun will be at C, where $AB=AC$. If PBD be the sun's circle of declination, CD will measure the equation of time, and it is clear that C and D will only coincide at the equinoxes and solstices, when the real and mean suns are on the same circle

of declination. From equinox to solstice C will be in advance of D, and behind it from solstice to equinox. Hence a meridian of the earth (which revolves on its axis in the direction of the

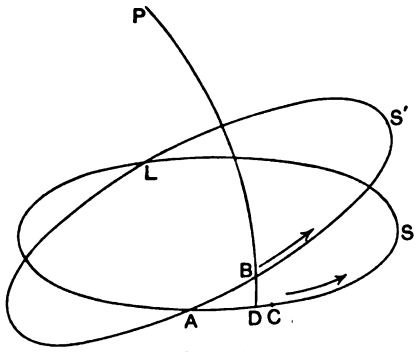


FIG. 50.

arrow) will arrive at D before it arrives at C, and apparent noon will precede mean noon, or the equation of time will be subtractive from equinox to solstice, and *vice versa* from solstice to equinox.

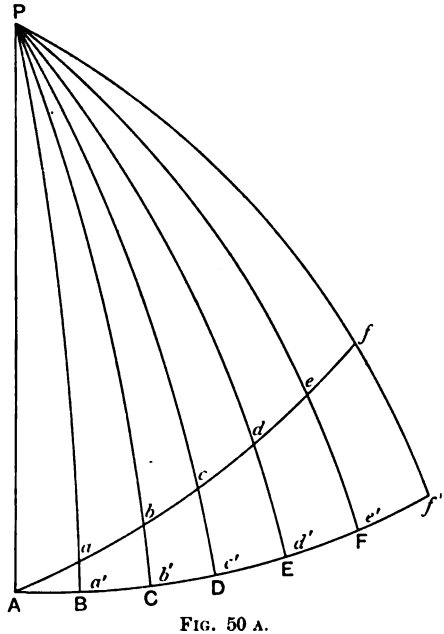


FIG. 50 A.

To illustrate this practically let Af represent the ecliptic and Af' the equinoctial between the vernal equinox and summer solstice.

Let Af and Af' be divided into six equal portions Aa, ab ... Aa', $a'b'$ ....

Through a, b, c ... draw the circles of declination PaB, PbC, PcD, etc.

It is then evident that though Aa' is greater than AB, ef is less than Ff', *i.e.* the sun's longitude at first changes more rapidly than its right ascension, but as it approaches the equinox the longitude changes more slowly than the right ascension. Hence the numerical value of the equation of time will increase to a maximum value and then decrease to 0 at the solstice, where the real and mean suns are again on the same circle of declination.

Solving the equation

$$\tan Aa = \tan AB \,.\, \sec 23^\circ\ 27'$$

by giving to AB successively the value 1 h., 2 h., ... 6 h., we obtain

	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h.
AB - - -	= 1 0 0	2 0 0	3 0 0	4 0 0	5 0 0	6
$Aa' = Aa$ - -	= 1 5 8	2 8 44	3 9 52	4 8 37	5 4 45	6
Equation of time =	5 8	8 44	9 52	8 37	4 45	0

Thus the maximum value occurs about half-way between the equinox and the solstice. The exact period may be obtained from the expression

$$\cos \text{sun's declination} = \sqrt{\cos \text{obliquity of ecliptic}}.$$

Similar results may be obtained for the other quadrants.

(2) Neglecting the obliquity of the ecliptic, consider the fact that the sun's path is elliptical.

§ 67. Let E be the earth at one focus of the ellipse; let P be perigee, and A apogee; B the place of the real sun, and C that of the mean sun or imaginary sun; B and C coinciding at P and A only.

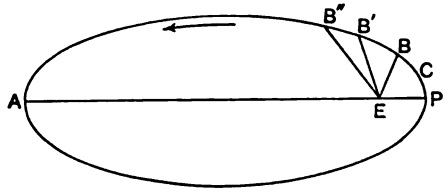


Fig. 51.

When it is nearest the earth, by the laws of planetary motion, the real sun has its greatest velocity. (If B, B', B'' be positions of B after equal intervals of time, the areas BEP, $B'EB$, $B''EB'$ are all equal; hence PB is greater than BB', BB' than $B'B''$, etc.) Hence B will be in advance of C from perigee to apogee, and behind it from apogee to perigee.

Hence from perigee to apogee the meridians of the earth will pass over the mean sun before they pass over the real sun, mean noon will take place before apparent noon, and the equation of time will be additive; and *vice versa* from apogee to perigee.

The above explanation is sufficient as an illustration. For the actual calculation of the equation of time, works on astronomy must be consulted.

§ 68. It is found that the greatest value of the equation of time, due to the obliquity of the ecliptic, is 10 minutes in time very nearly, while that due to the unequal motion in the orbit has 7 minutes as its greatest value.

Hence a simple graphical representation will show the value of the equation at different periods and also that it vanishes four times a year.

Draw a horizontal line XY to represent the time, equal intervals representing equal periods. Draw a curve $AAAA$ so that its ordinates, *i.e.* the perpendicular distances of points

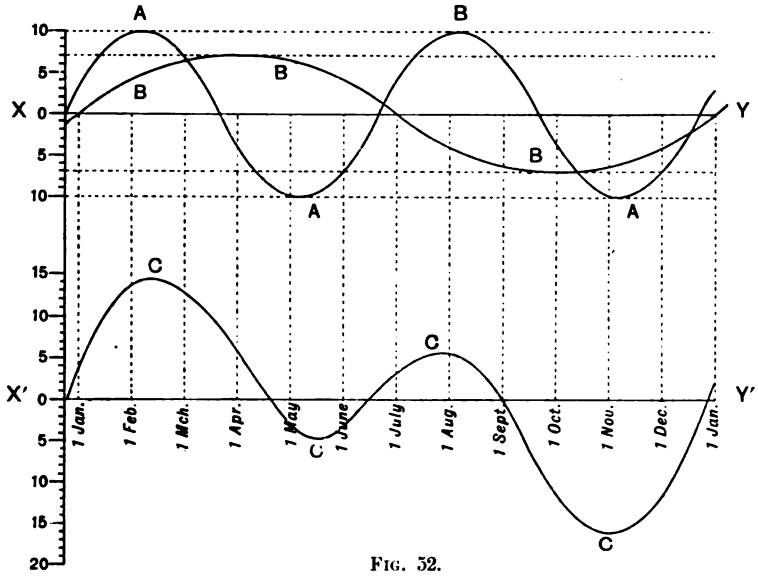


FIG. 52.

on it above and below XY, may represent that part of the equation of time for each day which depends on the obliquity of the ecliptic; while the curve $BBBB$ similarly represents that part due to the unequal motion in the orbit. $AAAA$ will cross the line XY at the equinoxes and solstices, and the

greatest value of the ordinates will be 10 m. at intermediate times. *BBBB* will cross the line *XY* at perigee and apogee, with 7 m. as the greatest value of the intermediate ordinates, positive values being measured above *XY*, negative values below.

The value of the equation of time due to the combined action of the two causes may be represented by a curve whose ordinates are the algebraic sum of the ordinates of the two former curves.

Let it be the curve *CCCC* drawn with respect to the line *X'Y'* (parallel, equal, and similarly divided to *XY*), to avoid confusing the figure. It will be seen that the equation of time vanishes about April 15th, June 15th, August 31st, December 24th, and has maximum values +14·5 m. about February 11th; −4 m. about May 14th; +6 m. about July 26th; −16·5 m. about November 3rd.

Difference in length between a solar and a sidereal day.

§ 69. Owing to the immense distance of the fixed stars, the earth's orbit by comparison becomes a mere point. Hence any given meridian revolves from a fixed star to the same star again in the same time that the earth takes to revolve on its axis, *i.e.* a sidereal day. This would be the case with the sun if the earth had no annual motion. But as the earth revolves round the sun, it advances almost a degree eastward in its orbit while it revolves once on its axis. If, then, the sun be on the meridian of any place on a given day, the earth must perform rather more than a complete revolution on its axis before the sun is again on the meridian on the day following. In other words, the earth must perform one complete revolution and as much in proportion of another as it has advanced in its orbit in that time, viz. about $\frac{1}{365}$ part of a revolution at a mean rate.

The earth, therefore, must perform about 366 revolutions in about 365 days, and the period of each revolution being a sidereal day, there must be about 366 sidereal days in a mean solar year.

And, generally, since the time of the rotation of a planet on its axis is *its* sidereal day, the number of sidereal days will always exceed by one the number of solar days, whatever it may be.

This may be illustrated by a figure.

Let E and E' be the positions of the earth in its orbit on two successive days, S the sun, F the direction of a fixed point, as a fixed star, which is on a meridian PP' of the earth at the same time as the sun when the earth is at E.

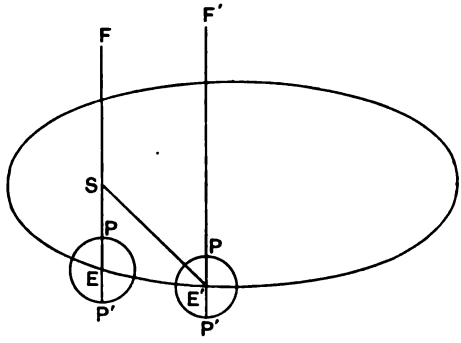


FIG. 53.

When the earth is at E' next day, and the fixed point is again on the meridian, PP' will have revolved through 360°, but it will have to revolve in addition through the angle $F'E'S$, or $E'SE$, before it again passes over the sun (PF' is parallel to PF owing to the immense distance of F).

The value of the arc EE' varies from about 61′ to about 57′. Its average value may be thus found.

§ 70. The mean sun describes the equinoctial in a mean solar year, and therefore describes $\frac{360^\circ}{365{\cdot}242242} = 59'\ 8{\cdot}33''$ in a mean solar day; that is, a meridian of the earth has to revolve through 360° 59′ 8·33″ in a mean solar day.

Equivalent sidereal and mean solar intervals.

§ 71. A mean solar year contains 365·242242 days; there are therefore 366·242242 sidereal days in the same time.

Hence if M and S be the measures of the *same interval* expressed in mean and sidereal time respectively,

$$\frac{M}{\text{Interval}} = \frac{365{\cdot}242242}{\text{one mean solar year}}$$

and

$$\frac{S}{\text{Interval}} = \frac{366{\cdot}242242}{\text{one mean solar year}},$$

$$\therefore \frac{M}{S} = \frac{365{\cdot}242242}{366{\cdot}242242} = 1 - x$$

and $$\frac{S}{M} = \frac{366{\cdot}242242}{365{\cdot}242242} = 1 + x',$$

where $x = {\cdot}00273043$

and $x' = {\cdot}00273791,$

$$\therefore M = {\cdot}99726957\ S$$

and $$S = 1{\cdot}00273791\ M.$$

From these expressions the tables of time equivalents in the *Nautical Almanac* are calculated; also the tables for acceleration of sidereal on mean time, and retardation of mean on sidereal time.

§ 72. The same results may be obtained by considering that a meridian of the earth revolves through 360° in a sidereal day, and through 360° 59′ 8·33″ in a mean solar day.

Hence $$\frac{M}{S} = \frac{360^\circ}{360^\circ\ 59'\ 8{\cdot}33''},$$

$$\frac{S}{M} = \frac{360^\circ\ 59'\ 8{\cdot}33''}{360^\circ},$$

which expressions give exactly the same results as before obtained for M and S in terms of each other.

Examples.—Convert 17 h. 27 m. 47 s. mean time into the equivalent in sidereal time.

By Time Equivalents.

		h.	m.	s.	
17 h.	mean time	= 17	2	47·56	sidereal time.
27 m.	„	=	27	4·44	„
47 s.	„	=		47·13	„
		17	30	39·13	„

By Tables.

	h.	m.	s.
	17	27	47
Correction for 17 h.,		2	47·56
„ 27 m.,			4·44
„ 47 s.,			·13
	17	30	39·13

Convert 19 h. 27 m. 18 s. sidereal time into the equivalent mean time.

By Time Equivalents.

		h.	m.	s.	
19 h. sidereal time	=	18	56	53·24	mean time.
27 m. „	=		26	55·58	„
18 s. „	=			17·95	„
		19	24	6·77	„

By Tables.

	h.	m.	s.	
Correction for 19 h.,		3	6·76	
„ 27 m.,			4·42	
„ 18 s.,			·05	
Total correction,		3	11·23	to be subtracted.
	19	27	18	
	19	24	6·77	

Leap Year.

§ 73. As the mean solar year consists of 365·242242 days, while it is necessary for the ordinary purposes of life that the civil year shall consist of an exact number of days, a plan has been devised to render the error in the calendar thus produced as small as possible.

The mean solar year containing very nearly 365¼ days, three consecutive years are considered to consist of 365 days, and each fourth year (with the exceptions to be mentioned) to contain 366 days. This is named "Leap Year."

The error thus produced would amount to about 3·1 days in 400 years. The extra day is therefore not added in the case of three out of four "century" years, thus reducing the error to about ·1 of a day in 400 years, or one day in about 4000 years.

Thus to find leap year, divide the number of the year by 4. If there is no remainder it is leap year, except in the case of a "century" year, when the divisor must be 400. Thus the year 1900 was not a leap year.

CHAPTER IX.

TO CONVERT ARC INTO TIME, ETC. GREENWICH DATE. CORRECTION OF ELEMENTS FROM "NAUTICAL ALMANAC."

To convert arc into time and the converse.

§ 74. The longitude of a place is defined to be the smaller arc of the equator intercepted between the first meridian and that of the place. It may also be considered as a difference in time. Thus the interval between two consecutive passages of the mean sun over any meridian is 24 mean solar hours. Hence 24 hours in time correspond to 360° of arc; from which we obtain that

1 hour	of time	corresponds to	15° of arc.
1 minute	„	„	15′ „
1 second	„	„	15″ „

Also that

15 degrees	of arc	„	1 h. of time.
1 degree	„	„	4 m. „
1 minute	„	„	4 s. „
1 second	„	„	$\frac{1}{15}$ s. „

by means of which arc can be turned into time and the converse.

Examples.—Convert 115° 17′ 45″ into time.

		h.	m.	s.
115°	$= \frac{115}{15}$ h. $= 7\frac{10}{15}$ h.	= 7	40	0
17′	$= \frac{17}{15}$ m. $= 1\frac{2}{15}$ m.	=	1	8
45″	$= \frac{45}{15}$ s. $= 3$ s.	=		3
115° 17′ 45″		= 7	41	11

Convert 9 h. 47 m. 23 s. into arc.

9 h.	$=9\times15°=$	135°	0′	0″
47 m.	$=\frac{47}{4}°=11\frac{3}{4}°=$	11	45	0
23 s.	$=\frac{23}{4}'=5\frac{3}{4}'=$		5	45
9 h. 47 m. 23 s.		146	50	45

§ 75. Hence we obtain the practical rules:

To convert arc into time, "Divide the degrees, etc., by 15 and multiply the remainder by 4." This will give hours and minutes, minutes and seconds, seconds and fractions of seconds respectively.

To convert time into arc, "Multiply the hours by 15, divide the minutes, etc., by 4, and multiply the remainders by 15." This will give degrees, degrees and minutes, minutes and seconds respectively.

Examples.—Convert the following arcs into time:

(1)	47°	19′	15″	(4)	163°	46′	50″
(2)	98	47	30	(5)	147	29	42
(3)	119	26	45				

Convert the following times into arc:

	h.	m.	s.		h.	m.	s.
(6)	3	25	16	(9)	10	19	47·5
(7)	4	17	19	(10)	8	16	29·3
(8)	9	27	27				

In a similar manner hours, etc., of right ascension, or sidereal time, may be turned into the corresponding degrees, etc., of arc.

In some Nautical Tables the tables of log sines, etc., and log haversines are calculated for time as well as arc, and the conversion of time into arc, or the converse, can be effected by inspection.

§ 76. From the above considerations it appears that the longitude of a place shows the difference in time between that place and Greenwich, from which longitude is reckoned; for the longitude converted into time shows the interval which *will* elapse between the passage of the mean sun over the meridian of Greenwich and that of the given place, if in west longitude; or which *has* elapsed between its passage over a meridian in east longitude and that over the meridian of Greenwich. As the earth revolves from west to east, easterly meridians will pass over the mean sun earlier than that of Greenwich; westerly meridians will pass later. Hence the

time at a place east of Greenwich is before Greenwich time, and after it at a place west of Greenwich.

Further, the difference of longitude between any two places converted into time will give the difference between the local times at those places at any given instant.

The Greenwich date.

§ 77. If, then, we wish to obtain the time at Greenwich corresponding to the time at any other place, we must apply to the local time the longitude turned into time, adding it if the longitude is west, subtracting if the longitude is east.

If it is required to obtain the time at any place corresponding to a given Greenwich time, the longitude in time must be subtracted from the Greenwich time if the longitude is west, and added if the longitude is east.

The Greenwich time thus found, to the nearest minute, is called the Greenwich date. The knowledge of it is required in almost every nautical problem, because the right ascensions, declinations, etc., of the various heavenly bodies are tabulated in the *Nautical Almanac* for certain instants of Greenwich time. Before, therefore, we can make use of these elements in the solution of Nautical Astronomy problems, it is necessary that we should know the Greenwich time corresponding to the times of observation, in order that the proper values of the elements may be obtained.

The Greenwich time may be found more accurately by means of a chronometer whose error is known. The known error on Greenwich mean time being applied to the chronometer time will produce the Greenwich mean time required. It is, however, necessary to know the local time and the longitude, at any rate approximately, as chronometers are only marked up to 12 hours, and it could not be decided whether or no the correct Greenwich mean time was more or less than 12 hours without a knowledge of the local time and the longitude.

Examples.—Feb. 15th in longitude 45° W, find the Greenwich dates corresponding to local times 8 h. A.M. and 3 h. 15 m. P.M.

	h.	m.			
Feb. 14th, - - -	20	0	Feb. 15th, - - -	3	15
Long. in time, - -	3	0 W	Long. in time, - -	3	0 W
Greenwich date, Feb. 14th,	23	0	Greenwich date, Feb. 15th,	6	15

Feb. 15th in longitude 117° E, find the Greenwich dates corresponding to local times 8 h. A.M. and 3 h. 15 m. P.M.

	h.	m.
Feb. 14th, - - -	20	0
Long. in time, - -	7	48 E
Greenwich date, Feb. 14th,	12	12

	h.	m.
Feb. 15th, - - -	3	15
Long. in time, - -	7	48 E
Greenwich date, Feb. 14th,	19	27

In the second part of this example, as the longitude to be subtracted is greater than the local time, it is necessary to add 24 hours, 27 h. 15 m. on Feb. 14th corresponding to 3 h. 15 m. on Feb. 15th.

Feb. 15th in longitude 45° W, at about 7 A.M. local time a chronometer showed 9 h. 47 m. 45 s., its error on Greenwich mean time being 13 m. 15 s. slow. Required the Greenwich mean time.

	h.	m.	s.
Chronometer time, -	9	47	45
Error, slow, - -		13	15+
	10	1	0
	12	0	0
Greenwich mean time,	22	1	0

	h.	m.
Feb. 14th, - - -	19	0
Long. in time, - - -	3	0
Greenwich date, Feb. 14th,	22	0

In this case the Greenwich date shows that 12 hours must be added to the 10 h. 1 m. obtained by applying the error of the chronometer to the chronometer time.

To take out the Sun's Declination.

§ 78. The sun's declination is tabulated in the *Nautical Almanac* for each day at noon at Greenwich.

If it is required to find its value at the time any observation of the sun is taken, the Greenwich date must first be obtained, and then the hourly variation multiplied by the hours and fractions of an hour in the Greenwich date. This will give the correction to be applied to the declination at noon at Greenwich, added if the declination is increasing, subtracted if decreasing.

This is the general principle of the rule, but there are modifications which require notice.

The hourly variation represents the rate of change of the declination *at noon* of the given day, but this rate is not constant throughout the 24 hours, as may be seen from the fact that the hourly variation varies from day to day.

When, therefore, the Greenwich date is more than 12 hours, the declination should be taken out for the *nearer* Greenwich

noon, and corrected for the difference between the Greenwich date and 24 hours, the correction obtained being *subtracted* if the declination is *increasing*, and *vice versa*.

If great accuracy is desired, the hourly variation used should be that for the time midway between the Greenwich date and the *nearer* Greenwich noon; but for the ordinary purposes of navigation, the hourly variation at the nearer noon is usually sufficient.

Example.—Find the sun's declination on Feb. 16th at 10 h. A.M. in longitude 79° W.

	h.	m.		Declination.	Variation in one hour.
Feb. 15th,	22	0	Feb. 16th, at Greenwich mean noon,	12° 18′ 55″ S	52·1″
Long.,	5	16+		2 52−	3·3
Greenwich date, Feb. 16th,	3	16	declination required,	12 16 3	15·63 156·3
					171·93
					2′ 52″

The hourly variation is multiplied by 3·3, as 16 m. is very nearly ·3 of an hour (6 minutes is $\frac{1}{10}$ or ·1 of an hour, so that the number of minutes of the Greenwich date divided by 6 gives the decimal part of an hour).

The change is subtracted from the noon declination as the declination is decreasing.

Next let us take an example in which the Greenwich date is over 12 hours.

Example.—Find the sun's declination on June 19th at 11 A.M. in longitude 33° E.

	h.	m.		Declination.	Hourly variation.
June 18th,	23	0	June 18th, at Greenwich mean noon,	23° 25′ 12·3″ N	3·31″
Long. E,	2	12−		1 8·8	20·8
Greenwich date, June 18th,	20	48	declination required,	23 26 21	2·648 66·20
					68·848
					1′ 8·8″

The hourly variation, having been taken for noon on June 18th, and multiplied by the hours, etc., of the Greenwich date produces a result which is *greater* than the declination for noon June 19th, *i.e.* 23° 26′ 19″, showing that the principle used must be incorrect.

Taking, therefore, 20 h. 48 m. from 24 h., we obtain 3 h. 12 m. as the interval that will elapse before noon June 19th when the hourly variation is 2·28″.

2·28″ × 3·2 = 7·3″, the correction which must be subtracted from 23° 26′ 19″ to obtain a more correct value of the declination at the given time.

The same method applies to the sun's apparent right ascension, which is not, however, often required in navigation.

§ 79. The sun's declination is tabulated for both apparent and mean noon at Greenwich.

As the time kept on board ship at sea is apparent time, if the Greenwich mean time of an observation, such as latitude by meridian altitude, deviation by altitude azimuth, etc., where the G. M. T. is not required to be known accurately, is wanted, the ship time would have to be corrected for the equation of time before finding the Greenwich date. But by applying the longitude directly to the ship time, the Greenwich apparent time can be found at once, and the declination for apparent noon corrected, so as to find the declination at the time of observation.

The practical difference will not be great, but, as a matter of principle and showing an understanding of what is being done, the point is worth notice.

Examples.—Required the sun's declination:

	Day.	Mean time. h. m.	Longitude.
(1)	April 6th,	11 15 P.M.,	74° 30′ W
(2)	April 27th,	9 A.M.,	69 40 E
(3)	June 5th,	3 P.M.,	117 30 E
(4)	June 23rd,	7 A.M.,	47 0 W

	Day.	Apparent time.	Longitude.
(5)	Dec. 7th,	9 A.M.,	147° 0′ W
(6)	Dec. 29th,	4 P.M.,	87 20 E

To take out the Sun's Semi-diameter.

§ 80. This changes so slowly that no correction is needed and the semi-diameter tabulated for Greenwich mean noon may be considered as correct for any hour of the day.

To take out the Equation of Time.

§ 81. Obtain the Greenwich date, take out the equation of time for the *nearest* Greenwich mean noon, multiply the variation in one hour by the hours and parts of an hour of the Greenwich date, or of the interval between the Greenwich date and 24 h., when the Greenwich date is over 12 h., and apply the correction thus obtained; adding if the equation of time is increasing, subtracting if it is decreasing, in the first case, and *vice versa* in the second case.

Examples.—Required the equation of time at 6 h. 18 m. P.M. on June 6th in longitude 53° 20′ W.

	h. m. s.		Equation of time. m. s.	Variation in 1 hour.
June 6th, - - - -	6 18 0	Noon, June 6th,	1 37·41	·445
Long. W, - - - -	3 33 20+		4·38−	9·85
Greenwich date, June 6th,	9 51 20		1 33·03	2225
				3560
				4005
				4·38325

Required the equation of time at 6 h. 18 m. A.M. on June 6th in longitude 53° 20′ W.

	h. m. s.		Equation of time. m. s.	Variation.
June 5th, - - - -	18 18 0	Noon, June 6th,	1 37·41	·445
Long. W, - - - -	3 33 20		·96	2·15
Greenwich date, June 5th,	21 51 0		1 38·37	2225
	24 0 0			445
	2 9 0			890
				·95675

Here the correction is + because the equation of time is decreasing, but the Greenwich date is before noon on June 6th.

Care must be taken to mark the equation of time as additive to or subtractive from mean or apparent time, as the case may be, at the time that it is taken out.

It should always be taken out for mean noon. When it is necessary to reduce ship apparent time to ship mean time for the purpose of finding the Greenwich date, it is sufficient, in practice, to apply the equation of time to the nearest minute

Examples.—(1) Required the equation of time at 11 h. 13 m. A.M. in longitude 117° 29′ E on June 8th.

(2) Required the equation of time at 9 h. 14 m. A.M. in longitude 104° 23′ W on April 15th.

(3) Required the equation of time at 4 h. 45 m. P.M. in longitude 165° 19′ E on December 1st.

To take out the Right Ascension of the Mean Sun.

§ 82. The right ascension of the mean sun at Greenwich mean noon is found in the column headed "Sidereal Time" on page II. of each month in the *Nautical Almanac.*

Since sidereal time is the hour angle of the first point of Aries, or the right ascension of the meridian when the mean sun is on the meridian of Greenwich, the mean sun's right ascension is that of the meridian, and is therefore the same as sidereal time at Greenwich mean noon.

As the right ascension of the mean sun increases regularly 3 m. 55·55 s. in every 24 hours, its value at any other time than Greenwich mean noon can be found by simple proportion, or more easily by the tables of time equivalents in the *Nautical Almanac,* or by the table "Acceleration of Sidereal on Mean Time" in Nautical Tables.

Example.—Required the right ascension of the mean sun at 6 h. 15 m. P.M. in longitude 49° W on June 15th.

	h. m.		h. m. s.
June 15th,	- 6 15	Sidereal time at Greenwich mean noon,	5 34 4·20
Long. W,	- 3 16	Increase for 9 hours, - - - -	1 28·71
		„ „ 31 minutes, - - -	5·09
Greenwich date,	9 31	Right ascension of the mean sun, - -	5 35 38

Examples.—(1) Required the right ascension of the mean sun at 3 h. 57 m. P.M. in longitude 94° 25′ W on April 6th.

(2) Required the right ascension of the mean sun at 6 h. 18 m. A.M. in longitude 114° 33′ W on June 14th.

(3) Required the right ascension of the mean sun at 2 h. 15 m. P.M. in longitude 76° 21′ E on December 19th.

To take out the Moon's Right Ascension and Declination.

§ 83. The moon's R.A. and declination are tabulated for each hour of the day, and the change in 10 minutes of time at the commencement of each hour is also given.

Therefore, find the Greenwich date, take out the R.A. or declination for the hour of the Greenwich date, multiply the change in 10 m. by the number of minutes in the Greenwich date, and mark off one more decimal place in the result. Add the correction so found, in the case of the R.A., and add or subtract in the case of the declination according as the declination is increasing or decreasing.

If the change in 10 m. is changing rapidly, take out the R.A. or the declination for the *nearest* hour, and apply the correction accordingly, as in the case of the sun's declination.

Examples.—Required the moon's right ascension and declination at 5 h. 25 m. A.M. in longitude 41° 27′ W on April 6th.

	h.	m.	s.
April 5th, - - - - -	17	25	0
Long. W, - - - - -	2	45	48+
Greenwich date, April 5th, - -	20	10	48

	Moon's R.A. h.	m.	s.	Variation in 10 m.	Moon's declination.			Variation in 10 m.
At 20 h.,	10	19	11·26	22·48	12°	21′	27·9″ N	161·1″
			24·28	10·8		2	53·9 –	10·8
	10	19	35·54	242·784	12	18	34	1739·88
				24·28				173·9

	h.	m.	s.
Moon's R.A., - - - -	10	19	35·54
„ declination, - - -	12	18	34 N

Required the moon's declination at 10 h. 15 m. P.M. in longitude 49° 30′ E on April 1st.

	h.	m.	Moon's declination.				Variation in 10 m.
April 1st, - -	10	15	At 6 h.,	23°	39′	17·8″ N	4·94
Long., - - -	3	18				28·15	57
Greenwich date, -	6	57		23	39	45·95	281·58
							28·15

The result thus obtained is greater than the declination at 7 h., which should therefore be taken and corrected for the 3 m. which will elapse between 6 h. 57 m. and 7 h. This is, of course, an extreme case, taken to illustrate the principle.

Examples.—(1) Required the moon's R.A. and declination at 11 h. 18 m. P.M. in long. 95° 21′ W on June 12th.

(2) Required the moon's R.A. and declination at 9 h. 43 m. P.M. in long. 72° 47′ E on December 23rd.

To take out the Moon's Semi-diameter and Horizontal Parallax.

§ 84. These are tabulated for noon and midnight of each day.

The Greenwich date having been found, the correction is obtained by simple proportion for the hours that have elapsed since Greenwich noon or midnight.

Example.—Required the moon's semi-diameter and horizontal parallax at 9 h. 15 m. A.M. in long. 116° 10′ E on June 4th.

	h.	m.	s.	Moon's semi-diameter.	
June 3rd, - - - - -	21	15	0	Midnight, June 3rd,	15′ 52·3″
Long. E., - - - - -	7	44	40	Noon, June 4th, -	15 48·8
Greenwich date, June 3rd,	13	30	20	Change in 12 h., -	3·5
				„ 1½ h., -	·44

Semi-diameter required, 15′ 51·86″.

Moon's horizontal parallax.		
Midnight, June 3rd, - - - - -	58′	8·9″
Noon, June 4th, - - - - -	57	56·1
Change in 12 h., - - - - -		12·8
„ 1½ h., - - - - -		1·6

Horizontal parallax required, 58′ 7·3″.

The horizontal parallax inserted in the *Nautical Almanac* is the equatorial horizontal parallax, which is greatest at the equator and decreases as the latitude increases; but as the greatest value of the difference is about 12″, for almost all purposes of Nautical Astronomy the parallax thus found may be considered as the value at the place of observation.

Examples.—(1) Required the moon's semi-diameter and horizontal parallax at 7 h. 15 m. P.M. in long. 87° E on December 17th.

(2) Required the moon's semi-diameter and horizontal parallax at 3 h. 14 m. A.M. in long. 116° 27′ W on June 11th.

(3) Required the moon's semi-diameter and horizontal parallax at 9 h. 17 m. A.M. in long. 43° E on April 10th.

To take out a Planet's Right Ascension and Declination.

§ 85. These are tabulated for Mean Noon and for Time of Transit each day at Greenwich.

The Greenwich date having been found, the proportional part of the change in 24 hours will give the required correction

to the values inserted for Mean Noon. The correction may also be obtained from the variation of R.A. or dec. in one hour of longitude.

To take out the Right Ascension and Declination of a fixed Star.

§ 86. Look out the R.A. of the star in the Table "Mean places of stars," turn to the corresponding R.A. in the Table "Apparent places of stars," and take out the R.A. and dec. for the nearest day.

Example.—The R.A. of Capella on December 27th differs by 7 s. from its mean value. Hence using the mean value would cause an error of nearly two miles of longitude, if the star were observed in order to determine the longitude.

CHAPTER X.

PROBLEMS ON TIME. MERIDIAN PASSAGES. HOUR ANGLES.

Given apparent time and the equation of time, to find mean time; or given mean time and the equation of time, to find apparent time.

§ 87. Since the equation of time is the difference between mean and apparent time, its value, corrected for the Greenwich date, and applied with the proper sign to apparent or mean time, will produce mean or apparent time respectively.

Given mean time, to find sidereal time.

§ 88. Let QAQ' represent the celestial equator, QPQ' being the meridian, A the first point of Aries, m the mean sun, west of the meridian in (1), east in (2). The sidereal time

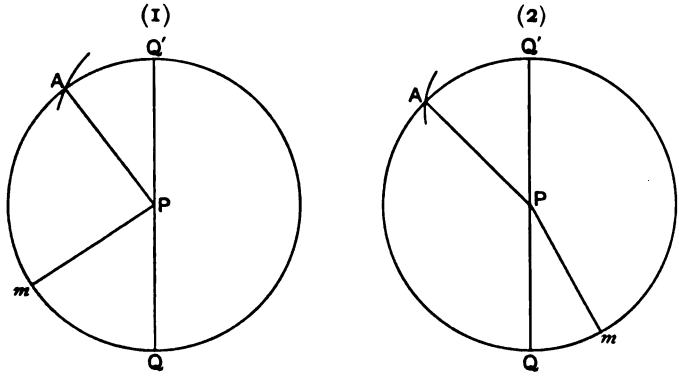


FIG. 54.

is measured by the angle APQ or the arc AQ, as it is the westerly hour angle of the first point of Aries. or the time that has elapsed since the first point of Aries was on the meridian.

In (1)

Sidereal time $= AQ = Am + mQ$

= Right ascension of the mean sun (corrected for the Greenwich date) + mean time.

In (2)

Sidereal time $= AQ = Am - mQ$

= R.A. mean sun (corrected for Greenwich date) $-$ (24 h. $-$ mean time)

= R.A. mean sun + mean time $-$ 24 h.

Hence, generally,

Sidereal time = R.A. mean sun + mean time,

24 hours being subtracted from the result if over 24 hours.

Examples.—April 18th, in long. 49° E, the mean time was 5 h. 6 m. 18 s. P.M.; required the sidereal time.

	h. m. s.		R.A.M.S. h. m. s.
S.M.T., April 18th, - -	5 6 18		1 45 23·89
Long., - - - - - -	3 16 0 −	Correction for 1 h.,	9·86
		„ „ 50 m.,	8·21
Greenwich date, April 18th,	1 50 18		1 45 41·96

	h. m. s.
Corrected R.A.M.S., - -	1 45 42
S.M.T., - - - - - -	5 6 18
Sidereal time, - - - -	6 52 0

December 19th, in long. 77° W, the mean time was 2 h. 16 m. 15 s. A.M.; required the sidereal time.

	h. m. s.		R.A.M.S. h. m. s.
S.M.T., Dec. 18th, - -	14 16 15		17 47 23·67
Long., - - - - - -	5 8 0 +	Correction for 19 h.,	3 7·27
		„ „ 24 m.,	3·94
Greenwich date, Dec. 18th, -	19 24 15		17 40 34·88

	h. m. s.
Corrected R.A.M.S., - -	17 40 35
S.M.T., - - - - - -	14 16 15
	31 56 50
	24 0 0
Sidereal time, - - -	7 56 50

If the time given is *apparent*, as in the case when it is required to find the sidereal time in observation of the pole star at sea, where the time kept is *apparent*, it must be reduced to *mean* by the application of the equation of time, which may for all practical purposes of navigation be taken out to the nearest minute.

Examples.—(1) April 24th, in long. 119° E, given ship mean time 8 h. 14 m. 21 s. P.M.; required sidereal time.

(2) June 11th, in long. 57° W, given ship mean time 7 h. 19 m. 16 s. A.M.; required sidereal time.

(3) June 28th, in long. 115° 50′ E, given ship apparent time 5 h. 21 m. A.M.; required sidereal time.

(4) December 16th, in long. 73° 20′ W, given ship apparent time 11 h. 19 m. P.M.; required sidereal time.

(5) December 27th, in long. 23° E, given ship apparent time 6 h. 19 m. A.M.; required sidereal time.

Given mean time or apparent time, to find what heavenly body will pass the meridian next after that time.

§ 89. Let AQ represent the celestial equator, PQ the celestial meridian, A the first point of Aries, m the mean sun, X a heavenly body passing the meridian.

Then mQ is the given mean time, or the given apparent time corrected for the equation of time, and Am the R.A. mean sun (corrected for the Greenwich date).

The R.A. of $X = AQ$

$= Am + mQ$

= R.A. mean sun + mean time.

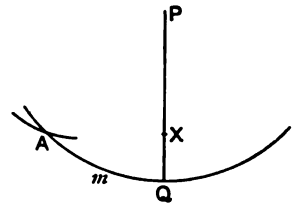


FIG. 55.

Hence that star in the catalogue in the *Nautical Almanac* whose right ascension is equal to that found will be on the meridian at the given time, or if there is no star with that R.A., the one whose R.A. is the next greater will be the first to pass the meridian after the given time.

It is often convenient to know what bright stars will pass the meridian between two given times. The R.A. of the meridian must be found, as above, corresponding to each of the given times, and the stars whose R.A's. lie between the sidereal times thus determined will be the stars required.

Stars that are thus on the meridian will not necessarily be available for observation. This depends on the latitude, and it will be easily deduced from the method of finding the latitude by observations on the meridian, that the zenith distance of a heavenly body is equal to the difference between the latitude and declination when they are of the same name, and the sum when they are of different names.

Stars, therefore, whose declination of opposite name is greater than the complement of the latitude, will be *below* the horizon when on the meridian. And further, in practice, stars cannot as a rule be observed unless their altitude is greater than 5°.

Examples.—On April 25th, what bright star will be the first to pass the meridian after 11 P.M. apparent time, in longitude 15° W?

	h.	m.	s.
April 25th,	11	0	0
Equation of time,		2	11
S.M.T.,	10	57	49
Long.,	1	0	0
Greenwich date, April 25th,	11	57	49

	R.A.M.S. h.	m.	s.
	2	12	59·8
Correction for 11 h.,	1	48	4
„ „ 58 m.,		9	5
	2	14	57
S.M.T.,	10	57	49
R.A. of meridian,	13	12	46

α Virginis (Spica), whose R.A. is 13 h. 19 m., is the star required.

Which bright stars will be available for observation on the meridian between the hours of midnight, Feb. 10th, and 6 A.M., Feb. 11th (apparent time), in latitude 47° N, longitude 124° W?

	h.	m.
Feb. 10th,	12	0
Long.,	8	16+
Greenwich date, Feb. 10th,	20	16

Equation of time.
14 m. 26 s. +apparent time.

	R.A.M.S. h.	m.	s.
	21	21	14·8
Correction for 20 h.,		3	17·1
„ „ 16 m.,			2·6
	21	24	33
S.M.T.,	12	14	26
	9	38	59

	h.	m.
Feb. 10th,	18	0
	8	16
Greenwich date, Feb. 11th,	2	16

Equation of time.
14 m. 26 s.

	R.A.M.S. h.	m.	s.
	21	25	11·3
Correction for 2 h.,			19·7
„ „ 16 m.,			2·6
	21	25	33
	18	14	26
	15	39	59

The bright stars whose Right Ascensions lie between 9 h. 38 m. 59 s. and 15 h. 39 m. 59 s. are α Leonis (Regulus), α Crucis, α Virginis (Spica), β Centauri, α Boötes (Arcturus), and α Centauri; but of these, α Crucis, declination 62° S, β Centauri, 60° S, and α Centauri, 60° S, will be below the horizon.

Hence Regulus, Spica, and Arcturus are the principal stars required.

N.B.—In the 1896 *Nautical Almanac* the magnitudes of the stars are arranged on a new system. Hence by "bright stars" is meant "stars of magnitude not lower than the second."

Examples.—(1) Which bright stars will pass the meridian between the hours of 8 P.M. and midnight, apparent time, longitude 62° W on April 18th?

(2) Which bright stars will pass the meridian between the hours of midnight, June 12th, and 4 A.M., June 13th, apparent time, longitude 73° E?

(3) Which bright stars will pass the meridian between the hours of midnight, December 14th, and 6 A.M., December 15th, apparent time, longitude 145° E?

(4) Which bright stars will be available for observation on the meridian between the hours of 8 P.M. and midnight, apparent time, December 1st, in latitude 35° N, longitude 136 W?

Given sidereal time, to find mean time or apparent time.

§ 90. Let $QmAQ'$ represent the celestial equator, QQ' the meridian, P the pole, A the first point of Aries, m the mean sun, west of the meridian in (1), east in (2).

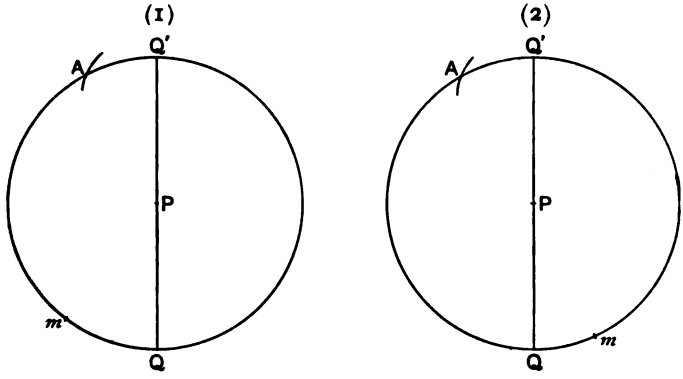


FIG. 56.

Then AQ is the right ascension of the meridian, or the given sidereal time, Am the right ascension of the mean sun, mQ in (1) and 24 h. $-Qm$ in (2) the mean time required.

$$mQ = AQ - Am, \quad \text{fig. (1)},$$

$\therefore$ mean time = sidereal time $-$ *R.A.* mean sun.

$$Qm = Am - AQ, \quad \text{fig. (2)},$$

$\therefore$ 24 h. $-$ mean time = *R.A.* mean sun $-$ sidereal time,

or mean time = 24 h. + sidereal time $-$ *R.A.* mean sun.

Hence we have, generally,

mean time = sidereal time $-$ *R.A.* mean sun,

24 hours being added, if necessary, to the sidereal time.

But as we cannot obtain the R.A. mean sun until we know the mean time, we must obtain an approximate value of the mean time by using in the above formula the R.A. mean sun for the day as tabulated in the *Nautical Almanac*. To this approximate mean time apply the longitude, and get a Greenwich date with which to obtain a more correct value of the R.A. mean sun. This being subtracted from the given sidereal time will give a more correct value of the mean time, by means of which a still more correct value of the R.A. mean sun may be obtained; and so on to any desired degree of approximation, the second being usually sufficient for navigation purposes.

Example.—Given sidereal time 9 h. 47 m. 10 s., find mean time at the same instant on December 18th in longitude 67° W.

	h.	m.	s.
Sidereal time,	9	47	10
R.A. mean sun at noon, 18th,	17	47	24
Ship mean time, nearly, 17th,	15	59	46
Long.,	4	28	0
Greenwich date, 17th,	20	27	46

R.A.M.S.	h.	m.	s.
Noon, 17th,	17	43	27·11
Correction for 20 h.,		3	17·13
„ 27 m.,			4·436
„ 46 s.,			·128
	17	46	48·804

Second Approximation.

	h.	m.	s.
Sidereal time,	9	47	10
R.A. mean sun,	17	46	48·8
Ship mean time,	16	0	21·2
	4	28	0
Greenwich date,	20	28	21

R.A.M.S.	h.	m.	s.
	17	43	27·11
Correction for 20 h.,		3	17·13
„ 28 m.,			4·60
„ 21 s.,			·058
	17	46	48·898

The new value of the R.A. mean sun differs, therefore, only by ·094 of a second from that already found. Hence 16 h. 0 m. 21 s. is the mean time required, or 4 h. 0 m. 21 s. A.M.

If apparent time is required, the corrected equation of time is to be applied to the mean time thus found.

Examples.—(1) Given sidereal time 14 h. 16 m. 29 s. on June 28th in longitude 74° 20′ W. Find the apparent time.

(2) Given sidereal time 1 h. 14 m. 37 s. on April 26th in longitude 114° E. Find the apparent time.

To find the time when a heavenly body will be on the meridian.

§ 91. This is a particular case of the preceding problem. When a heavenly body is on the meridian its right ascension is equal to the right ascension of the meridian or sidereal time.

Hence the mean time of meridian passage of a heavenly body is found by subtracting the right ascension of the mean sun from that of the body, increased, if necessary, by 24 hours, the right ascension of the mean sun being corrected to any desired degree of accuracy.

The apparent time required to find the time of a star's meridian passage at sea is obtained by application of the equation of time.

Example.—Find the apparent time of the meridian passage of Regulus (α Leonis) in longitude 73° W on June 12th.

	h.	m.	s.
Star's R.A.,	10	2	48
R.A. mean sun, approximately,	5	22	14
	4	40	34
Long.,	4	52	0
Greenwich date,	9	32	34

	R.A.M.S.		
	h.	m.	s.
	5	22	14·52
Correction for 9 h.,		1	28·71
,, 33 m.,			5·42
	5	23	48·65

	h.	m.	s.
Star's R.A.,	10	2	48
R.A. mean sun,	5	23	49
Mean time of passage,	4	38	59
			23
apparent time,	4	39	22

Equation of time.
s.
28
5 −
23 + to mean time.

§ 92. It is necessary to remember that the ship's clock shows correct apparent time only at noon, or, in ships of high speed, at such other times as may be found convenient, and that it will be too fast if the ship has run to the westward from noon or such other time till the time of observation, and too slow if to the eastward, four minutes of time for each degree of longitude.

Examples.—(1) Find the apparent times of the meridian passages of α Boötes, α Scorpii, α Aquilae on June 14th in longitude 27° W.

(2) Find the apparent times of the meridian passages of α Tauri, α Orionis, α Argus on December 16th in longitude 39° E.

(3) Find the time which will be shown by a clock, put right at apparent noon, when the star α Leonis (Regulus) is on the meridian on April 6th in longitude 97° E, the ship having changed her longitude 67 miles to the eastward at the time of the observation.

To find the time of the Moon's Meridian Passage in any longitude.

§ 93. The time of the moon's meridian passage at Greenwich is tabulated for each day in the *Nautical Almanac*; and, if the moon were like a star with a constant right ascension, the Greenwich time of passage would give the local time of passage at any other meridian, allowing for a small change in the right ascension of the mean sun. If the time of a star's passage were, say, 8 P.M., the local time at that instant, at a meridian 45° west of Greenwich, would be 5 P.M.; by the time the meridian came to the star, 3 hours due to the difference of longitude would have elapsed, and the star's meridian passage would be 8 P.M. in local time at that place also; and so for any other meridian.

But the moon's R.A. increases much more rapidly than that of the sun, on account of the motion of the moon in her orbit. Therefore the time of the moon's meridian passage is considerably later each day, the amount, 40 m. to 66 m., depending on the number of minutes by which the increase of R.A. of the moon exceeds that of the mean sun.

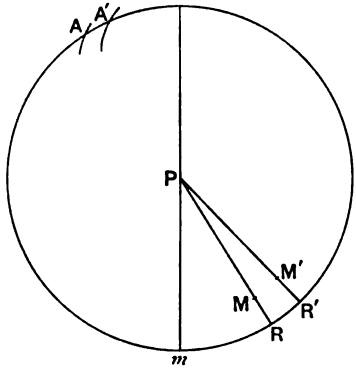


FIG. 57.

If m be the mean sun on the meridian, A and M the corresponding positions of the first point of Aries and of the moon; when m is on the meridian next day, A will be at A

($AA'=4$ m. nearly), while M will be at M', RR' being (very nearly) the moon's change of R.A. and (very nearly) the retardation of the meridian passage. This is to say, the moon is farther to the eastward each day with regard to the meridians, which revolve from west to east with the earth.

Hence the interval between the moon's meridian passage over an easterly meridian and that of Greenwich will be greater than that due to the difference of longitude, because the meridian will have to revolve through the difference of longitude, and, in addition, the amount the moon has meanwhile moved to the eastward.

In other words, the local time of passage over an easterly meridian is earlier than the time tabulated for the meridian passage at Greenwich; and, similarly, the local time will be later at a westerly meridian.

To find the amount of the correction.

§ 94. Let L be the longitude, D the difference between the tabulated times of the moon's meridian passage, between which the required time of passage lies.

Then, as the retardation D takes place in 24 hours or 360° of longitude, by proportion

$$306 : L :: D : \text{correction}.$$

$$\therefore \text{correction} = \frac{L}{360} \times D,$$

and is subtractive for east longitude, additive for west longitude.

Hence the rule: "Take out the times of the moon's meridian passage for the given day and the day *before* for east longitude; or for the given day and the day *after* for west longitude; calculate or take from the tables the correction, subtract it from the time for the given day for east longitude, or add it for west longitude." The result will be the local (not the Greenwich) mean time of passage over the given meridian.

The longitude in time must be applied in the usual way to obtain the Greenwich date.

It is to be noticed that the day, and day before or after, are to be understood as *astronomical* day, etc., a point often overlooked in the working of examples.

The "Lower Meridian Passage" is used in finding the time of high water, and *not* in finding the latitude.

Examples.—Find the time of the moon's meridian passage on April 5th in longitude 47° W.

	h.	m.	
Mer. passage, April 5th, - - -	8	58·5	
,, ,, 6th, - - -	9	50·2	
		51·7	
Correction from table, - - -		6	
S.M.T. of passage, - - - -	9	4·5	
Long., - - - - - -	3	8+	
Greenwich date, - - - - -	12	12	April 5th.

Find the time of the moon's meridian passage on April 16th in longitude 53° E.

	h.	m.	
Mer. passage, April 16th, - - -	17	43·6	April 15th.
,, ,, 15th, - - -	16	50·8	,, 14th.
		52·8	
Correction, - - - - - -		8	
S.M.T. of passage, - - - -	17	35·6	,, 15th.
Long., - - - - - -	3	32–	
Greenwich date, - - - -	14	3·6	,, 15th.

Find the ship mean time and the Greenwich mean time of the moon's meridian passage:

(1) April 6th, longitude 73° W.
(2) April 8th, longitude 49° E.
(3) April 20th, longitude 89° E.
(4) June 8th, longitude 116° W.
(5) June 27th, longitude 127° E.
(6) Dec. 5th, longitude 81° W.

To find the time of a Planet's Meridian Passage in any longitude.

§ 94.* The principle is the same as in the case of the moon, except that the time of passage on any day is not always later than that on the preceding day. The moon's right ascension constantly increases from 0 h. to 24 h., and the daily change of R.A. is always greater than that of the mean sun; hence the time of the meridian passage is *always later* day by day. But, as the *apparent* motion of a planet, as seen from the earth, is a *combination* of the actual motions of the two bodies in their respective orbits, the right ascension of a planet is sometimes increasing and sometimes decreasing. So long, therefore, as the planet's R.A. is increasing more rapidly than that of the mean sun, the time of its meridian

passage will be *later* day by day. When the rate of increase is equal to that of the mean sun the time of passage will be practically the same on successive days; when it is less, or when the planet's R.A. is decreasing, the time of passage will be *earlier* day by day. These changes can be easily traced by inspection of a planet's elements in the *Nautical Almanac.*

Thus, in 1895, the R.A. of Venus increased at a gradually decreasing rate from January 1st until about August 26th, after which it decreased until about October 8th, and then increased until the end of the year. The time of meridian passage was *later* day by day until June 28, when it remained the same for three days; it was then *earlier* day by day until November 29th, when it remained the same for five days, after which it was *later* again day by day until the end of the year.

It is therefore necessary, in finding the time of a planet's meridian passage, to notice whether the time of passage is *accelerated* or *retarded.*

N.B.—For the mathematical investigation of a planet's *stationary points* and *retrograde motion*, see Godfray's *Astronomy*, 4th edition, pp. 276, *seqq.*

Given the latitude of the place, and the zenith distance and declination of a heavenly body, to find its hour angle, and thence the mean time.

§ 95. Let *NEWS* represent the horizon, *NS* the meridian, *P* the pole, *Z* the zenith, *X* the heavenly body, west of the meridian in (1), east in (2).

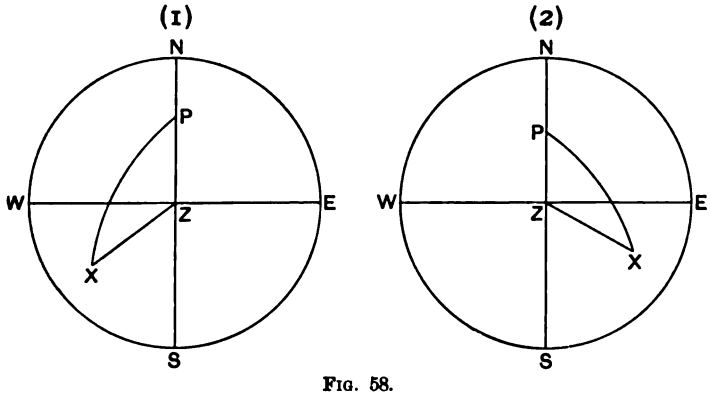


FIG. 58.

Then *ZPX* is the body's hour angle in (1), and 24 h.—the hour angle in (2).

Let $$ZPX = h,$$
$$PZ = 90° - \text{lat.} = c,$$
$$PX = 90° \pm \text{dec.} = p,$$
$$ZX = 90° - \text{alt.} = z.$$

In the triangle ZPX the three sides are known, to find the angle ZPX.

$\therefore$ by the well-known formula of Spherical Trigonometry, hav ZPX

$$= \operatorname{cosec} PZ \cdot \operatorname{cosec} PX \sqrt{\operatorname{hav}(ZX + PZ \sim PX) \operatorname{hav}(ZX - PZ \sim PX)}.$$

NOTE.—It is better for beginners to use this formula, as the quantities used represent the sides of the triangle PZX, and the *difference* of PX and PZ is *always* to be found.

The work is, however, usually shortened by the use of the formula

$$\operatorname{hav} h = \sec l \cdot \sec d \sqrt{\operatorname{hav}(z + l \pm d) \operatorname{hav}(z - l \pm d)},$$

since $PZ \sim PX = l \sim d$ when latitude and declination are of the same name, and $l + d$ when they are of different names.

If the table of haversines is not at hand, the formula can be adapted to the ordinary table of log sines, etc., by writing

$$\sin^2 \frac{\theta}{2} = \operatorname{hav} \theta,$$

$$\therefore \sin^2 \frac{h}{2} = \operatorname{cosec} PZ \cdot \operatorname{cosec} PX \sin \frac{ZX + \overline{PZ \sim PX}}{2} \sin \frac{ZX - \overline{PZ \sim PX}}{2}.$$

If the haversine table is used, the hour angle is taken from the top of the page when the heavenly body is west of the meridian, and from the bottom when the heavenly body is east of the meridian.

The hour angle having been found, which is the apparent time in the case of the sun, the application of the equation of time will produce the mean time required.

A different method is necessary in the case of any other heavenly body.

Let QAQ' represent the celestial equator, QQ' the meridian, m the mean sun, X the heavenly body, AR its right ascension, A being the first point of Aries.

QPR or the arc QR is the hour angle in (1), and is 24 h. $-$the hour angle in (2).

mean time $= mQ = AQ - Am$ in (1),
$= RQ + AR - Am$
$=$ heavenly body's hour angle $+$ its right ascension $-$ R.A. mean sun.

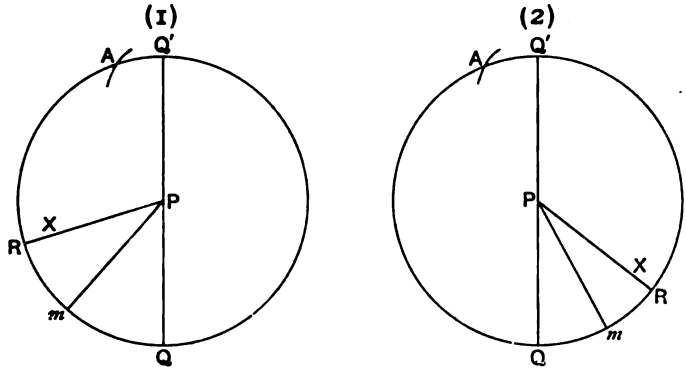


FIG. 59.

24 h. − mean time = mQ
= $QR - Rm$ in (2),
= $QR - (AR - Am)$
= 24 h. − body's hour angle − its right ascension + R.A. mean sun,

or mean time = hour angle + right ascension − R.A. mean sun.

By drawing figures for other cases it will be found that the above expression (24 hours being added or subtracted if necessary) holds good generally.

It is really this expression that is used in the case of the sun, as the equation of time is the difference between the right ascensions of the mean and apparent suns.

Examples will be found in the next chapter, on finding the longitude.

To find the length of the day at any place.

§ 96. Let l be the latitude, d the sun's declination, h the sun's hour angle at rising or setting. Then $2h$ = length of day.

In the quadrantal triangle PXZ (fig. 60),

$\cos h = -\tan l \,.\, \tan d$, if l and d are of the same names.

$\cos h = \tan l \,.\, \tan d$, if l and d are of different names.

If $l = 0$, $\cos h = 0$,

$\therefore\ h = 90°$ or 6 hours;

at the equator, therefore, the day is always 12 hours long.

If $d = 0$, $\cos h = 0$,

$\therefore\ h = 90°$ or 6 hours;

at the time of the equinox, therefore, the sun rises at 6 o'clock all over the earth, and day and night are each 12 hours long.

If $l = 90° - d$, $\cos h = -1$,

$\therefore h = 180°$ or 12 hours;

the sun, therefore, does not set, but just reaches the horizon below the elevated pole, at midnight.

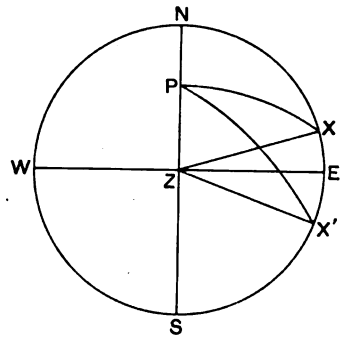


FIG. 60.

If $l > 90° - d$, $\cos h > -1$, which is impossible; the sun, therefore, neither rises nor sets, but continues above the horizon, and there is continuous daylight.

If $l = 90° - d$, when l and d have different names, $\cos h = 1$, $\therefore h = 0$, and the sun comes to the horizon when on the meridian and does not rise at all.

If $l > 90° - d$, when l and d have different names, $\cos h > 1$, which is impossible; the sun, therefore, neither rises nor sets, but continues below the horizon, and there is continuous darkness.

The formulæ $\cos h = -\tan l \,.\, \tan d$,

i.e. $\cos(180° - h) = \tan l \,.\, \tan d$

(when l and d are of the same name),

and $\cos h' = \tan l \,.\, \tan d$

(when l and d are of different name):

show that $\cos(180° - h) = \cos h'$,

$\therefore$ 12 hours $- h = h'$,

$\therefore$ $h + h' = 12$ hours,

i.e. when the latitude and declination have the same name, the sun rises as much before 6 o'clock as it rises after 6 o'clock when they are of different names.

In the foregoing calculations, the theoretical rising and setting of the sun's centre have been considered. But, when the effect of refraction is taken into account, it is evident that there will be rather more daylight than the calculations account for, *e.g.* the sun will really rise a little before 6 o'clock at the vernal equinox in north latitude, and will rise at 6 o'clock when the sun's declination, of opposite name to the latitude, is equal to the correction in altitude at the horizon.

Similarly, there will be some sunlight at each pole for a little more than half the year.

Formulæ in "Time Problems," collected for reference.

(1) Mean time = apparent time ± equation of time.

(2) Sidereal time = right ascension of mean sun + mean time (24 h. being subtracted, if necessary).

(3) Right ascension of that heavenly body which will pass the meridian at a given time = right ascension of mean sun + given mean time.

(4) Mean time = sidereal time − right ascension of mean sun (24 h. being added to sidereal time, if necessary).

(5) Mean time of meridian passage of a heavenly body = R.A. of body − R.A. of mean sun (24 h. being added to R.A. of body, if necessary).

(6) Hour angle of heavenly body

$$\text{hav}\, h = \text{cosec}\, c\, \text{cosec}\, p \sqrt{\text{hav}\,\{z+(p\sim c)\}\,\text{hav}\,\{z-(p\sim c)\}},$$

where c = co-latitude, p = polar distance, z = zenith distance; or

$$\sin^2 \frac{\text{hour angle}}{2} = \text{cosec}\, c\, \text{cosec}\, p \sin \frac{z+(p\sim c)}{2} \sin \frac{z-(p\sim c)}{2}.$$

(7) Mean time = hour angle of heavenly body + R.A. of heavenly body − R.A. of mean sun (24 h. being added or subtracted, if necessary).

(8) Equation of equal altitudes (cf. § 99),

$$E = \frac{dp}{15} \sec d \cot PXZ,$$

dp being the change of declination in ½ elapsed time, E being *positive* if the polar distance is *increasing*, and *negative* if the polar distance is *decreasing*; or

$$E = \frac{dp}{15} \tan d \cot \tfrac{1}{2} \text{ elapsed time} - \frac{dp}{15} \tan l \,\text{cosec}\, \tfrac{1}{2} \text{ elapsed time},$$

the signs of dp, $\tan l$, and $\tan d$ being *positive* when the sun is moving to the north, and l and d are north; otherwise *negative*.

CHAPTER XI.

LONGITUDE. ERROR OF CHRONOMETER: (1) BY SINGLE ALTITUDE; (2) BY EQUAL ALTITUDES.

Longitude.

§ 97. From what has already been said, it will be gathered that the longitude of any place on the earth's surface, expressed in time, is the difference between local mean time and Greenwich mean time at the same instant.

Local mean time may be found by the methods of § 95; Greenwich mean time is obtained from a chronometer, or from the mean of several chronometers whose errors on Greenwich mean time are known, a chronometer being a timekeeper specially constructed to keep accurate time, and whose error on Greenwich time varies only slightly and with great regularity from day to day.

Another method of finding the longitude, by lunar observations, will be discussed later.

Example.—(1) Local or ship mean time, found from observation of the sun E of meridian.

April 18th, in latitude 40° 16′ N, longitude by account 35° 45′ W, about 7 h. 10 m. ship apparent time, the true altitude of the sun's centre (E of meridian) was 19° 50′ 30″, when a chronometer showed 11 h. 18 m. 19 s. On April 8th at Greenwich mean noon the chronometer was 1 h. 47 m. 15 s. fast on Greenwich mean time, and its daily rate was 4·3 s. gaining. Required the longitude.

Greenwich date.	h.	m.		Rate of chronometer. April 17th at 21½ hours.		
S.A.T., - - -	19	10	Apl. 17th.	„ 8th „ 0 „		
Long. W, - - -	2	23	+	interval, - -	9 3/8	days
Greenwich app. time,	21	33				= 9·9 days.
equation of time, -		1	−	daily rate, - - -	4·3	
Greenwich date, -	21	32	Apl. 17th.		297	
	24	0			396	
	2	28		accumulated rate, -	42·57	

Greenwich mean time.

	h.	m.	s.
chronometer, - -	11	18	19
error, - - -	1	47	58
	9	30	21
	12	0	0
Greenwich mean time,	21	30	21

	h.	m.	s.	
error, April 8th, -	1	47	15	fast.
rate, - - -			43	gained.
error at time of observation,	1	47	58	fast.

To find zenith distance.

	90°	0′	0″
altitude, - -	19	50	30
zen. dist., - -	70	9	30

Sun's declination.				Change per hour.	Eq. of time. s.	Change. s.
April 18th,	10°	50′	9″ N	52·4″	41·25	·58
		2	11 –	2·5	1·45	2·5
	10	48	0	131	39·8	1·45
	90	0	0	2′ 11″	Sub. fr. app. time.	
polar dist.,	79	12	0			

To find the sun's hour angle.

co-lat., -	49°	44′	0″	cosec 10·117450
polar dist.,	79	12	0	cosec 10·007762
diff., - -	29	28	0	
zen. dist., -	70	9	30	
sum, - -	99	37	30	½ hav 4·883057
diff., - -	40	41	30	½ hav 4·541187
				hav h = 9·549456

Sun's hour angle, 19 h. 7 m. 44 s.

To find the longitude.

	h.	m.	s.
ship app. time, -	19	7	44
equation of time, -			40 –
ship mean time, -	19	7	4
Greenwich mean time,	21	30	21
long. in time, - -	2	23	17
Long.,	35°	49′	15″ W

As the Greenwich time is greater than the ship time, the longitude is west.

If the ordinary table of log sines is used, the work is as follows:

co-lat., - -	49°	44′	0″	cosec 10·117450	
polar dist., -	79	12	0	cosec 10·007762	
diff., - -	29	28	0		
zen. dist., -	70	9	30		
sum, - -	99	37	30		
diff., - -	40	41	30		
½ sum, - -	49	48	45	sin 9·883057	$\frac{h}{2}$ = 2 h. 26 m. 8 s.
½ diff., - -	20	20	45	sin 9·541187	2
				2)19·549456	4 52 16
				sin $\frac{h}{2}$ = 9·774728	24 0 0

Sun's hour angle, 19 7 44 as before.

(2) Local or ship mean time found from observation of the sun, W of meridian.

June 29th, in latitude 39° 25′ S, longitude by account 58° 15′ W, about 2 h. 50 m. P.M. ship apparent time, the true altitude of the sun's centre (W of meridian) was 15° 47′ 20″, when a chronometer showed 5 h. 45 m. 19 s. On June 19th at Greenwich mean noon the chronometer was 1 h. 2 m. 15 s. slow on Greenwich mean time, and its daily rate was 3·8 s. gaining. Required the longitude.

Greenwich date.

	h. m.	
S.A.T.,	2 50	June 29th.
Long. W,	3 53+	
Greenwich app. time,	6 43	
equation of time,	3+	
Greenwich date,	6 46	June 29th.

Greenwich mean time.

	h. m. s.
chronometer,	5 45 19
error,	1 1 36
Greenwich mean time,	6 46 55

To find zenith distance.

	90° 0′ 0″
altitude,	15 47 20
zen. dist.,	74 12 40

Rate of chronometer.

June 29th at 7 hours.
„ 19th „ 0 „
interval, $10\frac{7}{24}$ days
$= 10{\cdot}3$ days.

daily rate,	3·8 s.
	824
	309
accumulated rate,	39·14

	h. m. s.
error, June 19th,	1 2 15 slow.
rate,	0 0 39 gained.
error at time of observation,	1 1 36 slow.

Sun's declination.	Change.
23° 14′ 45″ N	8·3″
56 –	6·8
23 13 49	56·44
90 0 0	
polar dist., 113 13 49	

Eq. of time.	Change.
m. s.	s.
3 9·4	6·8
3·4	·5
3 13	3·4

additive to app. time.

To find the sun's hour angle.

co-lat.,	50° 35′ 0″	cosec 10·112074
polar dist.,	113 13 50	cosec 10·036715
diff.,	62 38 50	
zen. dist.,	74 12 40	
sum,	136 51 30	½ hav 4·968465
diff.,	11 33 50	½ hav 4·003214
		hav h 9·120468

To find the longitude.

	h. m. s.
ship apparent time,	2 50 25
equation of time,	3 13+
ship mean time,	2 53 38
Greenwich mean time,	6 46 55
longitude in time,	3 53 17

Long., 58° 19′ 15″ W.

Sun's hour angle, 2 h. 50 m. 25 s.

As the Greenwich mean time is greater than the ship mean time, the longitude is west.

(3) Longitude by altitude of the moon, west of meridian.

April 1st, about 8 h. 30 m. P.M. ship apparent time, in latitude 38° 15′ N, longitude by account 15° E, the true altitude of the moon's centre (W of meridian) was 47° 9′ 50″, when a chronometer showed 5 h. 56 m. 23 s. On March 22nd, at Greenwich mean noon, the chronometer was 1 h. 37 m. 21 s. slow on Greenwich mean time ; daily rate, 4 s. losing.

	Greenwich date. h. m.		Rate of chronometer.	
S.A.T., - - -	8 30	April 1st.	April 1st at $7\frac{1}{2}$ hours.	
Long. E, - -	1 0 −		March 22nd „ 0 „	
Greenwich app. time,	7 30		interval, - -	$10\frac{15}{48}$ days
equation of time, -	4			=10·3 days.
				4
Greenwich date, -	7 34	April 1st.	accumulated rate, - -	41·2

	Greenwich mean time. h. m. s.		h. m. s.
chronometer, - -	5 56 23	error, March 22nd,	1 37 21 slow.
error, - - -	1 38 2+	rate, - - -	41 losing.
G.M.T., - -	7 34 25	error at time of observation,	1 38 2 slow.

Moon's R.A. h. m. s.	Change. s.	Moon's declination.	Change.	R.A. mean sun. m. s.
5 49 19·5	28·87	28° 39′ 42″ N	2·1″	38 22·47
1 27·9	3·4	7	3·4	1 9
				5·59
5 50 47·4	87·958	28 39 49	7·14	·07
		90 0 0		
		polar dist., 61 20 11		39 37·13

To find zenith distance.

90° 0′ 0″
47 9 50 altitude.

42 50 10 zen. dist.

To find the moon's hour angle.

			To find the longitude. h. m. s.
co-lat., -	51° 45′ 0″	cosec 10·104955	Moon's hour angle, - 3 22 52·3
polar dist.,	61 20 10	cosec 10·056780	„ R.A. - - 5 50 47·4
diff., -	9 35 10		9 13 39·7
zen. dist.,	42 50 10		R.A.M.S., - - 39 37
sum, -	52 25 20	½ hav 4·645108	S.M.T., - - - 8 34 2·7
diff., -	33 15 0	½ hav 4·456528	G.M.T., - - - 7 34 25
		hav h 9·263471	Long. in time, - - 59 37·7

h=3 h. 22 m. 52·3 s. Long. 14° 54′ 30″ E.

Greenwich time being less than ship time, the longitude is east.

(4) Longitude by altitude of a star, east of meridian.

December 7th, about 1 h. 30 m. A.M. ship apparent time, in latitude 32° 27′ S, longitude by account 117° 30′ W, the true altitude of α Leonis (east of meridian) was 21° 21′ 30″, when a chronometer showed 10 h. 57 m. 43 s. On November 7th, at Greenwich mean noon, the chronometer was 1 h. 44 m. 41 s. fast on Greenwich mean time; daily rate, 3·2 s. gaining.

Greenwich date.

	h.	m.	
S.A.T.,	13	30	Dec. 6th.
Long. W,	7	50	+
	21	20	
equation of time,		8	−
Greenwich date,	21	12	Dec. 6th.

Rate of chronometer.

Dec. 6th at 21 hours.
Nov. 7th ,, 0 ,,
Interval, $29\frac{7}{8}$ days
$= 29{\cdot}9$ days.
3·2
accumulated rate, 95·68

Greenwich mean time.

	h.	m.	s.
chronometer,	10	57	43
error,	1	46	17
	9	11	26
	12	0	0
Greenwich mean time,	21	11	26

	h.	m.	s.	
error, Nov. 7th,	1	44	41	fast.
rate,		1	36	
error at time of observation,	1	46	17	fast.

To find zenith distance.

	90°	0′	0″
	21	21	30
zen. dist.,	68	38	30

R.A mean sun.

h.	m.	s.
17	0	5
	3	27
		2
17	3	34

α Leonis R.A., 10 h. 2 m. 51 s. declination, 12° 28′ 30″ N
polar dist., 102 28 30

To find the star's hour angle.

co-lat.,	57°	33′	0″	cosec 10·073730
polar dist.,	102	28	30	cosec 10·010377
diff.,	44	55	30	
zen. dist.,	68	38	30	
sum,	113	34	0	½ hav 4·922520
diff.,	23	43	0	½ hav 4·312796
				hav h 9·319423

hour angle = 20 h. 22 m. 34 s.

To find the longitude.

	h.	m.	s.
Star's hour angle,	20	22	34
,, R.A.,	10	2	51
	30	25	25
R.A. mean sun,	17	3	34
S.M.T.,	13	21	51
G.M.T.,	21	11	26
Long. in time,	7	49	35

Long. 117° 23′ 45″ W.

Greenwich time being greater than ship time, the longitude is west.

Examples.—(1) April 27th, about 7 h. 30 m. A.M. apparent time, in latitude 53° 18′ 30″ N, longitude by account 5° 45′ W, the obs. alt. sun's L.L. was 24° 15′ 30″, when a chronometer showed 7 h. 23 m. 23 s. On April 16th, at G.M. noon, the chronometer was 0 h. 27 m. 59 s. slow on G.M.T.; daily rate, 6·6 s. gaining. The index error was 2′ 50″−, and the height of the eye 18 feet.

(2) June 4th, about 9 A.M. apparent time, in latitude 16° 47′ S, longitude by account 62° 10′ E, the obs. alt. sun's L.L. was 30° 44′ 40″, when a chronometer showed 6 h. 7 m. 43 s. On April 27th, at G.M. noon, the chronometer was 1 h. 17 m. 6 s. fast on G.M.T.; daily rate, 2·8 s. gaining. The index error was 1′ 40″+, and the height of the eye 23 feet.

(3) April 18th, about 3 h. 20 m. P.M. apparent time, in latitude 24° 40′ S, longitude by account 26° W, the obs. alt. sun's L.L. was 29° 12′ 30″, when a chronometer showed 3 h. 51 m. 35 s. On March 15th, at G.M. noon, the chronometer was 1 h. 15 m. 43 s. slow on G.M.T.; daily rate, 3·1 s. gaining. The index error was 3′−, and the height of the eye 26 feet.

(4) June 10th, about 6 h. 25 m. A.M. apparent time, in latitude 40° 20′ N, longitude by account 42° 30′ W, the obs. alt. sun's L.L. was 19° 17′ 20″, when a chronometer showed 9 h. 45 m. 50 s. On May 20th, at G.M. noon, the chronometer was 0 h. 29 m. 42 s. fast on G.M.T.; daily rate, 3·8 s. gaining. The index error was 2′ 40″−, and the height of the eye 20 feet.

(5) June 28th, about 3 h. 20 m. P.M. apparent time, in latitude 19° 15′ S, longitude by account 169° 45′ E, the obs. alt. sun's L.L. was 25° 7′ 20″, when a chronometer showed 7 h. 9 m. 46 s. On May 31st, at G.M. noon, the chronometer was 3 h. 6 m. 46 s. fast on G.M.T.; daily rate, 1·9 s. losing. The index error was 2′ 10″+, and the height of the eye 24 feet.

(6) December 10th, about 7 h. 30 m. A.M. apparent time, in latitude 32° 40′ S, longitude by account 71° 10′ W, the obs. alt. sun's L.L. was 30° 12′ 40″, when a chronometer showed 10 h. 17 m. 25 s. On November 9th, at G.M. noon, the chronometer was 1 h. 51 m. 17 s. slow on G.M.T.; daily rate, 3·3 s. gaining. The index error was 1′ 20″+, and the height of the eye 22 feet.

(7) April 6th, about 6 h. 40 m. P.M. apparent time, in latitude 33° 12′ S, longitude by account 71° 20′ E, the obs. alt. moon's L.L. (E of meridian) was 28° 57′ 40″, when a chronometer showed 3 h. 49 m. 16 s. On March 8th, at G.M. noon, the chronometer was 1 h. 50 m. 58 s. fast on G.M.T.; daily rate, 2·3 s. gaining. The index error was 2′ 50″+, and the height of the eye 20 feet.

(8) December 23rd, about 10 h. P.M. apparent time, in latitude 16° 16′ N, longitude by account 67° W, the obs. alt. noon's U.L. (W of meridian) was 26° 40′, when a chronometer showed 3 h. 51 m. 35 s. On November 30th, at G.M. noon, the chronometer was 1 h. 23 m. 36 s. fast on G.M.T.; daily rate, 2·5 s. gaining. The index error was 2′ 10″+, and the height of the eye 26 feet.

(9) April 25th, about 6 h. 15 m. A.M. apparent time, in latitude 36° 14′ S, longitude by account 76° 30′ E, the obs. alt. α Scorpii (W of meridian)

was 38° 23′ 40″, when a chronometer showed 3 h. 47 m. 20 s. On March 31st, at G.M. noon, the chronometer was 2 h. 39 m. 28 s. fast on G.M.T.; daily rate, 3·4 s. gaining. The index error was 2′ 40″+, and the height of the eye 19 feet.

(10) April 2nd, about 6 h. 30 m. P.M. apparent time, in latitude 28° 14′ N, longitude by account 17° 30′ W, the obs. alt. α Tauri (W of meridian) was 49° 53′ 30″, when a chronometer showed 5 h. 47 m. 11 s. On March 17th, at G.M. noon, the chronometer was 1 h. 57 m. 22 s. slow on G.M.T.; daily rate, 5·2 s. gaining. The index error was 1′ 30″−, and the height of the eye 22 feet.

(11) June 1st, about 9 h. 30 m. P.M. apparent time, in latitude 51° 10′ N, longitude by account 2° 50′ E, the obs. alt. α Lyrae (E of meridian) was 43° 15′ 20″, when a chronometer showed 11 h. 14 m. 39 s. On may 20th, at G.M. noon, the chronometer was 1 h. 57 m. 10 s. fast on G.M.T.; daily rate, 4·2 s. gaining. The index error was 3′ 10″−, and the height of the eye 17 feet.

(12) June 18th, about 7 h. 30 m. P.M. apparent time, in latitude 32° 16′ N, longitude by account 142° 35′ E, the obs. alt. α Scorpii (E of meridian) was 16° 37′ 50″, when a chronometer showed 7 h. 49 m. 47 s. On May 21st, at G.M. noon, the chronometer was 2 h. 9 m. 6 s. slow; daily rate, 3·8 s. losing. The index error was 2′ 20″+, and the height of the eye 24 feet.

(13) April 15th, about 7 h. 20 m. P.M. apparent time, in latitude 52° 18′ N, longitude by account 47° 13′ W, the obs. alt. Jupiter's centre (W of meridian) was 46° 37′ 30″, when a chronometer showed 7 h. 31 m. 16 s. On March 20th, at G.M. noon, the chronometer was 2 h. 58 m. 32 s. slow; daily rate, 2·5 s. gaining. The index error was 2′ 20″−, and the height of the eye 19 feet.

(14) December 2nd, about midnight apparent time, in latitude 7° 55′ S, longitude by account 148° E, the obs. alt. Jupiter's centre (E of meridian) was 22° 34′ 50″, when a chronometer showed 4 h. 19 m. 35 s. On November 1st, at G.M. noon, the chronometer was 2 h. 19 m. 42 s. fast on G.M.T.; daily rate, 4·2 s. gaining. The index error was 1′ 40″+, and the height of the eye 22 feet.

Error and rate of Chronometer by single altitude.

§ 98. When the longitude is known accurately, the error of the chronometer can be determined. The mean time is found in the usual manner from the observed altitude of a heavenly body. To this the longitude in time is applied, and the Greenwich mean time determined; the difference between this and the chronometer time of observation is the error required.

Two errors having been thus determined, with a sufficient number of days' interval between the observations, the daily rate is found by dividing the difference of the errors by the days and parts of a day (if any) contained in the interval.

These observations are usually taken in the artificial horizon, the sea horizon being seldom sufficiently reliable when *accurate* results are required.

In all observations it is better to take several altitudes at small intervals, and use the mean of the altitudes as *the* observed altitude, and the mean of the times as *the* chronometer time, since this mean is more likely to be accurate than any single altitude.

For ordinary observations three altitudes are usually taken; but for error and rate of chronometer *less* than five should not be taken. An odd number is preferable, as the mean should then nearly agree with the middle observation, and this affords a check on error.

Examples.

N.B.—The altitudes are to be considered as the mean of several.

(1) April 4th, about 8 h. A.M. mean time, in latitude 50° 22′ N, longitude 4° 10′ 15″ W, the obs. alt. sun's L.L. in artificial horizon was 46° 0′ 44″, when a chronometer showed 11 h. 47 m. 16 s. Index error 2′ 30″+.

(2) April 23rd, about 7 h. 45 m. A.M. mean time, in latitude 22° 16′ 30″ N, longitude 114° 10′ E, the obs. alt. sun's L.L. in artificial horizon was 57° 27′, when a chronometer showed 9 h. 46 m. 48 s. Index error 2′ 10″−. Find the daily rate, if the chronometer was 2 h. 19 m. 49 s. slow on G.M.T. at G.M. noon, April 14th.

(3) April 30th, about 9 h. 35 m. P.M. mean time, in latitude 35° 53′ N longitude 14° 31′ E, the obs. alt. Procyon (W of meridian) in artificial horizon was 40° 53′ 30″, when a chronometer showed 10 h. 12 m. 3 s. Index error 3′−. Find the daily rate, if the chronometer was 1 h. 36 m. 27 s. fast on G.M.T. at G.M. noon, April 1st.

(4) June 26th, about 9 h. P.M. mean time, in latitude 34° 11′ 30″ S, longitude 18° 31′ 45″ E, the obs. alt. α Crucis (W of meridian) in artificial horizon was 100° 51′ 56″, when a chronometer showed 9 h. 18 m. 4 s. Index error 2′ 20″−.

(5) December 16th, about 8 h. 40 m. P.M. mean time, in latitude 7° 55′ 30″ S, longitude 14° 25′ 30″ W, the obs. alt. α Orionis (E of meridian) in artificial horizon was 71° 36′ 2″, when a chronometer showed 7 h. 49 m. 37 s. Index error 1′ 40″+. Find the daily rate if the chronometer was 1 h. 48 m. 35 s. slow on G.M.T. at G.M. noon, December 1st.

(6) December 5th, about 4 h. 30 m. P.M. mean time, in latitude 33° 52′ S, longitude 151° 12′ 45″ E, the obs. alt. sun's L.L. in artificial horizon was 56° 32′ 44″, when a chronometer showed 2 h. 14 m. 37 s. Index error 40″−. Find the daily rate, if the chronometer was 4 h. 9 m. 14 s. slow on G.M.T. at 8 A.M., November 22nd, in longitude 115° 45′ E.

Error of Chronometer by equal altitudes of the sun.

§ 99. If the sun's declination did not alter, the mean of the chronometer times at which the sun has the same altitude on opposite sides of the meridian would be the chronometer time of apparent noon. But the sun's declination does alter, and a correction called the "Equation of equal altitudes" must be applied to the mean of the chronometer times. The difference between the chronometer time of apparent noon, thus found, and the Greenwich time of apparent noon is the error of the chronometer on Greenwich mean time.

In §§ 185, 186 it is shown that the equation of equal altitudes is to be found—

(1) from the expression

$$E \text{ in time} = \frac{dp}{15} \sec \text{dec.} \cot PXZ,$$

dp being the change of declination in half the elapsed time between the observations, cot PXZ being found from the values of the latitude, declination, and zenith distance; while the sign of E is positive if the polar distance is increasing, and negative if the polar distance is decreasing;

(2) from the expression

$$E = \frac{dp}{15} . \tan \text{dec.} \cot \tfrac{1}{2} \text{ elapsed time}$$

$$- \frac{dp}{15} . \tan \text{lat.} \operatorname{cosec} \tfrac{1}{2} \text{ elapsed time,}$$

dp being change of declination in half the elapsed time: the sign of dp being + or − according as the sun is moving to the north or the south; of tan l and tan d respectively + or − according as l or d are N or S; cf. § 186*.

Example.—April 16th, in lat. 30° N, long. 75° W, the sun had equal altitudes 33° 30′ (approximately) at the following times by chronometer:

A.M.	P.M.
h. m. s.	h. m. s.
8 46 57	4 22 45

Required the error of the chronometer.

The equation of time is required for the G.M.T. of apparent noon, and the declination for the G.M.T. of the first observation; but no appreciable error will be caused by using the declination for the G.M.T. of apparent noon.

	h.	m.	s.
A.M. time, - -	8	46	57
P.M. time, - -	4	22	45
elapsed time, -	7	35	48
½ elapsed time, -	3	47	54

h.	m.	
0	0	apparent time of apparent noon.
0	0	equation of time to nearest minute.
0	0	mean time of apparent noon.
5	0	longitude.
5	0	Greenwich date of apparent noon to correct equation of time.
3	48	½ elapsed time.
1	12	Greenwich date of first observation to correct declination.

Declination.	Change.
10° 7′ 8″ N	53″
1 4	1·2
10 8 12	63·6

Equation of time. s.	Change. s.
13·09	·6
3	5
16 – app. time.	3·0

$dp = 53'' \times 3{\cdot}8 = 201{\cdot}4''$.

By first method.

To find *PXZ*.

80	cosec 10·006649
56	cosec 10·081426
24	
60	
84	½ hav 4·825511
36	½ hav 4·489982
	9·403568

PXZ = 60° 26′

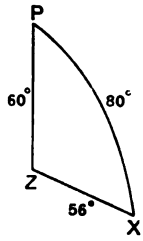


FIG. 61.

To find *E*.

dp	2·304059
sec *d*	10·006828
cot *PXZ*	9·753820
	2·064707
15 - -	1·176091
	·888616

E = 7·74.

E is negative, as the polar distance is decreasing.

	h.	m.	s.
A.M. time, - - -	8	46	57
½ elapsed time, - -	3	47	54
	34	51	
eqn. of equal altitudes,		8	
	34	43	

= chronometer time of app. noon.

h.	m.	s.	
0	0	0	app. time of app. noon.
		16	equation of time.
23	59	44	mean time.
5	0	0	
4	59	44	Greenwich mean time.
	34	43	chronometer time.
4	25	1	chronometer slow on Greenwich mean time.

(2) By second method.

dp, - - - - -	2·304059
tan dec., - - -	9·252191
cot ½ elapsed time, -	9·812932
	1·369182
15 - - - - -	1·176091
	·193091
	1·56 s.

dp, - - - - -	2·304059
tan lat., - - - -	9·761439
cosec ½ elapsed time,	10·076532
	2·142030
15 - - - - -	1·176091
	·965939
	9·246 s.

The sun is moving towards the north, the latitude and declination are both north.

$$\therefore E = 1{\cdot}56 \text{ s.} - 9{\cdot}246 \text{ s.}$$

$$= -7{\cdot}686 \text{ s.},$$

which differs from the value obtained by the first method by ·054 s.

The error of chronometer is then found as before.

§ 100. The method of observation is to fix the index at successive exact divisions of the limb, usually every 10′, and note the time when the sun has these altitudes. Then in the afternoon the time is noted when the sun has the same altitudes in reverse order.

The best time for this, as for all other observations for time, is when the sun is near the prime vertical, but, as this is not always practicable, observations may be made at any time, provided that there is not more than thirty seconds interval between the successive 10′ of altitude in artificial horizon.

§ 101. The great advantage of this method is that it eliminates the effect of errors of the instrument and personal errors of observation, which affect both altitudes in the same way; in the first method the altitude is not required to be known exactly, and in the second method it is not required at all, but only the *times* when the sun had the same altitude, whatever it may be.

A small error in the latitude also will not produce an appreciable effect; hence this method of finding error of chronometer is preferable to that by single altitude, if the latitude is not exactly known.

§ 102. It is necessary to know approximately at what time by chronometer the afternoon observations should commence.

To the chronometer time of the last A.M. observation apply the error of the chronometer on local mean time. (This error is easily found.) The result, corrected for the equation of time, will give the apparent time, which, subtracted from 24 hours, will give, approximately, the apparent time of the first P.M. observation. This, corrected for the equation of time and error of chronometer on local mean time, will give the approximate chronometer time of the first P.M. observation.

Example.—The chronometer time of the last A.M. observation for equal altitudes of the sun was 11 h. 14 m. 57 s.; the chronometer being approximately 2 h. 15 m. fast on mean time at the place, and the equation of time

6 m. 20 s. additive to mean time. Find approximately the time by chronometer when the P.M. observations should commence.

h.	m.	s.	
11	14	57	chronometer time.
2	15	0	error fast on mean time.
20	59	57	mean time of last A.M. observation.
	6	20	equation of time.
21	6	17	apparent time of last A.M. observation.
24	0	0	
2	53	43	apparent time of first P.M. observation.
	6	20	
2	47	23	mean time.
2	15	0	
5	2	23	approximate chronometer time of first P.M. observation.

To find the error of Chronometer by equal altitudes of a star.

§ 103. The mean of the chronometer times will be the chronometer time of the star's meridian passage. The Greenwich time of the star's meridian passage being calculated by the ordinary method, the difference of the times is the error of the chronometer.

Examples.—(1) April 5th, in latitude 13° 4′ N, longitude 59° 37′ W, the sun had equal altitudes (about 30°) at the following times by chronometer:

A.M.	P.M.
h. m. s.	h. m. s.
9 27 35	5 33 31

(2) April 20th, in latitude 35° 54′ N, longitude 14° 31′ 30″ E, the sun had equal altitudes (about 37°) at the following times by chronometer:

A.M.	P.M.
h. m. s.	h. m. s.
11 14 45	6 13 15

(3) June 7th, in latitude 15° 55′ S, longitude 5° 44′ W, the sun had equal altitudes (about 29°) at the following times by chronometer:

A.M.	P.M.
h. m. s.	h. m. s.
7 14 27	1 41 35

(4) December 2nd, in latitude 34° 11′ 30″ S, longitude 18° 31′ 30″ E, the sum had equal altitudes (about 48°) at the following times by chronometer:

A.M.	P.M.
h. m. s.	h. m. s.
11 14 27	5 23 29

(5) June 29th, in latitude 7° 55′ S, longitude 14° 25′ 30″ W, the sun had equal altitudes (about 35° 30′) at the following times by chronometer:

A.M.	P.M.
h. m. s.	h. m. s.
6 4 17	0 10 13

(6) April 10th, in latitude 32° 52′ S, longitude 151° 12′ 45″ E, the sun had equal altitudes (about 35° 40′) at the following times by chronometer:

A.M.	P.M.
h. m. s.	h. m. s.
10 53 39	3 56 19

CHAPTER XII.

LATITUDE.

MERIDIAN ALTITUDES. EX-MERIDIAN ALTITUDES. REDUCTION TO MERIDIAN. DOUBLE ALTITUDES. POLAR ANGLES. CORRECTION FOR RUN. POLE STAR.

To show that the altitude of the pole above the horizon is equal to the latitude of the place.

§ 104. Let *epp'* represent the earth, *z* any place on its surface, *eq* the equator, *p* the elevated pole, *HO* the celestial horizon, *EQ* the celestial equator, *P* the pole of the heavens, *Z* the zenith.

The arc $ZO = 90°$ and arc $PE = 90°$, therefore, taking arc *PZ* from each, the remainder *PO* = the remainder *EZ*, and *ez* also = *po*, But *ez* is the latitude, therefore *po* also = the latitude;

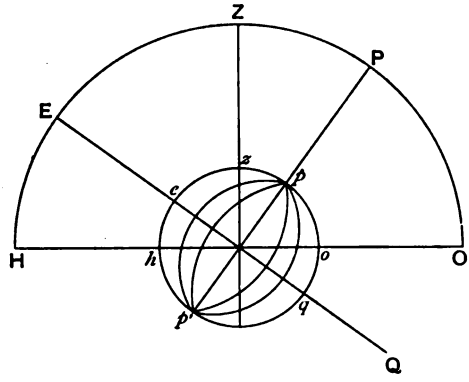


FIG. 62.

and *PO* which measures *po* on the celestial concave = the latitude. But *PO* is the elevation of the pole above the horizon.

Therefore the altitude of the pole above the horizon is equal to the latitude of the place.

Given the meridian altitudes of a heavenly body above and below the pole, to find the latitude.

§ 105. Let *NESW* represent the horizon, *P* the pole, *Z* the zenith, *NZS* the celestial meridian. Let *ABC* be the parallel of declination described by the body about the pole, *A* and *B* its places when on the meridian.

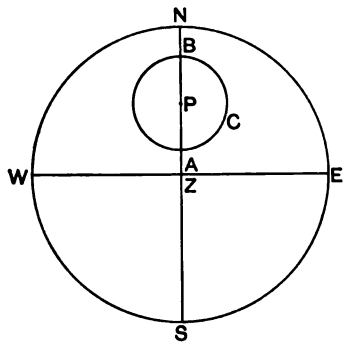


FIG. 63.

Then

AN = body's altitude above the pole.

BN = body's altitude below the pole.

PN = altitude of the pole = latitude.

Now

$$PN = PB + BN,$$

and $$PN = AN - PA.$$

$$\therefore\ 2PN = AN + BN.$$

since $PB = PA$.

$$\therefore\ PN = \frac{AN + BN}{2}.$$

or latitude = $\frac{1}{2}$ the sum of the altitudes above and below the pole.

Bodies whose meridian passages are both above the horizon are named "circumpolar." The figure shows that any body whose polar distance is less than the latitude is circumpolar.

In the case of sun, moon, or planets, an allowance would have to be made for the change of declination in the interval, *PA* and *PB* being equal only in the case of a fixed star.

Examples.—(1) The meridian altitudes (corrected) of Capella above and below the north pole respectively were 75° 25′ (zenith N) and 16° 21′ (zenith S); required the latitude.

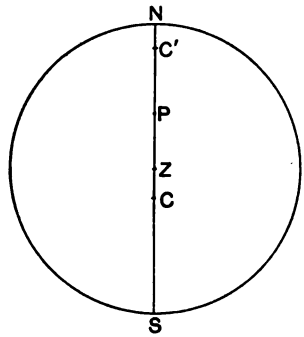


FIG. 64.

$$NC = 180° - 75°\ 25'$$
$$= 104°\ 35',$$
$$NC' = 16\ 21,$$
$$\therefore\ \text{latitude} = \frac{NC + NC'}{2}$$
$$= \frac{120°\ 56'}{2}$$
$$= 60°\ 28'\ \text{N.}$$

(2) The meridian altitudes (corrected) of α Centauri above and below the south pole respectively were 76° 31′ and 17° 41′ (zenith N in each case); required the latitude.

$$SC = 76° 31',$$
$$SC' = 17\ 41,$$
$$\therefore \text{ latitude} = \frac{SC + SC'}{2}$$
$$= \frac{94° 12'}{2}$$
$$= 47° 6' \text{ S.}$$

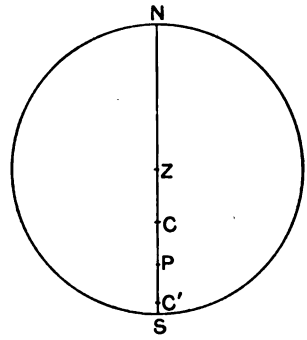


FIG. 65.

Examples.—(1) The obs. mer. alts. of Vega above and below the north pole were 67° 51′ 30″ (zenith N) and 9° 49′ 20″ (zenith S). Index error 2′ 10″ −; height of eye, 20 feet.

(2) The obs. mer. alts. of Capella above and below the north pole were 79° 32′ (zenith N) and 12° 5′ 10″ (zenith S). Index error, 3′ 10″+; height of eye, 23 feet.

(3) The obs. mer. alts. of α Centauri above and below the south pole (zenith N in each case) were 75° 41′ 30″ and 16° 31′ 40″. Index error, 1′ 40″+; height of eye, 22 feet.

(4) The obs. mer. alts. of Achernar above and below the south pole (zenith N in each case) were 82° 27′ 30″ and 18° 3′ 20″. Index error, 1′ 50″+; height of eye, 21 feet.

Given the meridian altitude of a heavenly body below the pole, to find the latitude.

§ 106. Let X be the body whose altitude on the meridian is less than that of the pole.

$$\text{Latitude} = PN$$
$$= PX + XN$$
$$= 90° - \text{dec.} + \text{mer. alt.}$$
$$= 90° + \text{mer. alt.} - \text{dec.}$$

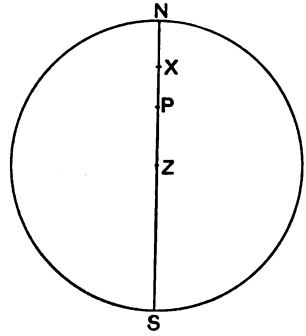


FIG. 66.

and is of the same name as the declination.

Examples.—(1) May 31st, the true meridian altitude of α Crucis below the south pole was 19° 57′ 30″. Required the latitude.

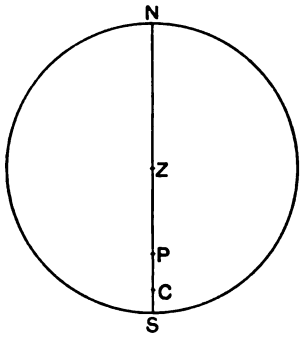


FIG. 67.

Meridian altitude,	-	19° 57′ 30″
		90 0 0
		109 57 30
α Crucis' declination,	-	62 31 30
		47 26 0

Latitude, 47° 26′ S.

(2) October 1st, the true meridian altitude of γ Draconis below the north pole was 15° 5′ 20″.

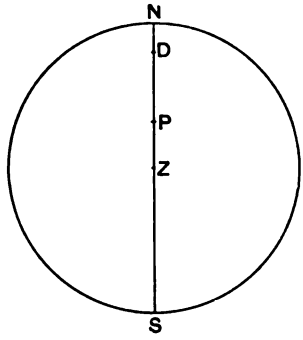


FIG. 68.

Meridian altitude,	-	15° 5′ 20″
		90 0 0
		105 5 20
γ Draconis' declination,		51 30 15
		53 35 5

Latitude, 53° 35′ N.

Examples.—(1) The obs. mer. alt. of Capella below the pole was 17° 27′ 10″. Index error, 1° 20′+; height of eye, 24 feet.

(2) The obs. mer. alt. of Vega below the pole was 14° 16′ 50″. Index error, 40″−; height of eye, 21 feet.

(3) The obs. mer. alt. of α Crucis below the pole was 16° 17′ 50″. Index error, 1′ 10″+; height of eye, 22 feet.

Given the meridian altitude of a heavenly body above the pole, and the bearing of the zenith from the body, to find the latitude.

§ 107. Let EQW be the equinoctial, X the place of the body whose declination is XQ. Then PQ and NZ each $= 90°$.

$\therefore$ $ZQ = PN =$ latitude required.

I. Let the latitude and declination be of the same name, and the body on the opposite side of the zenith from the pole, *i.e.* zenith distance same name as latitude (fig. 69).

$$\text{Latitude} = ZX + XQ$$
$$= \text{mer. zen. dist.} + \text{dec.}$$

and is of the *same* name as the declination.

II. Let X' be the place of the body when latitude and declination are of opposite names, zenith distance being of the same name as the latitude.

$$\text{Latitude} = ZX' - X'Q$$
$$= \text{mer. zen. dist.} - \text{dec.}$$

and is of *opposite* name to the declination.

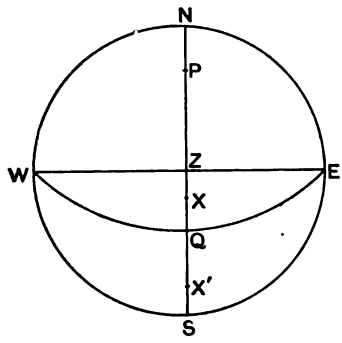


FIG. 69.

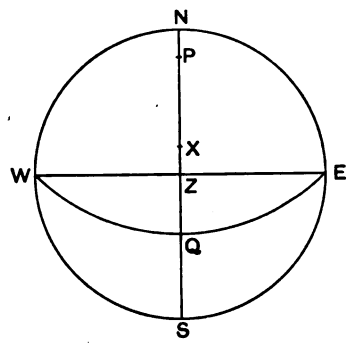


FIG. 70.

III. Let X be the position of the body when it is between the pole and the zenith, *i.e.* zenith distance and declination of different names (fig. 70).

$$\text{Latitude} = XQ - XZ$$
$$= \text{dec.} - \text{mer. zen. dist.}$$

and is of the *same* name as the declination.

Hence we have, generally,

$$\text{Latitude} = \text{mer. zen. dist.} \pm \text{dec.}$$

+ when zenith distance and declination are of the *same* name;
− when they are of *opposite* names.

In actual practice there can be no doubt as to whether the sum or difference of zenith distance and declination is to be taken, as the position must be approximately known.

In the working of examples the careful construction of a figure will always show which to take. This is a much better method than the trusting to rules, which are not always remembered when required.

Examples. (1) *Latitude by meridian altitude of the sun.*

April 15th, in longitude 115° E, the true meridian altitude of the sun's centre was 47° 39′ 40″ (zenith N of the sun). Required the latitude.

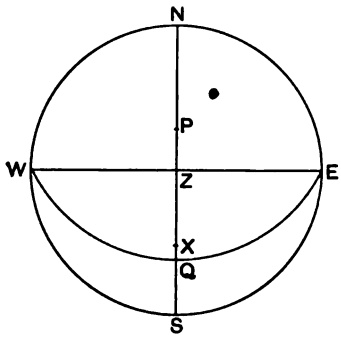


FIG. 71.

	h.	m.
April 15th,	0	0
Long.,	7	40 –
Greenwich app. time,	16	20 April 14.

Declination.	Change.
9° 46′ 36″	53·6″
6 53	7·7
9 39 43	3752
	3752
Semi-diameter.	412·72
15′ 58″	6′ 53″

True mer. alt.,	47° 39′ 40″
	90 0 0
zen. dist.,	42 20 20 N
declination,	9 39 43 N
latitude,	52 0 3 N

(2) *Latitude by meridian altitude of the moon.*

April 15th, in longitude 78° W, the true meridian altitude of the moon's centre was 42° 0′ 10″ (zenith N of the moon). Required the latitude.

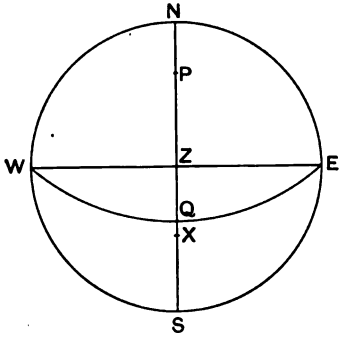


FIG. 72.

	h.	m.	
Moon's mer. pass. on 15th,	16	51	April 14.
,, ,, 16th,	17	44	,, 15.
		53	
correction,		11	
S.M.T. of moon's mer. pass.,	17	2	April 14.
Long.,	5	12	
Greenwich date,	22	14	April 14.

	Moon's declination.	Change.
At 22 h.,	28° 12′ 55″ S	26·7″
	37	1·4
	28 12 18	1068
		267
		37·38

True mer. alt.,	42° 0′ 10″
	90 0 0
zen. dist.,	47 59 50 N
dec.,	28 12 18 S
	19 47 32

Latitude, 19° 47′ 33″ N.

(3) *Latitude by meridian altitude of a star.*

Oct. 19th, the true meridian altitude of Vega was 69° 4′ 40″ (zenith S of the star). Required the latitude.

True mer. alt., - -	69°	4′ 40″
	90	0 0
zen. dist., - - -	20	55 20 S
dec., - - - -	38	41 20 N
	17	46 0

Latitude, 17° 46′ N.

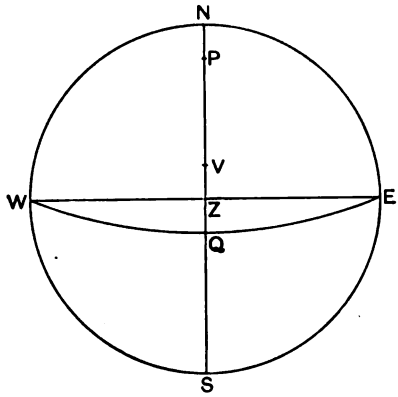


FIG. 73.

Examples. *Latitude by Sun Meridian Altitude.*

(1) April 4th, in longitude 64° 33′ W, the obs. mer. alt. sun's L.L. was 65° 19′ 20″ (zenith N of the sun). Index error, 2′ 10″+ ; height of eye, 22 feet.

(2) April 24th, in longitude 94° 25′ E, the obs. mer. alt. sun's L.L. was 44° 17′ 40″ (zenith S). Index error, 2′ 50″− ; height of eye, 24 feet.

(3) December 6th, in longitude 22° 30′ E, the obs. mer. alt. sun's L.L. was 31° 39′ 50″ (zenith N). Index error, 2′ 40″− ; height of eye, 23 feet.

(4) June 8th, in longitude 127° 45′ E, the obs. mer. alt. sun's L.L. was 87° 0′ 10″ (zenith N). Index error, 1′ 50″− ; height of eye, 22 feet.

(5) June 28th, in longitude 125° 30′ W, the obs. mer. alt. sun's L.L. was 75° 23′ 10″ (zenith N). Index error, 2′ 10″+ ; height of eye, 18 feet.

(6) December 2nd, in longitude 73° 16′ W, the obs. mer. alt. sun's L.L. was 50° 46′ 10″ (zenith N). Index error, 1′ 30″− ; height of eye, 23 feet.

(7) December 4th, in longitude 137° 42′ E, the obs. mer. alt. sun's L.L. was 77° 24′ 40″ (zenith N). Index error, 1′ 50″+ ; height of eye, 26 feet.

Latitude by Moon Meridian Altitude.

(8) April 4th, in longitude 119° W, the obs. mer. alt. moon's L.L. was 59° 18′ 40″ (zenith N of the moon). Index error, 3′ 10″+ ; height of eye, 24 feet.

(9) April 15th, in longitude 72° 30′ E, the obs. mer. alt. moon's L.L. was 45° 16′ 20″ (zenith N). Index error, 2′ 50″− ; height of eye, 21 feet.

(10) June 3rd, in longitude 22° 30′ E, the obs. mer. alt. of the moon's U.L. was 69° 14′ 40″ (zenith S). Index error, 1′ 20″− ; height of eye, 22 feet.

(11) June 12th, in longitude 68° 15′ W, the obs. mer. alt. moon's L.L. was 38° 17′ 20″ (zenith N). Index error, 2′ 10″+ ; height of eye, 21 feet.

(12) December 3rd, in longitude 39° 10′ W, the obs. mer. alt. moon's L.L. was 40° 25′ 10″ (zenith S). Index error, 30″− ; height of eye, 26 feet.

(13) December 24th, in longitude 117° 20′ E, the obs. mer. alt. moon's L.L. was 47° 27′ 40″ (zenith S). Index error, 1′ 10″+ ; height of eye, 24 feet.

Latitude by Star Meridian Altitude.

(14) The obs. mer. alt. of Aldebaran was 61° 6′ 20″ (zenith N). Index error, 2′ 10″− ; height of eye, 20 feet.

(15) The obs. mer. alt. of Aldebaran was 42° 16′ 50″ (zenith S). Index error, 1′ 50″+ ; height of eye, 24 feet.

(16) The obs. mer. alt. of Antares was 73° 21′ 40″ (zenith S). Index error, 1′ 10″− ; height of eye, 18 feet.

(17) The obs. mer. alt. of Spica was 42° 19′ 0″ (zenith N). Index error, 2′ 20″− ; height of eye, 19 feet.

(18) The obs. mer. alt. of Vega was 66° 17′ 10″ (zenith S). Index error, 50″+ ; height of eye, 22 feet.

To find the Latitude by observations near the Meridian.

§ 108. (I.) *Using the estimated latitude.*

FIG. 74.

Let $PZ = 90° - l = c,$

$PS = 90° \pm d = p,$

$ZS = z,$

$ZPS = h,$

Then by a well-known formula of Spherical Trigonometry,

$$\text{vers } z = \text{vers } (p \sim c) + 2 \sin c \,.\, \sin p \,.\, \text{hav } h.$$

$$\therefore \quad \text{vers } (p \sim c) = \text{vers } z - 2 \sin c \,.\, \sin p \,.\, \text{hav } h.$$

But $(p \sim c) = l \pm d,$

$$\therefore \quad \text{vers } (l \pm d) = \text{vers } z - 2 \sin c \,.\, \sin p \,.\, \text{hav } h.$$

Now l is not known exactly, but $l \pm d$ is the meridian zenith distance at a place in the *same* latitude as that of the observer at the time that the declination is d; and as the declination will only change very slightly between the time of observation and noon, it may be considered as the meridian zenith distance at the place of observation.

$\therefore$ vers meridian zen. dist.

$$= \text{vers obs. zen. dist.} - 2 \sin c \,.\, \sin p \,.\, \text{hav } h.$$

Let $\text{vers}\,\theta = 2\sin c \cdot \sin p \cdot \text{hav}\,h.$
$= 2\cos l \cdot \cos d \cdot \text{hav}\,h.$

Then vers mer. Z.D. $= \text{vers}\,z - \text{vers}\,\theta.$

θ may be found thus:

$$\text{vers}\,\theta = 2\cos l \cdot \cos d \cdot \text{hav}\,h,$$
$$\therefore\ \text{hav}\,\theta = \cos l \cdot \cos d \cdot \text{hav}\,h.$$

The meridian zenith distance at the place of observation being thus found, the latitude may be obtained by applying the declination as in the "Latitude by meridian altitude" problem.

The true bearing of the heavenly body being observed or obtained from the azimuth tables, the ship is *actually* on a "line of position," passing through the position obtained with the assumed longitude and the latitude calculated from it, *at right angles to the bearing of the body*, as the longitude by account may not be actually correct. If another line of position or the bearing of a known point of land, etc., can be obtained, the position can be accurately fixed. Otherwise the latitude obtained must be considered as approximately correct, with sufficient accuracy for the purposes of navigation, a correction for the "run" giving the latitude at noon.

N.B.—Care must be taken always to subtract vers θ from vers z. No mistake can be made if it is remembered that the meridian zenith distance at a place *must* be less than any other.

Example.—On May 24th, in latitude by account 47° N, longitude 23° 53′ W, the true altitude of the sun's centre near the meridian was 64° 25′ (zenith N of the sun), when a chronometer showed 11 h. 47 m. 25 s., its error on G.M.T. being 1 h. 24 m. 37 s. slow. Required the latitude.

	h. m. s.		Declination.	Change.
Chronometer, - - -	11 47 25		20° 46′ 38″ N	27·9″
error, - - - - -	1 24 37		33	1·2
	13 12 2		20 47 11	33·48
	12 0 0			
G.M.T., - - - - -	25 12 2	23rd.	Equation of time.	Change.
or	1 12 2	24th.	m. s.	s.
Long., - - - - -	1 35 32	W	3 24·9	·225
			·3	1·2
S.M.T., - - - - -	23 36 30		3 24·6	·270
Equation of time, - -	3 25			
S.A.T., - - - - -	23 39 55		Altitude, - -	64° 25′
	24 0 0			90 0
Sun's hour angle from the meridian, - - -	20 5		zen. dist., - -	25 35

Estimated lat.,	47°	cos 9·833783	
declination,	20° 47′ 11″	cos 9·970767	$\theta = 4°\ 0'\ 40''$
hour angle,	20 m. 5 s.	hav 7·282968	
		hav θ, 7·087518	

vers zen. dist.,	0098042	Mer. zen. dist.,	25° 15′ 25″	N
vers θ,	0002446	dec.,	20 47 11	N
vers mer. zen. dist.,	0095596	Lat.,	46 2 36	N
	45			
	51			

This result differs by 57½ miles from the latitude by account. Working again, therefore, with latitude 46° 2′ as estimated latitude, we have

cos lat.,	9·841509		
cos dec.,	9·970767	$\theta = 4°\ 2'\ 45''$	
hav hour angle,	7·282968		
	hav θ, 7·095244		

vers zen. dist.,	0098042	Mer. zen. dist.,	25° 15′ 1″	N
vers θ,	0002494	dec.,	20 47 11	N
	0095548	Lat.,	46 2 12	N
	45			
	3			

An error, therefore, of nearly a degree in the estimated latitude produces an error of less than half a mile in the latitude deduced from it. This shows the value of the method, the correct latitude used in composing the example being 46° 2′.

§ 109. Care must be taken to obtain a correct Greenwich date, the 12 hours being added, if necessary, to the time obtained when the chronometer error has been applied, *before* the longitude has been applied.

The application of the longitude should produce a *ship mean time* a little under 24 hours or a little over 0 hours or 24 hours; otherwise the Greenwich time must be 12 hours wrong, *e.g.* it would not be correct in the foregoing example to find the sun's hour angle thus

	h.	m.	s.	
	11	47	25	
	1	24	37	
G.M.T.,	13	12	2	
Long.,	1	35	32	
	11	36	30	
	12	0	0	
S.M.T.,	23	36	30	etc.

For although the same *SMT* is arrived at, the *GMT* is quite wrong, and the declination and equation of time deduced from it would be quite wrong also.

As *SMT* is being deduced from *GMT*, west longitude must be *subtracted* and east longitude *added*.

§ 110. If any other heavenly body is observed, the hour angle is to be found from the expression,

$$\text{hour angle} = SMT + \text{R.A. mean sun} - \text{R.A.},$$

the *RA* and the *RA* mean sun being of course corrected for the Greenwich date.

If a heavenly body be observed below the pole, the same method is to be used, with the substitution of 90°+alt. instead of zenith distance and 12 h.−hour angle for hour angle.

To find the Latitude by observations near the Meridian.

§ 111. (II.) *Without using the estimated latitude.*

From X drop XM perpendicular to PZ, or PZ produced.

Let $PM = x$,

$ZM = y$.

Then co-lat. $= x - y$ or $x + y$,

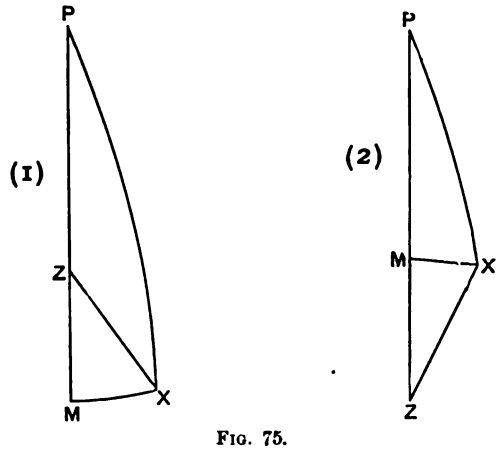


FIG. 75.

Let $PX = 90° \pm d = p$.

$ZX = 90° - \text{alt.} = z$.

$XPZ = \text{hour angle} = h$.

In right-angled triangle MPX

$$\cos h = \cot p \,.\, \tan x,$$

$$\therefore \quad \tan x = \cos h \,.\, \tan p \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also $$\cos p = \cos x \,.\, \cos XM,$$

$$\therefore \cos XM = \cos p \,.\, \sec x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In right-angled triangle MZX

$$\cos z = \cos y \,.\, \cos XM$$

$$= \cos y \,.\, \cos p \,.\, \sec x\,; \quad \text{from (2)},$$

$$\therefore \cos y = \cos z \,.\, \sec p \,.\, \cos x \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equations (1) and (3) determine x and y. In order to avoid the ambiguity in the signs of $\tan p$, $\tan x$, etc., let $90 \sim x = k$, and $90 \sim p = d$. Then, x and p being either both less or both greater than 90°,

(1) becomes $\cot k = \cos h \,.\, \cot d.$

(3) „ $\cos y = \sin \text{alt} \,.\, \operatorname{cosec} d \,.\, \sin k,$

and latitude $= k \pm y.$

In this case k does not differ much from d, and y is nearly the meridian zenith distance. This, or the construction of a figure, will show which sign is to be taken.

Working the foregoing example by this method we have

cot d	10·420680			sin alt.,	9·955186		
cos h	9·998330			cosec d	10·449908		
	10·419010			sin k	9·551549		
					9·956643		
k =	20°	51′	35″ N	y =	25°	10′	45″ N
y =	25	10	45 N				
Lat.,	46	2	20 N				

a result which nearly agrees with that found by the first method.

§ 112. The above method is not very reliable when the latitude and declination are small. In this case the logs of cot d, cosec d, cot k, and sin k all change rapidly, while those of cos y and sin alt. change very slowly. Hence a small error in the declination or the taking out the logs of d and k to the nearest 15″ may produce a large error in y, and, therefore, in the latitude.

Example.--Suppose that on April 2nd, 1894, in *correct* latitude 15° N, longitude 80° E, the true altitude of the sun's centre uear the meridian was 79° 39′ 0″ when the sun's hour angle from the meridian was 10 m.

	h. m.		Declination.	Change.
Ship app. time, - -	23 50	April 1st.	5° 1′ 57″ N	57·6″
	5 20		5 17	5·5
Greenwich app. time, -	18 30	April 1st.	4 56 40	316·8
	5 30			

	(1) *k* and *d* to nearest 15″.	(2) *k* and *d* accurate.
cos *h*	9·999586	9·999586
cot *d*	11·062803	11·062925
	11·062389	11·062511
	k = 4° 57′	*k* = 4° 56′ 57″
cosec *d*	11·064423	11·064545
sin *k*	8·935942	8·935869
sin alt.,	9·992875	9·992875
	9·993240	9·993289
y =	10° 4′ 45″	10° 2′ 45″
k =	4 57 0	4 56 57
Lat.,	15 1 45	14 59 42

i.e. neglect of 5″ in the declination and 3″ in *k* produces an error of 2 miles in the resulting latitude.

As the latitude decreases the error increases rapidly, a latitude of 15° being assumed to show that the error is appreciable even when the latitude is not very small.

Reduction to the Meridian.

§ 113. The work of finding the meridian altitude from the observed altitude in ex-meridian observations may be somewhat shortened by what is called the "Reduction to the meridian," which may be found in various ways, tables being calculated in order to further reduce the work.

In the triangle *PZS* (see fig. 74),

$$\cos ZS = \cos PZ \,.\, \cos PS + \sin PZ \,.\, \sin PS \,.\, \cos ZPS$$
$$= \cos PZ \,.\, \cos PS + \sin PZ \,.\, \sin PS\,(1 - \text{vers } ZPS),$$
$$\cos z = \cos (PZ \sim PS) - 2 \cos l \,.\, \cos d \,.\, \text{hav } h$$
$$= \cos \text{ mer. zen. dist.} - 2 \cos l \,.\, \cos d \,.\, \text{hav } h$$
$$= \cos (z - x) - 2 \cos l \,.\, \cos d \,.\, \text{hav } h$$

where x is the correction to be applied to ZS to obtain the meridian zenith distance.

$$\therefore \quad \cos(z-x)-\cos z = 2\cos l \,.\cos d \,.\operatorname{hav} h,$$

$$\therefore \quad 2\sin\left(z-\frac{x}{2}\right)\sin\frac{x}{2} = 2\cos l \,.\cos d \,.\operatorname{hav} h;$$

or, since x is a small arc,

$$x\sin 1'' = \frac{2\cos l \,.\cos d \,.\operatorname{hav} h}{\sin z}$$

or,
$$x\,(\text{in seconds}) = \frac{2\cos l \,.\cos d \,.\operatorname{hav} h}{\sin z \,.\sin 1''}.$$

Hence add together the logs of $\cos l$, $\cos d$, $\operatorname{cosec} z$, $\operatorname{hav} h$, and the constant quantity 5·615455, which is the log of $\frac{2}{\sin 1''}$.

§ 114. If the table of log haversines is not at hand, the expression may be adapted to the table of ordinary log sines as follows:

$$\text{since haversine } h = \frac{1-\cos h}{2} = \sin^2\frac{h}{2},$$

the expression may be written,

$$x = \frac{2\cos l \,.\cos d}{\sin z \,.\sin 1''} \,.\sin^2\frac{h}{2}$$

$$= \frac{\cos l \,.\cos d}{\sin z \,.\sin 1''} \,.\frac{\sin^2 h}{2}$$

$$\left(\text{since } \sin^2\frac{h}{2} = \frac{\sin^2 h}{4}, \text{ nearly, } h \text{ being a small angle}\right).$$

In this case add together the logs of $\cos l$, $\cos d$, $\operatorname{cosec} z$, twice the log of $\sin h$, and the constant quantity 5·013400, which latter is the arithmetical complement of log 2 sin 1″.

This will give the same result as before, the log of a natural number of seconds of arc, which, *subtracted* from the observed zenith distance, will give the meridian zenith distance, to which the declination is to be applied in the usual manner.

§ 115. The expression may also be written

$$x = \frac{2\cos l \,.\cos d}{\sin(l \pm d)\sin 1''} \cdot \left(\frac{h\sin 15'}{2}\right)^2,$$

since h is small, and z is nearly equal to the meridian zenith distance,

$$\therefore \; x = \frac{\cos l \,.\cos d \,.\sin^2 15'}{2\sin(l \pm d)\sin 1''} \,.h^2$$

$$= C \,.h^2,$$

where the quantities in C may be considered, without appreciable error, to have their meridian values. Tables are calculated giving the values of C for different latitudes and declinations. The reduction may then be found by multiplying the square of the number of minutes in the hour angle from noon by the requisite value of C.

Example.—With the latitude, declination, etc., already used in the example of finding latitude by ordinary ex-meridian method, we have

Estimated lat., - -	47°	cos 9·833783
declination, - -	20° 47′ 11″	cos 9·970767
hour angle, - -	20 m. 5 s.	hav 7·282968
zen. dist., - - -	25° 35′	cosec 10·364694
		5·615455
		3·067667
		6,0)116,9
		19′ 29″
zen. dist., - - -	25° 35′ 0″	
reduction, - -	19 29	
mer. zen. dist., - -	25 15 31	

which. agrees within 6″ with that obtained in the example.

Examples.—(1) June 3rd., in latitude by account 48° 20′ N, longitude 67° 13′ W, the obs. alt. sun's L.L. near the meridian was 63° 45′ 50″, when a chronometer whose error was 1 h. 6 m. 16 s. fast on G.M.T. showed 5 h. 18 m. 25 s. Index error, 2′ 50″ − ; height of eye, 20 feet.

(2) June 29th, in latitude by account 19° 50′ S, longitude 58° 25′ E, the obs. alt. sun's L.L. near the meridian was 46° 27′ 30″, when a chronometer whose error was 2 h. 43 m. 19 s. slow on G.M.T. showed 5 h. 8 m. 31 s. Index error, 1′ 30″ + ; height of eye, 24 feet.

(3) December 2nd, in latitude by account 47° 55′ S, longitude 179° 15′ E, the obs. alt. sun's L.L. near the meridian was 63° 41′ 30″, when a chronometer whose error was 2 h. 13 m. 35 s. fast on G.M.T. showed 1 h. 48 m. 21 s. Index error, 3′ 10″ − ; height of eye, 21 feet.

(4) June 5th, in latitude by account 54° 1′ N, longitude 4° 52′ W, the obs. alt. sun's L.L. near the meridian was 58° 13′ 20″, when a chronometer whose error was 1 h. 25 m. 59 s. fast on G.M.T. showed 1 h. 28 m. 29 s. Index error, 2′ 40″ + ; height of eye, 21 feet.

(5) April 25th, in latitude by account 35° S, longitude 20° 13′ E, the obs. alt. sun's L.L. near the meridian was 41° 22′ 50″, when a chronometer whose error was 2 h. 47 m. 29 s. slow on G.M.T. showed 8 h. 7 m. 56 s. Index error, 50″ − ; height of eye, 19 feet.

(6) April 26th, about 7 h. 55 m. P.M. apparent time, in latitude by account 54° 30′ N, longitude 3° 41′ W, the obs. alt. α Leonis near the meridian was 48° 8′, when a chronometer whose error was 26 m. 51 s. slow on G.M.T. showed 7 h. 40 m. 26 s. Index error, 2′ 50″ − ; height of eye, 18 feet.

(7) December 10th, about 7 h. 55 m. P.M. apparent time, in latitude by account 37° 20′ S, longitude, 115° 10′ E, the obs. alt. α Eridani near the meridian was 68° 56′ 50″, when a chronometer whose error was 2 h. 33 m. 44 s. fast on G.M.T. showed 2 h. 41 m. 16 s. Index error, 2′ 20″ + ; height of eye, 23 feet.

(8) December 5th, about 4 h. 20 m. A.M. apparent time, in latitude by account 41° 50′ N, longitude 47° 25′ W, the obs. alt. Jupiter's centre near the meridian was 66° 4′ 20″, when a chronometer whose error was 2 h. 17 m. 23 s. fast on G.M.T. showed 9 h. 38 m. 18 s. Index error, 1′ 40″ + ; height of eye, 20 feet.

Latitude by Double Altitude.

In this problem the latitude is obtained from two altitudes of the same or different heavenly bodies.

There are three cases to be considered:

§ 116. (I.) *When the same heavenly body is observed at different times.*

In this case let X and Y be the positions of the heavenly body. Then the polar angle XPY is evidently the difference

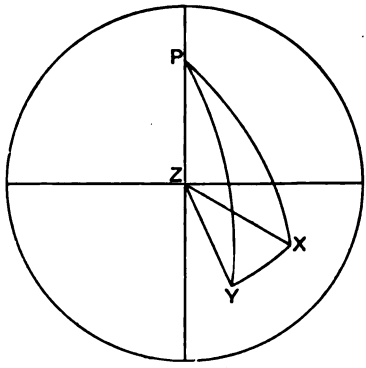


FIG. 76.

of the hour angles of X and Y. In the case of the sun this is the difference of the chronometer times, corrected for the change in the equation of time in the interval, if appreciable, and, if necessary, for the rate of the chronometer. These

corrections are not as a rule required. In the case of a star the elapsed time by chronometer should, for great accuracy, be turned into sidereal time.

§ 117. (II.) *When two heavenly bodies are observed at the same time.*

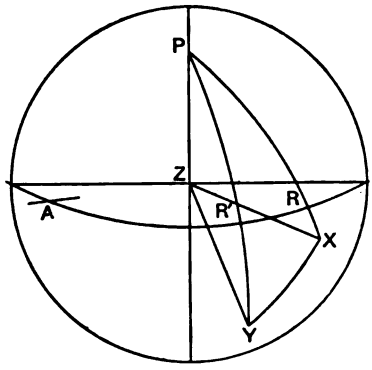


FIG. 77.

In this case the angle XPY is the measure of the arc RR', and is therefore equal to the difference of the right ascensions of the two bodies.

§ 118. (III.) *When two heavenly bodies are observed at different times.*

CASE I.—Let the easternmost star be first observed, as X in fig. 78 (1). After an interval, during which X has moved to X',

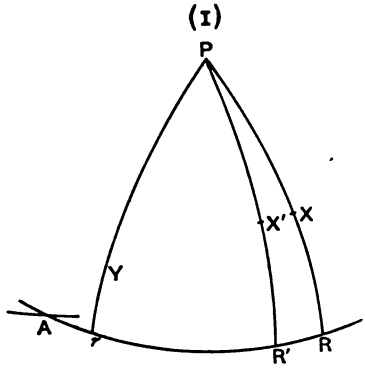


FIG. 78 (1).

let Y be observed. Let A be the position of the first point of Aries at the time of the second observation, and let AQ be the equator.

Draw the declination circles through X, etc., to meet the equator in R, R', r, then

AR' = right ascension of the body first observed.

Ar = „ „ second „

$R'R$ = sidereal interval

= elapsed time, as measured by chronometer, expressed in sidereal time.

Arc Rr = angle RPr

= polar angle required.

$Rr = AR' - Ar + RR'$

$= AR' + RR' - Ar$, ...(1)

CASE II.—Let the westernmost star be first observed, as Y in fig. 78 (2).

After Y has moved to Y', let X be observed.

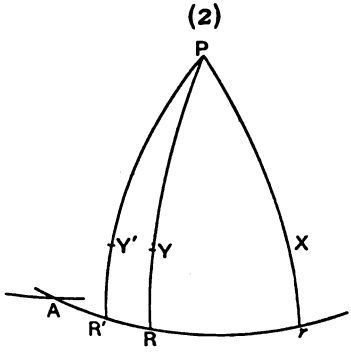


FIG. 78 (2).

Then Rr = angle RPr

= polar angle required.

$Rr = Ar - AR' - R'R$ .....(2)

$= Ar - (AR' + R'R)$ .....(3)

From expressions (1) and (3) we obtain a rule applicable to both cases, viz. "To the right ascension of the body first observed add the elapsed time expressed in sidereal time, and take the difference between the sum and the right ascension of the body last observed." The result is the polar angle required.

If the interval in time is sufficiently large for the rate of the chronometer to have any effect, the rate must be allowed

for, and the elapsed time is to be understood to mean "the difference of the chronometer times, corrected for the rate if necessary."

N.B.—The expression in the form (2) shows that it is *not* correct to say "add the elapsed time, expressed in sidereal time, to the difference of the right ascensions of the bodies observed."

Examples.—To find the polar angle when two heavenly bodies are observed at different times.

	Right ascension.	Time by chronometer.
	h. m. s.	h. m. s.
(1) α Pegasi, - - -	22 59 28	7 35 10
α Aquilæ, - - -	19 45 35	8 2 3
	h. m. s.	h. m. s.
(2) α Cygni, - - -	20 37 53	6 14 32
Vega, - - - -	18 33 26	6 25 2

(3) The stars Aldebaran (R.A. 4 h. 29 m. 50 s.) and Sirius (R.A. 6 h. 40 m. 29 s.) having been observed for double altitude, it was found that the polar angle was 2 h. 3 m. 54 s. What interval of mean time elapsed between the observations, and which was the star first observed?

NOTE.—If two observations of a planet are made, its R.A. must be calculated for the time of each observation, and then the rule applies as if two bodies had been observed at different times.

§ 119. The polar angle XPY having been determined by one of the foregoing methods, the solution proceeds as follows, by the ordinary methods of Spherical Trigonometry.

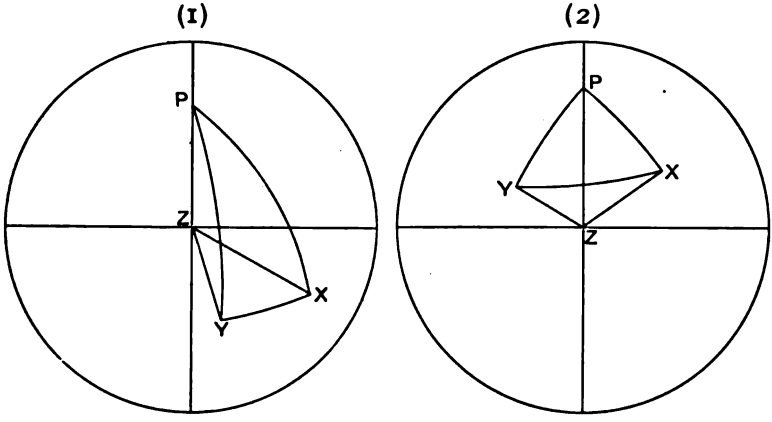


FIG. 79.

Join XY by the arc of a great circle, which, as in fig. 79 (1), would not pass between P and Z, but, as in fig. 79 (2), would do so, PX, PY, ZX, ZY, and the angle XPY are known.

In the spherical triangle PXY, with PX, PY, and the included angle XPY, find XY the third side; denoted arc (1).

In the same triangle with PX, XY, and PY, find the angle PXY; denoted arc (2).

In the triangle ZXY, with ZX, XY, and ZY, find the angle ZXY; denoted arc (3).

Then the angle PXZ will be the difference or the sum of arcs (2) and (3), according as XY would not or would pass between P and Z, and is denoted arc (4).

Finally, in the triangle PXZ are given PX, XZ, and the included angle PXZ, to find PZ, the co-latitude, and thence the latitude.

The same result would be arrived at by finding the angles at Y.

The question as to whether XY produced, if necessary, will or will not pass between the zenith and the pole can always be settled at the time of observation. In examples for practice the bearings are given, by means of which a figure may be drawn and the question settled.

Correction for Run.

§ 120. If the ship's position has changed during the time that has elapsed between the observations, a correction, called the "Correction for Run," must be applied to the first altitude in order to reduce it to what it would have been had it been taken at the same place as the second altitude.

The problem is then reduced to that of finding the latitude from two altitudes taken at the same place.

The correction may, of course, be applied to the second altitude to reduce it to an altitude taken at the place of the first observation, but this is seldom necessary, as the position at the place of the second observation is that most usually required.

The correction is obtained as follows:

Let Z be the zenith of the first place of observation, S the place of the sun when first observed, Z' the zenith of the second place of observation, PZP' the meridian. ZZ' is then the run of the ship in the interval, as the arc described on the surface of the earth must be measured by the arc on the celestial concave between the two positions of the zenith.

With centre S and distance SZ' describe a circle cutting ZS or ZS produced at D.

Then $ZD = ZS \sim ZS'$

= correction for run;

PZS = bearing of the sun at the first observation;

PZZ' = ship's course in the interval.

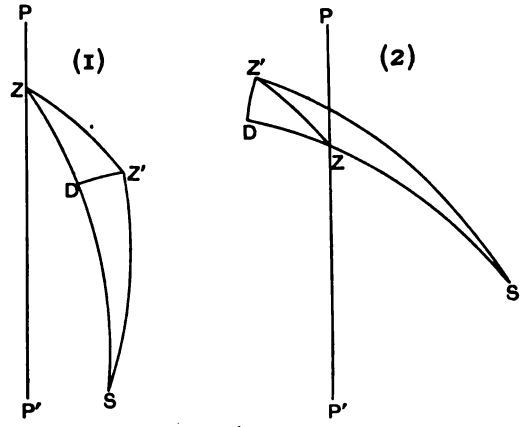


FIG. 80.

Let θ = the angle between these two directions.

Then $Z'ZD = \theta$ in (1),

and $= 180° - \theta$ in (2).

The triangle $Z'ZD$ may be considered as a plane right-angled triangle, ZZ' and ZD being, at largest, very small arcs of circles.

Then $ZD = ZZ' . \cos Z'ZD$

= distance run $\times \cos \theta$, in (1),

or = distance run $\times \cos (180° - \theta)$, in (2).

Now the diff. lat. column in the traverse table is simply the result obtained by multiplying the distance at the top by the cosine of the angle at the side. The traverse table can thus be used to find the correction by the following rule:

"With distance run as distance and angle between the bearing of the sun at the first observation and the course of the ship, or 180° − this angle when over 90° or 8 points, as a course, take out the diff. lat."

This will be the required correction, which the figures show to be subtractive from the first zenith distance or additive to the first altitude when θ is less than 90°, and additive to the first zenith distance or subtractive from the first altitude when θ is greater than 90°.

§ 121. If the ship has altered her course in the interval, the course and distance made good must be found in the usual way, and then the true bearing of the sun at the first observation must be also found by correcting the observed bearing for variation and deviation.

Otherwise the difference is to be taken between the compass bearing and the compass course, as they are equally affected by the compass error.

Considering the expression

$$\text{correction} = \text{distance run} \times \cos\theta,$$

we see that when $\theta = 0$ the correction is equal to the distance run and is additive to the first altitude, as the ship has run directly towards the first position of the sun.

As θ increases the correction decreases, and vanishes when $\theta = 90°$, the ship in this case having run neither towards nor away from the first position of the sun.

When θ is greater than 90° the correction increases, with a changed sign, as θ increases, until when $\theta = 180°$ the correction is again equal to the distance run, and is subtractive from the first altitude, as the ship has run directly away from the first position of the sun.

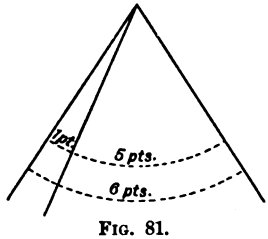


FIG. 81.

Examples.—(1) The bearing of the sun at the first observation being SEbE, and the run of the ship in the interval being SbW, 40 miles; find the correction for run.

SEbE = 5 points.
SbW = 1 „
$\theta = 6$ „

With course 6 points and distance 40′, the traverse table gives 15·3 as the diff. lat.

The correction for run is therefore 15′ 18″, to be added to the first altitude.

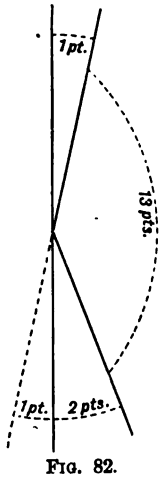


FIG. 82.

(2) The bearing of the sun at the first observation being SSE, and the run of the ship in the interval NbE, 35 miles; find the correction for run.

SSE = 2 points.
NbE = 1 „
3
16
$\theta = 13$ „
$\therefore\ 180° - \theta = 3$ „

With course 3 points and distance 35′, the traverse table gives 29·1 as the diff. lat.

The correction for run is therefore 29′ 6″, to be subtracted from the first altitude.

Examples. Correction for Run.

(1) The sun bore S 41° E at the first observation, and the run of the ship in the interval was S 29° W, 27 miles.

(2) The sun bore S 87° E at the first observation, and the run of the ship in the interval was N 16° W, 24 miles.

(3) The sun bore S 35° W at the first observation, and the run of the ship in the interval was N 75° E, 29 miles.

(4) The sun bore N 75° E at the first observation, and the run of the ship in the interval was S 63° E, 32 miles.

(5) If a ship's run between two observations was S 75° E, 21 miles, and the correction for run was 13·5′ subtractive from the first altitude, what was the bearing of the sun at the first observation?

(6) If the true bearing of the sun at the first observation was S 75° E, and the correction for run was 9·4′ subtractive from the first altitude what was the ship's course, the distance run being 24 miles?

Example.—Case I. December 24th, in latitude by account 46° 50′ N, longitude 9° W, the following double altitude of the sun was observed:

Ship app. time, nearly.	Chronometer time.	Obs. alt. sun's L.L.	Bearing.
h. m.	h. m. s.		
11 15 A.M.	2 41 7	18° 20′ 40″	SbE
2 35 P.M.	6 1 15	11 39 20	SWbS

The run of the ship in the interval was SSW 30 miles, the index error was 2′ 10″+, and the height of the eye 20 feet. Required the latitude.

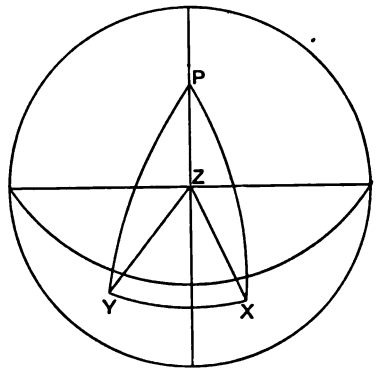


Fig. 83.

To find the polar angle.

h.	m.	s.
6	1	15
2	41	7
3	20	8 = *XPY*.

h.	m.	
23	15	Dec. 23rd.
	36	Long.
23	51	Greenwich app. time.

h.	m.	
2	35	Dec. 24th.
	36	Long.
3	11	Greenwich app. time.

Declination.			
23°	25′	53·8″	S
		·5	
23	25	54	
90	0	0	
113	25	54	$=PX$

Declination.			
23°	25′	53·8″	S
		10·8	
23	25	43	
90	0	0	
113	25	43	$=PY$

To find ZX.

18°	20′	40″	
	2	10	
18	22	50	
	4	24	
18	18	26	
	2	47	
18	15	39	
	16	18	
18	31	57	
	24	54	cor. for run.
18	56	51	
90	0	0	
71	3	9	$=ZX$

To find ZY.

11°	39′	20″	
	2	10	
11	41	30	
	4	24	
11	37	6	
	4	28	
11	32	38	
	16	18	
11	48	56	
90	0	0	
78	11	4	$=ZY$

To find the correction for run.

Bearing, SbE.

Ship's course, SSW.

Difference, 3 points.

Correction, 24·9′, to be added to the first altitude, as the difference is less than 8 points.

(1) *To find XY.*

PX	113° 25′ 54″	sin 9·962617
PY	113 25 43	sin 9·962631
diff.	11	
XPY = 3 h. 20 m. 8 s.		hav 9·252438

hav θ 9·177686

$\theta = 45° 39' 45''$

As the difference of PD in the case of the sun is always very small, θ is practically XY.

$\therefore \quad XY = 45° 39' 45''$

(2) *To find PXY.*

PX	113° 25′ 54″	cosec 10·037383
XY	45 39 45	cosec 10·145551
	67 46 9	
PY	113 25 43	
	181 11 52	½ hav 4·999976
	45 39 34	½ hav 4·588825

hav PXY 9·771735

$PXY = 100° 30' 30''$

(3) *To find ZXY.*

ZX	71° 3′ 9″	cosec 10·024194
XY	45 39 45	cosec 10·145551
	25 23 24	
ZY	78 11 4	
	103 34 28	½ hav 4·895269
	52 47 40	½ hav 4·647961

hav ZXY 9·712975

$ZXY = 91° 52' 45''$

(4) *To find PXZ.*

PXY =	100° 30′ 30″
ZXY =	91 52 45
PXZ =	8 37 45

(5) *To find PZ.*

PX	113° 25′ 54″	sin 9·962617
ZX	71 3 9	sin 9·975806
diff.	42 22 45	

$PXZ = 8° 37′ 45″$ hav 7·752811

hav θ		7·691234
θ	$= 8° 2′ 30″$	
vers θ		0009813
		20
vers diff.		0261152
		146
vers PZ		0271131
		031
		100
PZ	$= 43° 12′ 30″$	

(6) *To find the latitude.*

PZ	$= 43° 12′ 30″$
	90 0 0
Latitude	46 47 30 N

Formula used in finding XY, and similarly for PZ, in (1) and (5)

$$\text{vers } XY = \text{vers } (PX \sim PY) + \text{vers } \theta,$$

where hav $\theta = \sin PX \,.\, \sin PY \,.\, \text{hav } XPY$.

Formula used in finding PXY, and similarly for ZXY, in (2) and (3)

$$\text{hav } PXY = \text{cosec } PX \,.\, \text{cosec } XY \sqrt{\text{hav}(PY + PX \sim XY)\, \text{hav}(PY - PX \sim XY)}.$$

Note.—If tables containing versines and haversines are not at hand, XY may be found from the formula

$$\sin \frac{XY}{2} = \sqrt{\sin\left(\frac{PX+PY}{2} + \theta\right) \sin\left(\frac{PX+PY}{2} - \theta\right)}$$

where $\sin \theta = \sqrt{\sin PX \,.\, \sin PY \,.\, \cos^2 \frac{XPY}{2}}$,

and PXY from the formula

$$\sin^2 \frac{PXY}{2} = \text{cosec } PX \,.\, \text{cosec } XY \,.\, \sin \frac{PY + PX \sim XY}{2} \sin \frac{PY - PX \sim XY}{2}.$$

Case II.—June 10th, in latitude by account 35° S, the following simultaneous double altitude was observed (height of eye, 24 feet):

	Observed altitude.	Index error.	Bearing.
Achernar, - -	58° 24′ 10″	−2′ 50″	SWbS
Canopus, - -	57 37 10	+2 5	SE

Achernar, R.A., - - - - - -	1 h. 33 m. 49 s.
„ dec., - - - - - -	57° 45′ 45″ S
Canopus, R.A., - - - - - -	6 h. 21 m. 36 s.
„ dec., - - - - - -	52° 38′ 15″ S

To find the polar angle.

h. m. s.		
6 21 36	90° 0′ 0″	90° 0′ 0″
1 33 49	57 45 45	52 38 15
APC = 4 47 47	32 14 15 = PA	37 21 45 = PC

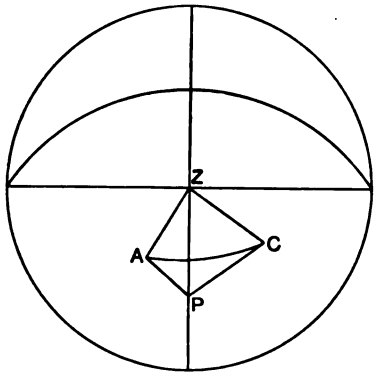


FIG. 84.

To find ZA.	*To find ZC.*
58° 24′ 10″	57° 37′ 10″
2 50	2 5
58 21 20	57 39 15
4 49	4 49
58 16 31	57 34 26
36	37
58 15 55	57 33 49
90 0 0	90 0 0
31 44 5 = ZA	32 26 11 = ZC

(1) *To find CA.*

PC	37° 21′ 45″	sin 9·783085	vers θ = 0223403
PA	32 14 15	sin 9·727077	vers. diff. 0003985
diff.	5 7 30		13
	h. m. s.		vers CA 0227401
APC =	4 47 47	hav 9·537872	266
		hav θ 9·048034	135
	θ = 39° 3′		CA = 39° 24′ 45″

(2) *To find PCA.*

PC	37° 21′ 45″	cosec 10·216915
CA	39 24 45	cosec 10·197295
	2 3 0	
PA	32 14 15	
	34 17 15	½ hav 4·469483
	30 11 15	½ hav 4·415640
		hav PCA 9·299333
	PCA = 53° 1′ 0″	

(3) *To find ZCA.*

ZC	32° 26′ 11″	cosec 10·270528
CA	39 24 45	cosec 10·197295
	6 58 34	
ZA	31 44 5	
	38 42 39	½ hav 4·520388
	24 45 31	½ hav 4·331188
		hav ZCA 9·319399
	ZCA =	54° 21′ 30″
	PCA =	53 1 0

(4) ∴ PCZ = 107 22 30

(5) *To find PZ.*

PC	37° 21′ 45″	sin 9·783085	vers θ 0422617
ZC	32 26 11	sin 9·729472	118
diff.	4 55 34		vers diff. 0003680
PCZ	107 22 30	hav 9·812454	13
		vers θ 9·325011	vers *PZ* 0426428
θ =	54° 44′ 30″		24
		PZ = 55° 0′ 1″	4
		90 0 0	
	Latitude,	34 59 59 S	

Case III.—June 5th, in latitude by account 56° N, the following double altitude was observed:

	Observed altitude.	Chronometer time.	Bearing.
α Cygni, - -	53° 45′ 40″	6 h. 14 m. 32 s.	E ¾ N
Vega - -	68 26 30	6 h. 25 m. 2 s.	SEbS

Index error, 2′ 30″ − ; height of eye, 21 feet.

Vega, R.A., - - -	18 h. 33 m. 25·5 s.	
„ dec., - - -	38° 41′ 0″ N	∴ *PV* = 51° 19′
α Cygni, R.A., - - -	20 h. 37 m. 53 s.	
„ dec., - - -	44° 54′ 0″ N	∴ *PC* = 45 6

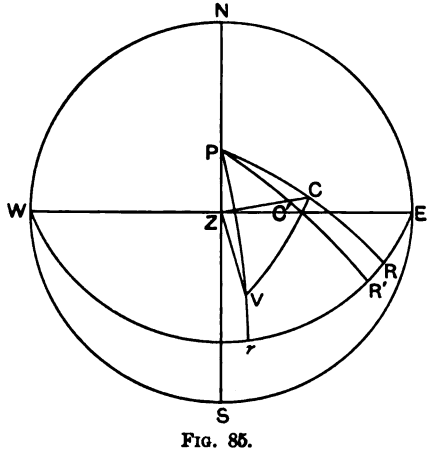


Fig. 85.

To find CPV, the polar angle.

	h. m. s.		h. m. s.	
Chron. time, -	6 14 32	R.A. α Cygni,	20 37 53	
„ „ -	6 25 2		10 1·6	elapsed time expressed
			30·1	in sidereal time.
Elapsed „ -	10 30		20 48 24·7	
		R.A. Vega,	18 33 25·5	
			2 14 59	= *CPV*

To find ZC.

53° 45′ 40″
2 30

53 43 10
4 31

53 38 39
43

53 37 56
90 0 0

36 22 4 = *ZC*

To find ZV.

68° 26′ 30″
2 30

68 24 0
4 31

68 19 29
23

68 19 6
90 0 0

21 40 54 = *ZV*

(1) *To find CV.*

PC 45° 6′ sin 9·850242
PV 51 19 sin 9·892435

diff. 6 13

CPV = 2 h. 14 m. 59 s. hav 8·925544

8·668221

vers θ 0093078
90
vers diff. 0005880

0099048

CV = 25° 43′ 0″

θ = 24° 55′ 45″

(2) *To find PCV.*

PC 45° 6′ cosec 10·149758
CV 25 43 cosec 10·362589

19 23
PV 51 19

70 42 ½ hav 4·762356
31 56 ½ hav 4·439456

hav *PCV* 9·714159

PCV = 92° 2′ 15″
ZCV = 40 9 0

∴ *PCZ* = 51 53 15 (4)

(3) *To find ZCV.*

ZC 36° 22′ cosec 10·226982
CV 25 43 cosec 10·362589

10 39
ZV 21 41

32 20 ½ hav 4·444720
11 2 ½ hav 3·982883

hav *ZCV* 9·017174

ZCV = 40° 9′

(5) *To find PZ.*

PC = 45° 6′ 0″ sin 9·850242
ZC = 36 22 0 sin 9·773018

diff. 8 44 0
PCZ = 51 53 15 hav 9·281933

hav θ 8·905193

θ = 32° 56′ 30″

vers θ 0160696
78
vers diff. 0011594

vers *PZ* 0172368
266

102

PZ = 34° 8′ 37″
90 0 0

Latitude, 55 51 23 N

Examples.—(1) April 30th, in latitude by account 45° N, longitude 140° 30′ W, the following double altitude of the sun was observed:

App. time, nearly.	Chronometer time.	Obs. alt. sun's L.L.	True bearing.
h. m.	h. m. s.		
2 12 P.M.	2 5 7	48° 50′ 30″	S 55° W
4 49 P.M.	4 41 25	23 2 40	West

The run of the ship in the interval was South (true), 6 miles per hour. Index error, 3′ 20″+; height of eye, 20 feet.

(2) June 3rd, in latitude by account 50° 30′ N, longitude 72° E, the following double altitude of the sun was observed:

App. time, nearly.	Chronometer time.	Obs. alt. sun's L.L.	True bearing.
h. m.	h. m. s.		
9 55 A.M.	7 43 27	51° 18′ 10″	S 50° E
11 15 A.M.	9 2 26	59 31 40	S 20° E

The run of the ship in the interval was N 79° W (true), 10 miles per hour. Index error, 1′ 10″−; height of eye, 18 feet.

(3) April 11th, in latitude by account 57° N, longitude 10° 30′ E, the following double altitude of the sun was observed:

App. time, nearly.	Chronometer time.	Obs. alt. sun's L.L.	True bearing.
h. m.	h. m. s.		
11 0 A.M.	10 21 28	40° 15′ 20″	S 20° E
2 0 P.M.	1 22 31	35 22 10	S 38° W

The run of the ship in the interval was E ½N (true), 11 miles per hour. Index error, 2′ 50″−; height of eye, 21 feet.

(4) June 29th, in latitude by account 33° S, longitude 17° E, the following double altitude of the sun was observed:

App. time, nearly.	Chronometer time.	Obs. alt. sun's L.L.	True bearing.
h. m.	h. m. s.		
8 10 A.M.	11 14 46	11° 38′ 50″	N 52° E
11 35 A.M.	2 39 36	33 26 30	N 12° E

The run of the ship in the interval was N 57° W, 10 miles per hour. Index error, 1′ 50″−; height of eye, 22 feet.

(5) December 27th, in latitude by account 36° 30′ N, the following simultaneous double altitude was observed:

Star.	Obs. alt.	Index error.	True bearing.
β Orionis, -	- 31° 25′ 30″	2′ 10″−	S 50° W
α Hydræ, -	- 40 16 10	2 40 +	S 40° E

Height of eye, 21 feet.

(6) June 15th, in latitude by account 41° 50′ N, the following simultaneous double altitude was observed:

Star.	Obs. alt.	Index error.	True bearing.
α Tauri, -	- 19° 31′ 20″	2′ 50″−	N 85° E
α Pegasi, -	- 62 43 40	3 10 +	S 15° E

Height of eye, 24 feet.

(7) June 5th, in latitude by account 37° N, the following double altitude was observed:

Star.	Obs. alt.	Chronometer time. h. m. s.	True bearing.
α Aquilæ, - -	19° 48′ 10″	7 14 26	S 86° E
α Scorpii, - -	24 53 0	7 29 48	S 17° E

Index error, 2′ 10″ −; height of eye, 24 feet; run of the ship in the interval, EbN (true), 12 miles per hour.

(8) December 17th, in latitude by account 34° 5′ S, the following double altitude was observed:

Star.	Obs. alt.	Chronometer time. h. m. s.	True bearing.
α Eridani, - -	65° 24′ 40″	6 57 54	S 13° E
α Piscis Australis,	59 36 50	7 18 27	S 87° W

Index error, 2′ 20″+; height of eye, 21 feet; run of the ship in the interval, NE (true), 12 miles per hour.

To find the latitude by observation of the Pole Star.

§ 122. It is shown (§ 183) that the latitude by altitude of the pole star is to be obtained from the expression

$$l = a - p\cos h + \frac{p^2 \sin^2 h}{2} \tan a \sin 1'',$$

a being the corrected altitude, p the polar distance, and h the hour angle.

To the corrected altitude, therefore, must be applied the corrections calculated from the above expression, and inserted in the *Nautical Almanac*, depending on the sidereal time of observation and the polar distance, 1′ being always subtracted from the corrected altitude, so as to allow the third correction to be always additive.

The work should be done in accordance with the example supplied in the *Nautical Almanac*, remembering, however, that on board ship the time kept is *apparent* time, which must be reduced to mean time by applying the equation of time to the nearest minute *before* the sidereal time is found.

The first correction being calculated for each 10 m., if the sidereal time found falls between two of the tabulated times the correction must be corrected for the interval. (The neglect of this is a frequent source of error in the working of examples.)

Example.—March 6th, in longitude 37° W, at about 7 h. 35 m. P.M. apparent time at ship, the observed altitude of the pole star was 46° 25′ 20″. Index error, 2′ 30″ −; height of eye, 20 feet. Required the latitude.

	h.	m.	s.	
ship apparent time,	7	35	0	
equation of time,		12	0+	
S.M.T., - -	7	47	0	
Long, - -	2	28	0	
Gr. date, - -	10	15	0	Mar. 6th.
obs. alt., - -	46°	25′	20″	
index error, -		2	30	
	46	22	50	
dip, - - -		4	24	
	46	18	26	
refraction, -			55	
	46	17	31	
		1	0	
Corrected alt., -	46	16	31	

	h.	m.	s.
R.A. mean sun, -	22	55	52
correction for 10 h.,		1	38·6
„ „ 15 m.,			2·4
	22	57	33
S.M.T., - - -	7	47	0
sidereal time, - -	6	44	33
corrected altitude, -	46°	16′	31″
1st correction, - -		11	34 –
	46	4	57
2nd „ - -			50+
3rd „ - -		1	14+
	46	7	1
Latitude, - -	46	7	0N

Some tables have a correction for the pole star inserted for ten year intervals, which give fairly correct results if care is taken to obtain a properly proportioned correction; but to obtain an accurate correction often involves more work than is expended in taking the separate corrections out of the *Nautical Almanac*.

Examples.—(1) April 5th, about 2 h. 15 m. A.M. apparent time, in longitude 67° 21′ W, the obs. alt. Polaris was 47° 45′ 30″. Index error, 2′ 20″+; height of eye, 21 feet.

(2) April 27th, about 11 h. 24 m. P.M. apparent time, in longitude 141° 30′ E, the obs. alt. Polaris was 40° 13′ 30″. Index error, 2′ 30″ –; height of eye, 24 feet.

(3) June 7th, about 9 h. 27 m. P.M. apparent time, in longitude 124° 10′ W, the obs. alt. Polaris was 46° 33′ 10″. Index error, 1′ 40″ +; height of eye, 22 feet.

(4) June 25th, about 1 h. 2 m. A.M. apparent time, in longitude 13° 53′ E, the obs. alt. Polaris was 40° 27′ 40″. Index error, 2′ 20″ +; height of eye, 25 feet.

(5) December 6th, about 11 h. 19 m. P.M. apparent time, in longitude 63° 19′ W, the obs. alt. Polaris was 32° 1′ 50″. Index error, 3′ 10″ +; height of eye, 18 feet.

(6) December 10th, about 1 h. 23 m. A.M. apparent time, in longitude 114° 11′ E, the obs. alt. Polaris was 21° 35′ 30″. Index error, 2′ 10″ –; height of eye, 19 feet.

Formulæ in "Latitude Problems," collected for reference.

(1) *Circumpolar body.*

lat. = ½ sum of mer. alts. above and below the pole (measured from the same point of the horizon).

(2) *Body below the pole.*

lat. = 90° + mer. alt. − dec.

(3) *Body above the pole.*

lat. = mer. zen. dist. $\pm$ dec.

(4) *Ex-meridian altitude (using estimated latitude).*

vers mer. zen. dist. = vers. obs. zen. dist. − vers θ;

where hav θ = cos est. lat. cos dec. hav hour angle;

then use (3).

If the body is below the pole, substitute 90° + mer. alt. for zen. dist., and 12 h. − hour angle for hour angle,

then use (2).

(5) *Ex-meridian altitude (not using estimated latitude).*

cot k = cos hour angle . cot dec.; cos y = sin alt. cosec dec. sin k;

then lat. = $k \pm y$.

(6) *Reduction to meridian.*

$$x \text{ (in seconds of arc)} = \frac{2 \cos l \,.\, \cos d \,.\, \text{hav}\, h}{\sin z \,.\, \sin 1''};$$

$$\text{or} \quad = \frac{\cos l \,.\, \cos d}{\sin z \,.\, \sin 1''} \,.\, \frac{\sin^2 h}{2}.$$

(7) *Double altitude.*

XY or arc (1).

$$\text{vers}\, XY = \text{vers}\,(PX \sim PY) + \text{vers}\,\theta;$$

where hav θ = sin PX . sin PY . hav XPY.

PZ or arc (5). Similar formula.

PXY or arc (2).

$$\text{hav}\, PXY = \text{cosec}\, PX \,.\, \text{cosec}\, XY \sqrt{\text{hav}\,(PY + PX \sim XY)\,\text{hav}\,(PY - PX \sim XY)}.$$

ZXY or arc (3). Similar formula.

PXZ or arc (4) = arc (2) $\mp$ arc (3) according as XY does not or does pass between P and Z.

If tables containing haversines and versines are not available, XY or arc (1) may be found from

$$\sin \frac{XY}{2} = \sqrt{\sin\left(\frac{PX + PY}{2} + \theta\right) \sin\left(\frac{PX + PY}{2} - \theta\right)};$$

where $\sin\theta = \sqrt{\sin PX \,.\, \sin PY \,.\, \cos^2 \frac{XPY}{2}}$.

Similarly for PZ or arc (5).

And PXY from the formula,

$$\sin^2 \frac{PXY}{2} = \text{cosec}\, PX \,.\, \text{cosec}\, PY \,.\, \sin \frac{PY + (PX \sim XY)}{2} \sin \frac{PY - (PX \sim XY)}{2}.$$

And similarly for ZXY or arc (3).

CHAPTER XIII.

COMPASS ERRORS. TRUE BEARING OF TERRESTRIAL OBJECT. MAXIMUM AZIMUTH.

To find the deviation when the true bearing has been found.

§ 123. Bearings should always be measured from the north, as the compass error is defined to be the angle between the directions of the true and compass north, or, more exactly, between the planes in which these directions lie.

If, therefore, a true bearing is found from the south, take it from 180° and measure it from the north. The compass error can, of course, be found equally well with reference to the south point, but it is better, for a beginner especially, to adhere to one method.

Draw a line to represent the *true* meridian. From the point in it which represents the zenith lay off the *true* bearing. This will give the direction of the sun or other heavenly body. Measure *back* from this direction an angle equal to the body's compass bearing, and so obtain the direction of the compass north.

The compass error will be the difference between the true and compass bearings, and will be east or west, according as the compass north falls to the right or the left of the true north.

Examples.—(1) The sun bore by compass N 60° W, the true bearing being N 75° W.

At Z in NZ make the angle $NZS = 75°$. S is in the direction of the sun. At the point Z in SZ make the angle $SZC = 60°$. ZC is clearly the direction of the north point of the compass, which, falling to the left of the true north, shows that the compass error is west. Fig. 86 (1).

75°
60
—
Compass error, 15° W
—

(2) The sun bore by compass N 80° W, the true bearing being S 120° W.

180°
120

N 60 W true bearing from north.

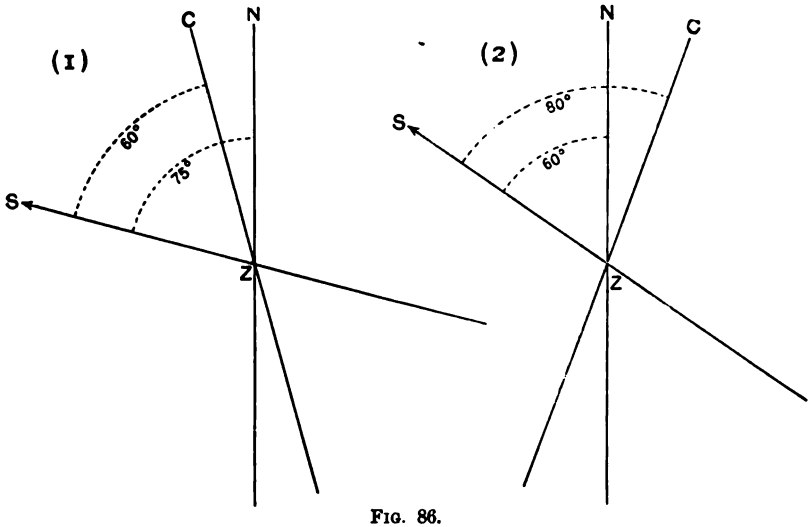


FIG. 86.

The figure being drawn similarly to the one in the last case, it is clear that the compass error is 20° E. Fig. 86 (2).

If a figure is always drawn in this manner no mistake can occur, whereas mistakes do constantly occur if verbal rules are given, which are usually derived from figures with two directions of the sun (which cannot be the case), and only one direction of the north point (of which there are two at least).

When the variation is given or, in practice, taken from the chart, and it is required to find the deviation, the figure is to be modified as follows:

Draw ZM to represent the direction of the magnetic north. The deviation will be east or west according as C falls to the right or the left of M, and its amount will be the algebraical difference between the variation and the compass error.

Thus in case (1) suppose that the variation is 17° W. Fig. 87 (1).

Here $NZM = 17°$,

$NZC = 15°$,

$\therefore$ deviation $= 2°$,

and is east because the compass north falls to the right of the magnetic north.

In case (2) suppose that the variation is 22° E. Fig. 87 (2).

Here $NZM = 22°$,

$NZC = 20°$,

$\therefore$ deviation $= 2°$,

and is west because the compass north falls to the left of the magnetic north.

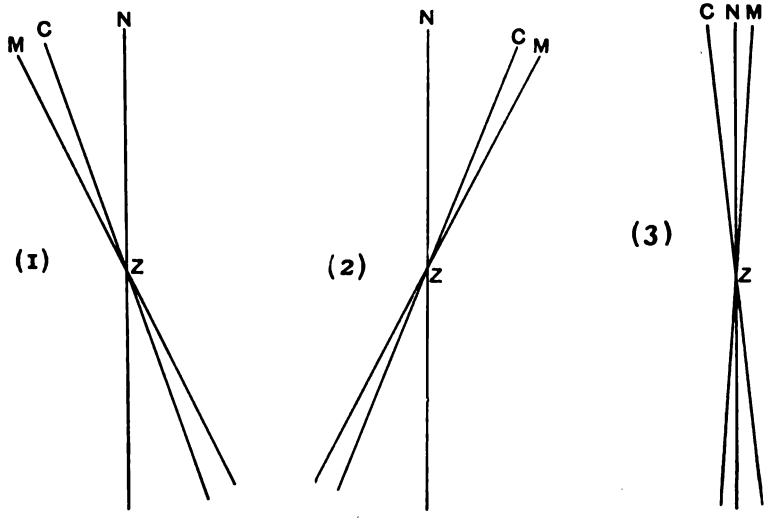


FIG. 87.

As a third case suppose that the compass error is 2° W, and the variation is 2° E. In this case the deviation is (fig. 87 (3))

$MZN + NZC = 4°$ (algebraical difference between 2° to the right and 2° to the left),

and is west as C falls to the left of M.

To find the compass error, or the deviation, by amplitude, or by the true bearing of the sun at rising or setting.

§ 124. Let X and X' be the position of the sun on the horizon, when the declination is of the same name as, or different name to, the latitude. Fig. 88.

PZX and PZX' are quadrantal triangles.

$$\therefore \quad \cos PX = \sin PZ \cdot \cos PZX$$

or
$$\cos PZX = \cos PX \cdot \operatorname{cosec} PZ \quad \ldots\ldots(1)$$

and
$$\cos PX' = \sin PZ \cdot \cos PZX',$$

or
$$\cos PZX' = \cos PX' \cdot \operatorname{cosec} PZ \quad \ldots\ldots(2)$$

But
$$\cos PZX = \sin EZX$$
$$= \sin \text{amplitude},$$
$$\cos PZX' = -\sin EZX'$$
$$= -\sin \text{amplitude},$$
$$\cos PX = \sin \text{dec.}$$
$$\cos PX' = -\sin \text{dec.}$$
$$\operatorname{cosec} PZ = \sec \text{lat.}$$

Therefore (1) and (2) each become

$$\sin \text{amplitude} = \sin \text{dec.} \sec \text{lat.},$$

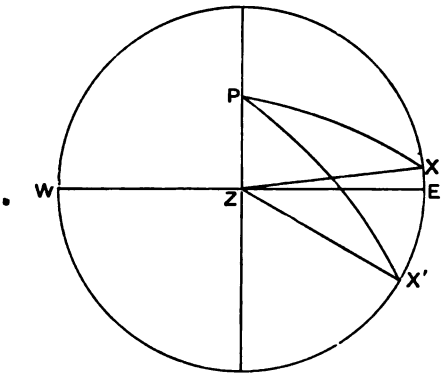


FIG. 88.

the amplitude being marked E or W according as the sun is rising or setting, and N or S according to the name of the declination.

The true bearing is then 90° − amplitude, or 90° + amplitude, according as the latitude and declination are of the same or different name, and the compass error or the deviation may be deduced, in accordance with the explanation just given.

Examples.—(1) April 13th, about 5 h. 15 m. A.M. apparent time, in latitude 52° 20′ N, longitude 11° 15′ W, the sun rose by compass E ¾ S. The variation being 24° W ; required the deviation.

NOTE.—As courses and bearings are now usually measured in *degrees* from the north or south point of the compass, it would appear to be an advantage to do away with the term "amplitude," in which the bearing is measured from the east or west point (confusion thus being frequently caused), and write the formula,

$$\cos \text{true bearing at rising or setting} = \sin \text{dec.} \sec \text{lat.},$$

remembering that the bearing must be taken from 180° when the declination is south, so as to obtain the true bearing from the north point.

	h. m.	Declination.	Change.
April 12th, - -	17 15	9° 3′ 25″ N	54·5″
Long., - - -	45 W	5 27	6
Greenwich app. time,	18 0	8 57 58	327·0
	24 0		
	6 0		

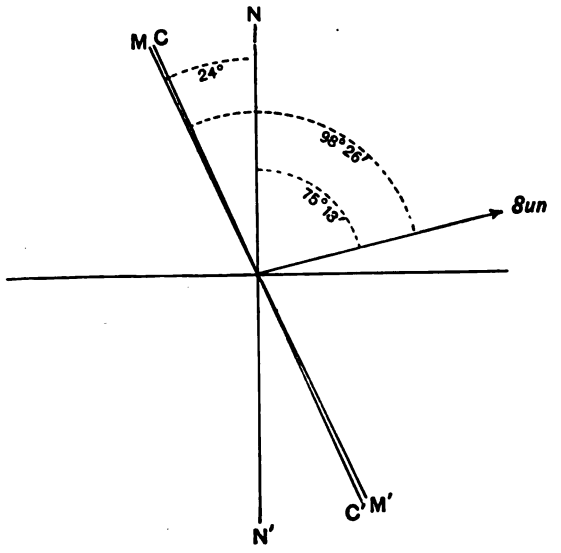


FIG. 89.

sin dec., - - -	9·192734
sec lat., - - -	10·213911
sin amp., or cos true bearing,	9·406645

Amplitude, - -	E 14° 46′ 45″ N
	90 0 0
True bearing, -	N 75 13 15 E
Compass „ -	N 98 26 15 E
Compass error. - -	23 13 0 W
Variation, - - -	24 0 0 W
Deviation, - - -	47 0 E

(2) April 30th, about 5 h. 5 m. P.M. apparent time, in latitude 41° 35′ S, longitude 85° 15′ E, the sun set by compass NWbW. The variation being 18° W; required the deviation.

	h. m.	Declination.	Change.
April 30th, - -	5 5	14° 46′ 55″ N	46″
Long., - - -	5 41 E	28	·6
Greenwich app. time,	23 24 April 29.	14 46 27	27·6

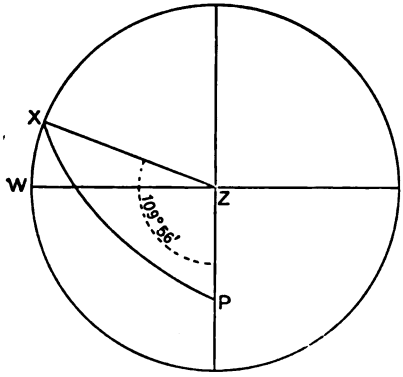


FIG. 90.

sin dec., - - -	9·406581
sec lat., - - -	10·126104
sin amp., or cos true bearing,	9·532685

True bearing = N 70° 4′ W.

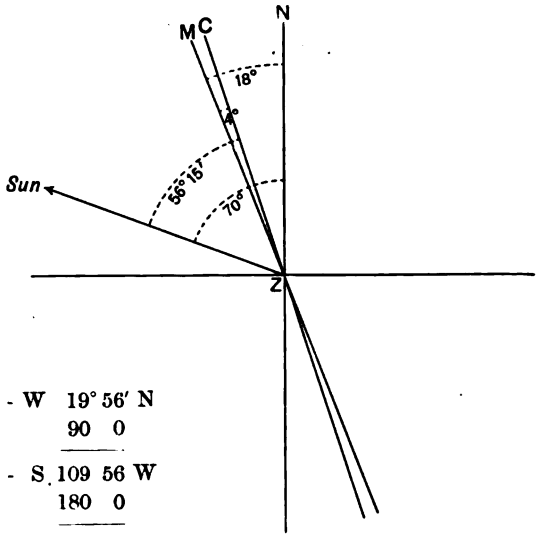


FIG. 91.

Amplitude, - - -	W	19° 56′ N
		90 0
True bearing, - -	S	109 56 W
		180 0
	or N	70 4 W
Compass bearing, - -	N	56 15 W
Compass error, - -		13 49 W
Variation, - - -		18 0
Deviation, - - -		4 11 E

Examples.—(1) April 4th, about 6 h. 20 m. P.M. apparent time, in latitude 41° 41′ N, longitude 35° 20′ W, the sun set by compass N 60° 20′ W. The variation being 24° W ; required the deviation.

(2) April 22nd, about 6 h. 30 m. A.M. apparent time, in latitude 27° 47′ S, longitude 93° 25′ E, the sun rose by compass east. The variation being 10° 15′ W ; required the deviation.

(3) June 5th, about 7 h. 15 m. P.M. apparent time, in latitude 37° 51′ N, longitude 129° 47′ W, the sun set by compass N 76° 20′ W. The variation being 17° 20′ E ; required the deviation.

(4) June 26th, about 6 h. 15 m. P.M. apparent time, in latitude 10° 15′ N, longitude 110° 22′ E, the sun set by compass N 65° 30′ W. The variation being 2° 20′ E ; required the deviation.

(5) December 2nd, about 6 h. 20 m. A.M. apparent time, in latitude 12° 5′ N, longitude 62° 50′ E, the sun rose by compass S 63° 30′ E. The variation being 1° 10′ W ; required the deviation.

To find the compass error or the deviation by altitude azimuth.

§ 125. Let X be the position of the sun or other heavenly body not on the horizon, the altitude and compass bearing being observed at the same time. In the triangle PZX the three sides are known. Hence the angle PZX may be determined by the usual formula,

haversine PZX

$$= \operatorname{cosec} PZ \cdot \operatorname{cosec} ZX \sqrt{\operatorname{hav}(PX + \overline{PZ \sim ZX}) \operatorname{hav}(PX - \overline{PZ \sim ZX})}.$$

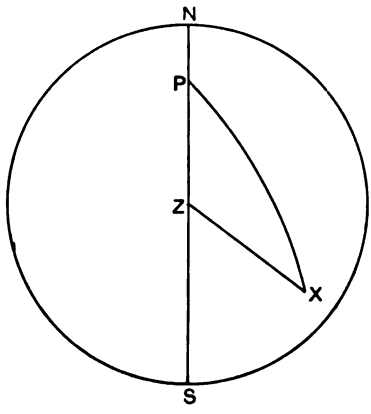


FIG. 92.

NOTE.—As in the case of longitude by chronometer, the formula

haversine PZX

$$= \sec \text{lat.} \sec \text{alt.} \sqrt{\operatorname{hav}(\text{P.D.} + \overline{\text{lat.} \sim \text{alt.}}) \operatorname{hav}(\text{P.D.} - \text{lat.} \sim \text{alt.})}$$

may be made use of.

The true bearing being thus found and marked N or S according to the latitude, and E or W according as the body is E or W of the meridian, the compass error or the deviation may be deduced as before.

The altitude azimuth problem is a convenient one, since the best time for longitude observations is also suitable for observations for azimuth. The compass bearing being taken at the same time as, or immediately after, the observation for longitude, the same altitude will determine the longitude and the compass error. It has also the advantage of being independent of the error of chronometer, the dead reckoning longitude being sufficiently accurate for the determination of the declination.

It is, however, seldom used in actual practice, the true bearing being taken from the "Azimuth Tables," which are calculated for every four minutes of ship apparent time, so long as the sun's altitude is less than 60°, for limits of latitude 60° N to 60° S, and declination 0° to 23°, the ship apparent time being found in working the observations for longitude or else deduced from the time by chronometer.

These tables have been calculated by "Time Azimuth."

Example.—June 1st, about 3 h. 10 m. P.M. apparent time, in latitude 27° 45′ S, longitude 104° 13′ E, the true altitude of the sun's centre was 22° 18′, when the sun bore by compass N 40° 20′ W. The variation being 5° W; required the deviation.

	h. m.	Declination.	Change.
June 1st, - - -	3 10	22° 3′ 54·8″ N	20·5
Long., - - -	6 57	1 18	3·8
Greenwich app. time, May 31st,	20 13	22 2 37	77·9
	24 0	90 0 0	
	3 47	112 2 37 = polar dist.	

co-lat., - - -	62° 15′ 0″	cosec 10·053063
zen. dist., - - -	67 42 0	cosec 10·033760
	5 27 0	
polar dist., - - -	112 2 37	
	117 29 37	½ hav 4·931902
	106 35 37	½ hav 4·904030
		9·922755

True bearing,	S 132° 23′ W
	180 0
	N 47 37 W
Compass bearing,	N 40 20 W
Compass error,	- 7 17 W
Variation, -	- 5 0 W
Deviation, -	- 2 17 W

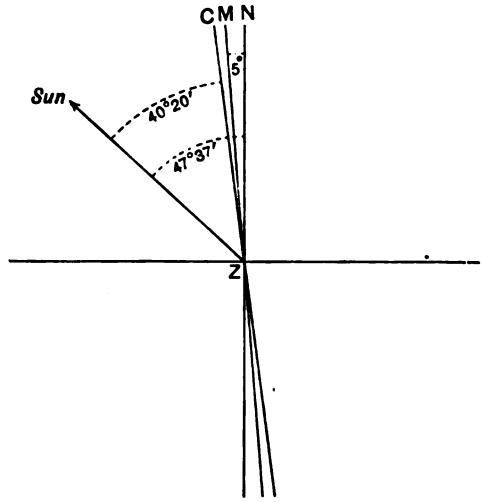


FIG. 93.

Examples.—(1) April 9th, about 8 h. 30 m. A.M. apparent time, in latitude 47° 27′ N, longitude 129° 31′ W, the sun bore by compass N 89° 40′ E, when the obs. alt. sun's L.L. was 30° 13′ 10″. Index error, 1′ 50″+ ; height of eye, 21 feet. The variation being 22° 30′ E ; required the deviation.

(2) June 12th, about 3 h. 20 m. P.M. apparent time, in latitude 20° 15′ S, longitude 37° 29′ W, the sun bore by compass N 36° 20′ W, when the obs. alt. sun's L.L. was 24° 34′ 50″. Index error, 1′ 30″− ; height of eye, 18 feet. The variation being 12° W ; required the deviation.

(3) December 3rd, about 7 h. 50 m. A.M. apparent time, in latitude 42° 15′ S, longitude 109° 50′ E, the sun bore by compass S 79° 20′ E, when the obs. alt. sun's L.L. was 34° 30′ 50″. Index error, 30″+ ; height of eye, 19 feet. The variation being 14° 30′ W ; required the deviation.

(4) December 9th, about 8 h. 50 m. A.M. apparent time, in latitude 23° 25′ N, longitude 123° 20′ E, the sun bore by compass S 45° 30′ E, when the obs. alt. sun's L.L. was 24° 33′ 20″. Index error, 2′ 40″− ; height of eye, 26 feet. The variation being 1° 50′ W ; required the deviation.

To find the compass error or the deviation by time azimuth.

§ 126. When the heavenly body's bearing by compass is observed, note the time by chronometer, and from it obtain the body's hour angle from the meridian. We then have in the triangle *ZPX*, *PZ*, *PX*, and the included angle *ZPX*, with which to find the true bearing *PZX* (fig. 92).

1st method, by the use of Napier's Analogies.

$$\tan \tfrac{1}{2}(Z+X) = \cos \tfrac{1}{2}(PX - PZ) \sec \tfrac{1}{2}(PX + PZ) \cot \frac{ZPX}{2},$$

$$\tan \tfrac{1}{2}(Z \sim X) = \sin \tfrac{1}{2}(PX \sim PZ) \operatorname{cosec} \tfrac{1}{2}(PX + PZ) \cot \frac{ZPX}{2},$$

$\frac{Z+X}{2}$ and $\frac{Z \sim X}{2}$ being found, PZX is their sum or their difference according as PX is greater or less than PZ.

It is to be noticed that when $\frac{1}{2}(PX+PZ)$ is greater than 90°, $\frac{1}{2}(Z+X)$ will also be greater than 90°. In this case we shall have $180^\circ - \frac{1}{2}(Z+X)$ instead of $\frac{1}{2}(Z+X)$.

2nd method, by first finding ZX and then calculating PZX from the three sides of the triangle PZX.

$$\operatorname{vers} ZX = \operatorname{vers}(PX \sim PZ) + 2 \sin PX \sin PZ \operatorname{hav} ZPX,$$

$$\operatorname{hav} PZX = \operatorname{cosec} PZ \operatorname{cosec} ZX \sqrt{\operatorname{hav}(PX + PZ \sim ZX) \operatorname{hav}(PX - PZ \sim ZX)}.$$

For the formulæ without using versines and haversines, see note, p. 175.

This method is sometimes preferred, as it uses well-known formulæ and has no distinction of cases. The former method is, however, much shorter.

The true bearing being found by either method, the compass error or the deviation may be deduced as usual.

Examples.—(1) April 8th, about 7 h. 50 m. A.M. apparent time, in latitude 30° 50′ N, longitude 16° 0′ E, the sun bore by compass S 66° 30′ E, when a chronometer, whose error on G.M.T. was 1 h. 2 m. 19 s. fast, showed 7° 49′ 54″. The variation being 19° 15′ W; required the deviation.

	h. m.
April 7th,	19 50
Long.,	1 4 E
Greenwich app. time,	18 46

Declination.	Change.
7° 12′ 57″ N	56·1
4 52	5·2
7 8 5	1122
90 0 0	2805
82 1 55 = PX	291·72
	4′ 52″

Equation of time.	Change.
m. s.	
1 53·6	·7
3·6	5·2
1 57 – M.T.	3·64

	h. m. s.
Chronometer,	7 49 54
error fast,	1 2 19
	6 47 35
	12 0 0
G.M.T.,	18 47 35
Long. E.,	1 4 0
S.M.T.,	19 51 35
eq. of time,	1 57 –
	19 49 38
	24 0 0
	4 10 22 sun's hr. angle.
	2 5 11 = $\frac{ZPX}{2}$.
Latitude,	30° 50′
	90 0
	59 10 = ZX

1st Method.

	$PX=$	82° 52′
	$PZ=$	59 10
sum,		142 2
diff.,		23 42
½ sum,		71 1
½ diff.,		11 51

To find PZX.

$\frac{1}{2}(Z+X)=$	78°	34′	30″
$\frac{1}{2}(Z\sim X)=$	19	39	30
$\therefore PZX=$ N	98	14	0 E

To find $\frac{1}{2}(Z+X)$.

$\cot\frac{ZPX}{2}$,	10·216161
$\cos\frac{1}{2}(PX-PZ)$,	9·990645
$\sec\frac{1}{2}(PX+PZ)$,	10·487725
	10·694531

$\frac{1}{2}(Z+X)=78°\ 34'\ 30''$

To find $\frac{1}{2}(Z\sim X)$.

$\cot\frac{ZPX}{2}$,	10·216161
$\sin\frac{1}{2}(PX-PZ)$,	9·312494
$\operatorname{cosec}\frac{1}{2}(PX+PZ)$	10·024287
	9·552942

$\frac{1}{2}(Z\sim X)=19°\ 39'\ 30''$

True bearing,	N	98° 14′ E
Compass „	N	113 30 E
Compass error,		15 16 W
Variation,		19 15 W
Deviation,		3 59 E

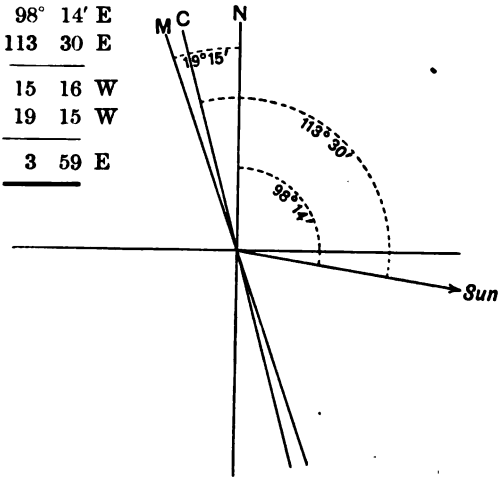


FIG. 94.

2nd Method.

To find ZX.

co-lat.,	59°	10′	sin 9·933822
polar dist.,	82	52	sin 9·996625
diff.,	23	42	
	h.	m.	s.
hour angle,	4	10	22 hav 9·431099
			9·361546

$\theta=57°\ 18'\ 15''$

S. N. N

vers θ,	0459760
	61
vers diff.,	0084337
	0544158
	3937
	221

$ZX=62°\ 52'\ 51''$

To find PZX.

co-lat., -	59° 10′	cosec 10·066178	
zen. dist., -	62 53	cosec 10·050571	
	3 43		
polar dist., -	82 52		
	86 35	½ hav	4·836142
	79 9	½ hav	4·804199
			9·757090

$PXZ = 98° 14'$ as before.

(2) In which $\frac{PX+PZ}{2}$ is greater than 90°.

July 23rd, about 4 h. 45 m. P.M. apparent time, in latitude 19° 30′ S, longitude 31° 35′ W, the sun bore NW ¾ W, when a chronometer, whose error was 1 h. 49 m. 16 s. fast on G.M.T., showed 8 h. 44 m. 21 s. The variation being 14° W ; required the deviation.

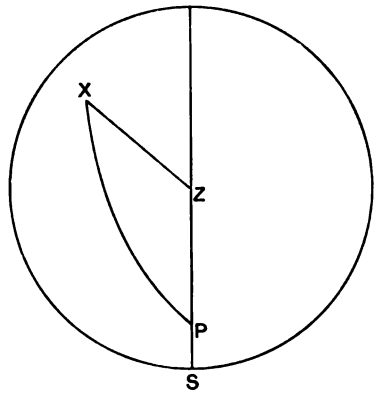


FIG. 95.

	h. m.		h. m. s.	Declination.	Change.
July 23rd, -	4 45	chron., -	8 44 21	20° 5′ 32″ N	30·7
Long., - -	2 6 W	error, - -	1 49 16	3 32	6·9
Gr. app. time,	6 51	G.M.T., -	6 55 5	20 2 0	2763
		Long., -	2 6 20	90 0 0	1842
		S.M.T., -	4 48 45	$PX = 110$ 2 0	211·83
		eq. of time,	6 15		3′ 32″
		sun's hour angle,	4 42 30	Equation of time. m. s.	
		$\frac{ZPX}{2} = 2$	21 15	6 14·8 – M.T.	
				·5	
				6 15	

PZ = 110° 2′	$\cot\frac{ZPX}{2}$, -	10·149742	$\cot\frac{ZPX}{2}$, -	10·149742
PX = 70 30				
sum, - 180 32	cos ½ diff., -	9·973625	sin ½ diff., -	9·529161
diff., - 39 32	sec ½ sum, -	12·332156	cosec ½ sum,	10·000005
½ sum, - 90 16		2·455523		9·678908
½ diff., - 19 46			$\frac{1}{2}(Z \sim X)$ = 25° 31′ 15″	

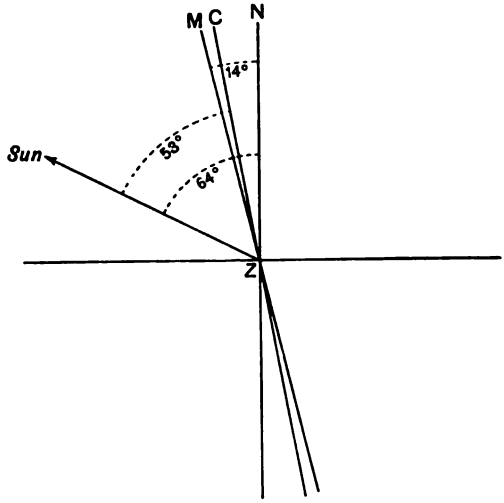


FIG. 96.

	89°	48′	0″
	180	0	0
$\frac{1}{2}(Z+X)$ =	90	12	0
$\frac{1}{2}(Z \sim X)$ =	25	31	15
PZX = S	115	43	15 W
	180	0	0
True bearing, - - N	64	16	45 W
Compass „ - - N	53	26	15 W
Compass error, - -	10	50	30 W
Variation, - - -	14	0	0 W
Deviation, - - -	3	9	30 E

As $\frac{PZ+PX}{2}$ = 90° 16′, the value found from the first column of logarithms must be taken from 180° to obtain $\frac{1}{2}(Z+X)$.

Examples.—(1) April 3rd, about 7 h. 50 m. A.M. apparent time, in latitude 35° 15′ N, longitude 13° 18′ W, the sun bore by compass S 61° 30′ E, when a chronometer, whose error on G.M.T. was 1 h. 24 m. 25 s. fast, showed 10 h. 11 m. 16 s. The variation being 19° W; required the deviation.

(2) April 19th, about 4 h. 15 m. P.M. apparent time, in latitude 27° 25′ S, longitude 39° 14′ E, the sun bore by compass N 43° 20′ W, when a chronometer, whose error on G.M.T. was 1 h. 54 m. 37 s. slow, showed 11 h. 42 m. 19 s. The variation being 22° W; required the deviation.

(3) June 8th, about 8 h. 20 m. A.M. apparent time, in latitude 15° 15′ S, longitude 80° 10′ W, the sun bore by compass N 46° 40′ E, when a chronometer, whose error on G.M.T. was 2 h. 3 m. 38 s. fast, showed 3 h. 43 m. 16 s. The variation being 12° E; required the deviation.

(4) June 26th, about 5 h. P.M. apparent time, in latitude 13° 42′ N, longitude 111° 12′ E, the sun bore by compass N 69° W, when a chronometer, whose error on G.M.T. was 1 h. 55 m. 13 s. slow, showed 7 h. 42 m. 18 s. The variation being 2° 20′ E; required the deviation.

(5) December 9th, about 8 h. A.M. apparent time, in latitude 11° 25′ N, longitude 58° 20′ E, the sun bore by compass S 61° 20′ E, when a chronometer, whose error on G.M.T. was 1 h. 1 m. 57 s. fast, showed 5 h. 1 m. 8 s. The variation being 1° 15′ W; required the deviation.

(6) December 27th, about 3 h. 50 m. P.M. apparent time, in latitude 17° 21′ S, longitude 81° 15′ W, the sun bore by compass S 64° W, when a chronometer, whose error on G.M.T. was 1 h. 26 m. 49 s. slow, showed 7 h. 49 m. 28 s. The variation being 12° E; required the deviation.

To find the true bearing of a terrestrial object.

§ 127. Let O be the object projected on the celestial concave, X the true place of the sun's centre.

To find PZO, the true bearing of O, we must measure the angular distance OX and the altitude of O. Hence the angle

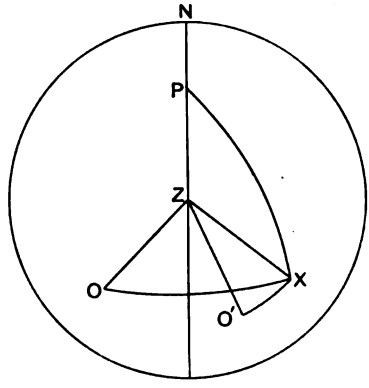


FIG. 97.

at the zenith OZX can be found. The sun's true bearing PZX having been calculated, PZO will equal $360° - (PZX + OZX)$.

If O had been E of the meridian, as at O', the true bearing of O' would be $PZX + O'ZX$, but the construction of a figure

will in each case determine the true bearing of O when PZX and OZX have been found.

The angular distance OX is, of course, measured from the apparent place of the sun, but in practice it is considered as being measured from the true place.

If O is on the horizon, $OZ = 90°$, and OZX becomes a quadrantal triangle.

True bearing and Mercatorial bearing.

§ 128. The azimuth or true bearing of a heavenly body is the angle at the zenith between the celestial meridian and the circle of altitude through the body; and similarly as to the true bearing of a terrestrial object.

The Mercatorial bearing is the bearing as calculated by the method of Mercator's sailing, and is the angle between the meridian and the line on the chart from the position of the observer to that of the object.

On account of the distortion of the Mercator's chart, these two bearings are not the same, but the difference is not appreciable unless the distance is more than a few miles. Thus, three distant peaks which appear to an observer to lie in the same line from him (*i.e.* on the same great circle) will not appear to lie on the same straight line from his position on the chart.

If, therefore, the true bearing of a mountain peak be calculated so as to obtain the compass error, the peak must not be at a *great* distance; it must be far enough to render inappreciable the effects of parallax as a ship swings, but not far enough to render appreciable the difference between the true and Mercatorial bearing.

An expression for this difference may be found as follows:

The true bearing of a distant object differs from the angle made by the rhumb line with the meridians by one-half the convergency of the meridians.

Let PAC, PBD be the meridians passing through two places, AB being the arc of the parallel of middle latitude between them, CD the arc of the equator, EA, EB tangents to PA, PB, E falling on the diameter produced.

The convergency of the meridians is measured by the angle AEB.

Since AB is very small, it may be considered as a straight line.

$\therefore$ since AE, BE are equal,

$$AB = 2\,AE \sin \tfrac{1}{2} AEB$$
$$= 2\,AO \tan AOE \sin \tfrac{1}{2} AEB$$
$$= 2r \cot \text{mid. lat.} \sin \tfrac{1}{2} AEB.$$

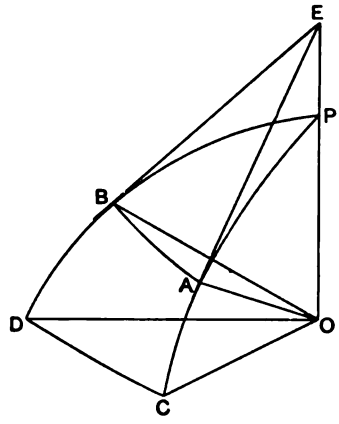


FIG. 97 A.

Again, since the triangle AOB is isosceles,

$$AB = 2\,AO \sin \tfrac{1}{2} AOB$$
$$= 2r \sin \tfrac{1}{2} \text{departure},$$

$\therefore$ $\cot \text{mid. lat.} \sin \tfrac{1}{2} AEB = \sin \tfrac{1}{2} \text{departure}$,

$\therefore$ $$\sin \tfrac{1}{2} \text{convergency} = \frac{\sin \tfrac{1}{2} \text{departure} \times \sin \text{mid. lat.}}{\cos \text{mid. lat.}}$$
$$= \sin \tfrac{1}{2} \text{diff. long.} \times \sin \text{mid. lat.}$$

Cf. Raper's *Practice of Navigation*, 10th Edition, § 395.

Example.—In latitude 28° N, at 4 h. 25 m. P.M. ship apparent time, an object O, whose true altitude was 5°, was observed to be 81° 40′ distant from the nearer limb of the sun, whose declination is 2° S and semi-diameter 16′. O is to the left of the sun, and E of the meridian; required the true bearing of O.

To find ZX.

co-lat., - - -	62°	sin 9·945935
polar dist., - -	92	sin 9·999735
diff.	30	
hour angle, 4 h. 25 m.		hav 9·475129
		9·420799

$\theta = 61° 46' 15''$ vers 0526937
64
30° vers 0133975
0660976

obs. dist., - - - 81° 40′
semi-diameter, - - 16
$OX = 81\ 56$

$ZX = 70° 11'$

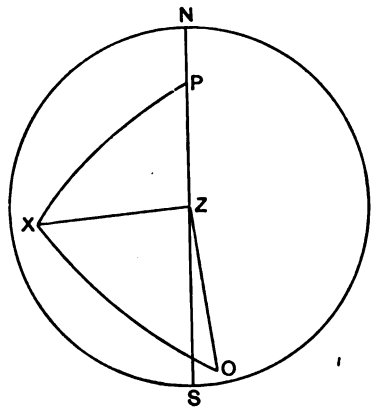


FIG. 98.

To find PZX.

co-lat., -	62° 0′	cosec 10·054065
zen. dist.,	70 11	cosec 10·026511
	8 11	
pol. dist.,	92 0	
	100 11	½ hav 4·884836
	83 49	½ hav 4·824738
		9·790150

To find OZX.

OZ, -	85° 0′	cosec 10·001656
XZ, -	70 11	cosec 10·026511
	14 49	
OX, -	81 56	
	96 45	½ hav 4·873616
	67 7	½ hav 4·742557
		9·644340

$PZX = 103° 30' 30''$
$OZX = 83\ 12\ 45$
$OZX = 83° 12' 45''$

186 33 15
360 0 0

$PZO = 173\ 26\ 45$

∴ true bearing of *O* is S 6° 33′ 45″ E.

If *O* had been on the horizon we should have had

$$\cos OZX = \cos OX \,.\, \operatorname{cosec} ZX,$$

and *OZX* would have been 81° 19′.

Examples.—(1) From the following data find the error of the compass:

Altitude sun's centre, - - -	45° 20′
Altitude moon's centre, - - -	30 50
Sun's declination, - - - -	8 8 N
Latitude, - - - - -	23 0 N
Lunar distance, - - - -	79 50
Moon's compass bearing, - - -	S 40 30 W
Time, - - - - - -	(about) 9 A.M.

(2) In latitude 50° 20′ N, at 8 h. 45 m. 58 s. A.M. ship apparent time, the angular distance of the nearer limb of the sun from an object O on the horizon was 79° 47′, O being to the right of the sun and west of the meridian. Find the true bearing of O, having given

Sun's semi-diameter, - - - - -	16′
Sun's declination, - - - - - -	8° 0 S

(3) At a place in latitude 30° 25′ N, when the sun's declination was 10° 29′ N, the angular distance of its centre from an object to the left of it, elevated 20° above the horizon, was found to be 85° at 3 h. 15 m. P.M. apparent time. Find the true bearing of the object.

Maximum Azimuth.

§ 129. When the declination of a heavenly body is greater than the latitude of the place, it will cross the meridian between the zenith and the pole.

If X be the position of such a body at rising, XSX' its parallel of declination, it is clear that the azimuth will increase until the body reaches the position S, at which the circle of altitude is a tangent to the parallel of declination, after which it will decrease until the body reaches the meridian, the azimuth, when the body is at T, being equal to that at rising.

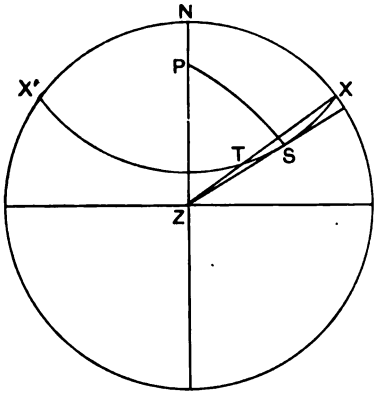


FIG. 99.

PSZ being a right angle, the hour angle when the azimuth is a maximum is found from the expression

$$\cos ZPS = \cot PZ \tan PS$$
$$= \tan \text{lat.} \cot \text{dec.},$$

and the maximum azimuth from the expression

$$\sin PS = \sin PZ \sin PZS$$

or

$$\sin PZS = \cos \text{dec.} \sec \text{lat.}$$

If a stick were placed at Z, its shadow at first would appear to move towards the west until the sun reached S, when it would return and finally point to the south, a similar result occurring in a reverse order in the afternoon.

The time during which the shadow would thus *retrograde* is the difference between the hour angles ZPX and ZPS.

At the time of the maximum azimuth, the heavenly body is said to be moving vertically in azimuth.

Examples.—(1) At a place in latitude 15° N, a star, whose right ascension is 23 h. 56 m. and declination 20° N, has its maximum azimuth at the same time that another star of declination 5° N is upon the prime vertical, both stars being east of the meridian. Find the right ascension of the second star.

(2) Find the time during which the shadow of a stick will retrograde in latitude 5° 30′ N, when the sun's declination is 15° N.

(3) In latitude 5° N, when the sun's declination is 19° 50′ N, find the sun's maximum azimuth, and at what time it will have this azimuth.

Formulæ in "Compass Error Problems," collected for reference.

(1) *Amplitude.*

Sin amplitude or cos true bearing = sin dec. sec lat.

(2) *Altitude azimuth.*

$$\text{hav azimuth} = \operatorname{cosec} c \,.\, \operatorname{cosec} z \sqrt{\operatorname{hav}(p+c\sim z)\operatorname{hav}(p-c\sim z)},$$

$$\text{or } \sin^2\frac{\text{azimuth}}{2} = \operatorname{cosec} c \,.\, \operatorname{cosec} z \,.\, \sin\frac{p+c\sim z}{2}\sin\frac{p-c\sim z}{2}.$$

(3) *Time azimuth.*

(*a*) $$\tan\tfrac{1}{2}(Z+X) = \frac{\cos\frac{p\sim c}{2}}{\cos\frac{p+c}{2}}\cot\tfrac{1}{2}\text{hour angle}.$$

$$\tan\tfrac{1}{2}(Z\sim X) = \frac{\sin\frac{p\sim c}{2}}{\sin\frac{p+c}{2}}\cot\tfrac{1}{2}\text{hour angle}.$$

$$\text{azimuth} = \tfrac{1}{2}(Z+X) \pm \tfrac{1}{2}(Z\sim X)$$

according as polar distance is greater or less than co-latitude.

(*b*) $$\operatorname{vers} z = \operatorname{vers}(p\sim c) + \operatorname{vers}\theta$$

$$\operatorname{hav}\theta = \sin p \,.\, \sin c \,.\, \operatorname{hav}\text{ hour angle}$$

$$\text{hav azimuth} = \operatorname{cosec} c \,.\, \operatorname{cosec} z \sqrt{\operatorname{hav}(p+c\sim z)\operatorname{hav}(p-c\sim z)}.$$

(*c*) Without versines or haversines.

$$\sin\frac{z}{2} = \sqrt{\sin\left(\frac{p+c}{2}+\theta\right)\sin\left(\frac{p+c}{2}-\theta\right)}$$

$$\text{where } \sin\theta = \sqrt{\sin p \,.\, \sin c \,.\, \cos^2\frac{\text{hour angle}}{2}}$$

$$\sin^2\frac{\text{azimuth}}{2} = \operatorname{cosec} c \,.\, \operatorname{cosec} z \,.\, \sin\frac{p+c\sim z}{2}\sin\frac{p-c\sim z}{2}.$$

CHAPTER XIV.

SUMNER'S METHOD.

§ 130. This method of obtaining a ship's position was proposed by Captain Thomas Sumner, of Boston, in the year 1837. The principle is as follows:

There is always one spot on the earth's surface which has the sun in its zenith, and that hemisphere of the earth which has this spot as its pole is illuminated by the sun, and may be called the hemisphere of illumination.

If small circles be drawn on the earth's surface parallel to the great circle which bounds the illuminated hemisphere, it is evident that the altitude of the sun will be the same at each point of any such small circle. These may, therefore, be called "Circles of equal altitude".

If, then, an altitude of the sun be observed, the observer must be somewhere on a circle of equal altitude; and if another altitude be observed when the sun has changed its bearing sufficiently, the observer is on another circle of equal altitude. His actual position must therefore be at one of the two points in which these circles intersect. These points being, as a rule, a long distance apart, his position by account will determine which point of intersection is the correct one.

In the figure S and S' are two positions of the sun or other heavenly body, HAB, $H'CD$ the corresponding hemispheres of illumination, ab, cd circles of equal altitude, e, f the points of intersection of these circles.

The positions of H and H' may be determined by the following considerations: The sun is vertical at noon at that place on the earth whose latitude is equal to the sun's declination of the same name. Hence the latitude and longitude of H and H' are respectively equal to the sun's declination and the Greenwich apparent times at the two observations.

If O be the centre of the earth, the arc Ha which measures the angle ZOS = the sun's zenith distance.

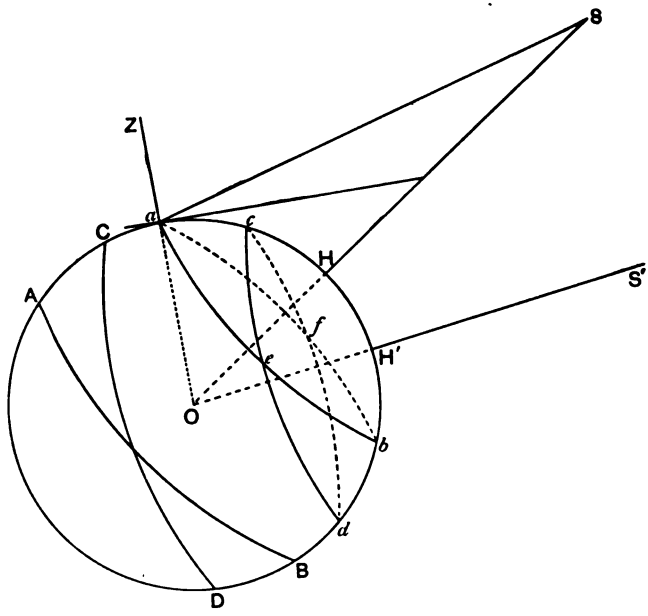


FIG. 100.

H and H' are not necessarily on the same meridian, but, as the meridians must be very close together, they are so represented in the figure to avoid confusion.

§ 131. If, now, these circles of equal altitude, or "circles of position," are transferred to a Mercator's chart, it will be found that they do not appear exactly as circles, on account of the increase in the length of the miles of latitude as you recede from the equator. They will, however, be represented by regular curves, any small portions of which may, for all practical purposes, be considered as straight lines, and are then named "lines of position."

§ 132. (I.) To obtain the ship's position, therefore, assume two latitudes not much differing—10 to 30 miles—from the latitude by account, on either side of it. With these latitudes and the first altitude obtain two longitudes. Mark the positions on a Mercator's chart and join them by a straight line. This will be a line of position (really a chord very near the arc).

Then, with the same two latitudes and the second altitude obtain two more positions, which, being joined, will give a second line of position, whose point of intersection with the first line will be the position of the ship; (1).

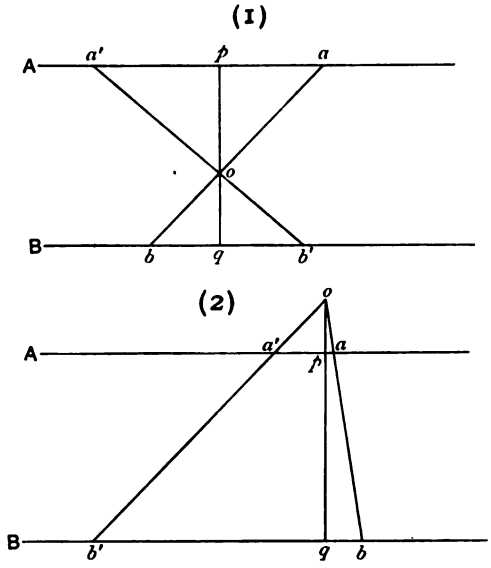


FIG. 101.

The point of intersection need not necessarily fall between the parallels of latitude chosen, but may be found as in (2).

§ 133. This method, however, is not much used, as the publication of the azimuth tables has enabled the true bearing of the sun to be obtained by inspection; thus the second method (§ 135) is much shorter and simpler.

§ 134. Since $dh = \frac{1}{15} dl$. sec lat . cot azimuth (§ 175), a small error in latitude will produce a large error in longitude when the body observed is near the meridian, especially in latitudes where Sumner's method is most useful. Hence the line of position will not so accurately represent the arc of the circle of position and the difference of longitude obtained will be so large that the line will probably be of unwieldy length.

But, on the other hand, a small error in the hour angle will produce only a small error in the latitude when the body is near the meridian.

Hence, when the second observation in Sumner's method is taken near the meridian, it is better to take the two longi-

tudes obtained from the first altitude; with them calculate two latitudes from the second altitude, by ex-meridian method and join the two positions thus found to obtain the second Sumner line, which will also be of moderate length.

As a general principle, therefore, observations suitable for latitude should be worked as such, and longitudes should not be calculated from observations which will give very small hour angles.

§ 135. (II.) The position may also be obtained on Sumner's principle by taking, as lines of position, tangents to the circles of equal altitude at points near to the true position, as these tangents do not differ appreciably from the arcs.

These tangents are at right angles to the radii of the small circles at their points of contact, and are, therefore, also at right angles to the true bearing of the sun, which must be in the direction of the centre of the circle (mercatorial bearing).

Or, from another point of view, since the planes of all great circles passing through S and H in fig. (100) must cut the circle of equal altitude at right angles, the tangent at any point is therefore at right angles to the bearing of the sun.

If, then, one position be found, with latitude by account, from the first altitude, and a straight line drawn through it at right angles to the true bearing of the sun at the time of observation (obtained from the azimuth tables, or by calculation), this straight line will be a line of position.

A second line being similarly found from the second altitude (with the same precautions as were referred to in § 134 if the body observed is near the meridian), the intersection of these two lines will give the position of the ship, which may be taken from the chart, or calculated.

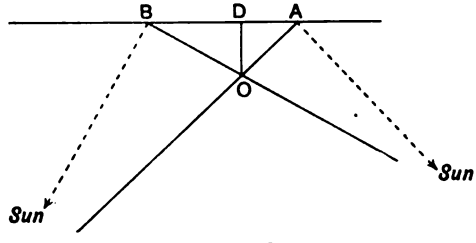


FIG. 102.

136. AO and BO being the Sumner lines at right angles to the dotted lines which represent the true bearing of the

sun, we have in the triangle AOB, the side AB, the difference of longitude between the two positions, and all the angles.

Hence $$AO = AB \cdot \frac{\sin ABO}{\sin AOB},$$

and then $$AD = AO \cdot \cos BAO,$$

$$OD = AO \cdot \sin BAO,$$

which determines the position of O, OD being the Mer. Diff. Lat.

§ 137. Thus far the ship's position has been considered as unchanged. But, if the position has changed in the interval, the first altitude may be corrected for "run" as in a double altitude, or the first Sumner line may be projected by laying off the run from any point of it, and drawing a line parallel to it through the extremity of the line representing the run.

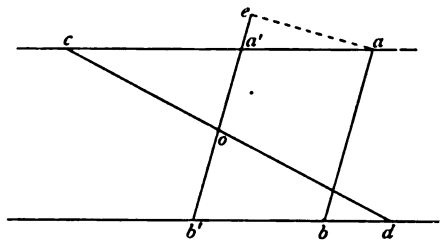


FIG. 103.

If ab represents the first Sumner line, $a'b'$ the first line projected for the run ae, cd the second Sumner line, then o will be the position of the ship.

§ 138. In very high latitudes, when the run is considerable, it might be necessary in projecting the Sumner line to take account of the varying length of the miles of latitude on the chart, which would cause the projected line to be not exactly parallel to the original one.

§ 139. The correctness of the position of the Sumner lines depends, of course, on the accuracy of the chronometer. If the chronometer is faster on G.M.T. than is supposed, the lines will be too far to the westward, and if slower too far to the eastward, but their direction will not be altered.

§ 140. If only one line of position can be obtained, it nevertheless may prove of great use; as the ship must be somewhere on that line at the time of observation, and, if the bearing of

a point of land, etc., can be obtained at the time or soon after, the intersection of the line or projected line with the line of the observed bearing will be the position of the ship.

A line of soundings also may be combined with a Sumner line. The position of a ship may also be very accurately fixed by combining a Sumner line found from observation of the sun, when its bearing from the meridian is considerable, with the meridian altitude observation. The latter puts the ship somewhere on a parallel of latitude, whose intersection with the projected Sumner line fixes the position very accurately.

Examples.—(1) In latitude by account 41° N, longitude 52° 30′ W, the following observations were taken to obtain a ship's position by Sumner's method:

Ship app. time, nearly. h. m.	Chron. time. h. m. s.	True altitude sun's centre.	True bearing.
9 30 A.M.	3 31 32	29° 6′ 50″	S 43° 30′ E
2 0 P.M.	8 1 6	32 37 30	S 37 0 W

The run of the ship in the interval was WSW (true) 8′ per hour; error of chronometer, 2 h. 45 m. 46 s. fast on G.M.T.; the sun's declination at the times of observation, 9° 5′ 30″ S and 9° 9′ 30″ S; and the equation of time, 14 m. 28 s. and 14 m. 30 s., subtractive from apparent time.

By projection, using the method of § 135.

1st Altitude.

co-lat., - -	49° 0′ 0″	cosec 10·122220
polar dist., - -	99 5 30	cosec 10·005491
	50 5 30	
zenith dist., -	60 53 10	
	10 47 40	½ hav 3·973405
	110 58 40	½ hav 4·915936
		9·017052

G.M.T. at 1st obs. h. m. s.		h. m. s.
3 31 32	S.A.T., - -	21 29 29
2 45 46	eqn. of time, -	14 28
45 46	S.M.T., - -	21 15 1
	G.M.T., - -	45 46
	Long., - -	3 30 45

Long. (*a*), 52° 41′ 15″ W.

2nd Altitude.

co-lat., - - -	49° 0′ 0″	10·122220
polar dist. - -	99 9 30	10·005572
	50 9 30	
zenith dist., - -	57 22 30	
	7 13 0	3·798897
	107 32 0	4·906667
		8·833356

G.M.T. at 2nd obs.		
h. m. s.		h. m. s.
8 1 6	S.A.T., - -	2 1 3
2 45 46	eqn. of time, -	14 30
5 15 20	S.M.T., - -	1 46 33
	G.M.T., - -	5 15 20
	Long., - - -	3 28 47

Long (*c*), 52° 11′ 45″ W.

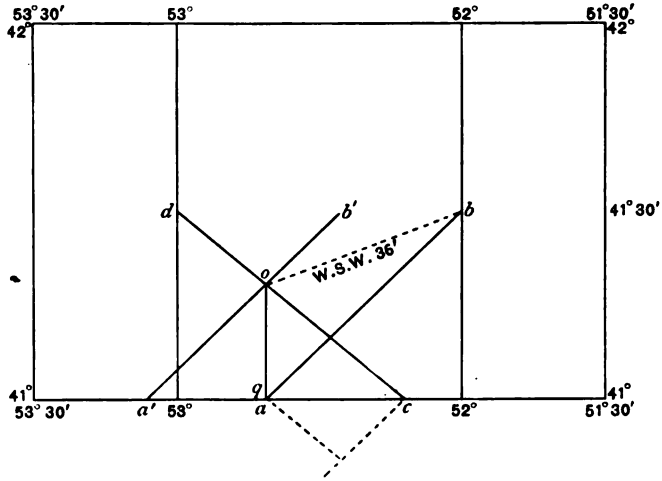


FIG. 104.

Marking on the figure the positions *a*, latitude 41° N, longitude 52° 41′ 15″ W, and *b*, *c*, latitude 41° N, longitude 52° 11′ 45″ W, the lines of position *ab* and *cd* are to be drawn at right angles to the dotted lines, which represent the true bearings of the sun respectively at the two observations. The line *ab* is then projected for the run of the ship into the position *a′b′*. The point *o* in which *a′b′* and *cd* intersect has the position latitude 41° 18′, longitude 52° 42′.

The result might have been obtained by the method of § 132, and then *b* and *d* in the figure would represent (very nearly) the positions when 41° 30′ was assumed to be the latitude. The position lines would then be

obtained by joining *ab*, *cd*. But when azimuth tables are at hand the method of § 135 is much shorter.

If it is desired to find the position by calculation, we have

Lat. of $a' = 41°\ 0'$
Long. „ $= 53\ 8$
$a'c = 56$
Angle $oa'c = 43\ 30$
„ $oca' = 37\ 0$

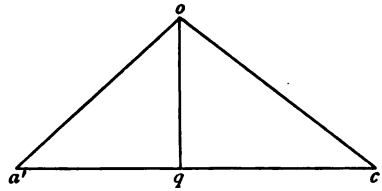


FIG. 105.

$oa' = 56 . \sin 37° . \operatorname{cosec} 99°\ 30'$

56	1·748188
sin 37° 0′	9·779463
cosec 99 30	10·005997
	1·533648

$oq = oa' . \sin 43°\ 30'$

	1·533648
	9·837812
	1·371460

$oq = 23·52$

Mer. parts for 41°, 2701·60
23·52
2725·12

Lat. of $o = 41°\ 18'$

Long. of $a' = 53°\ 8'\ 0''$
$a'q = 24\ 48$

Long. of $o = 52\ 43\ 12$

$a'q = oa' . \cos 43°\ 30'$

	1·533648
	9·860562
	1·394210

$a'q = 24·8$

which agrees exactly with the latitude and is about 1′ different from the longitude already obtained.

(2) *The second altitude being taken with a small bearing from the meridian.* Latitude by account 41° 50′ N, longitude 52° 30′ W, altitudes observed at 8 h. 30 m. A.M. and 11 h. 15 m. A.M. ship time; chronometer times, 10 h. 18 m. 25 s. and 1 h. 3 m. 18 s.; chronometer slow on G.M.T., 1 h. 30 m. 42 s.; altitudes of sun's centre, 9° 52′ 40″ and 25° 23′ 50″; declination, 22° 4′ S and 22° 5′ S; equation of time, 10 m. 50 s. and 10 m. 47 s. subtractive from apparent time; run of the ship in the interval, south 30′; bearings, S 48° 30′ E and S 11° 30′ E.

To find the longitude with latitude 41° 50′ N.

48° 10′ 0″	10·127792
112 4 0	10·033039
63 54 0	
80 7 20	
144 1 20	4·978233
16 13 20	4·149506
	9·288570

G.M.T. h. m. s.		h. m. s.
10 18 25	S.A.T., - -	20 30 44
1 30 42	eqn. of time, -	10 50
23 49 7	S.M.T., - -	20 19 54
	G.M.T., - -	23 49 7
	Long. (*a*),	3 29 13
		52° 18′ 15″ W.

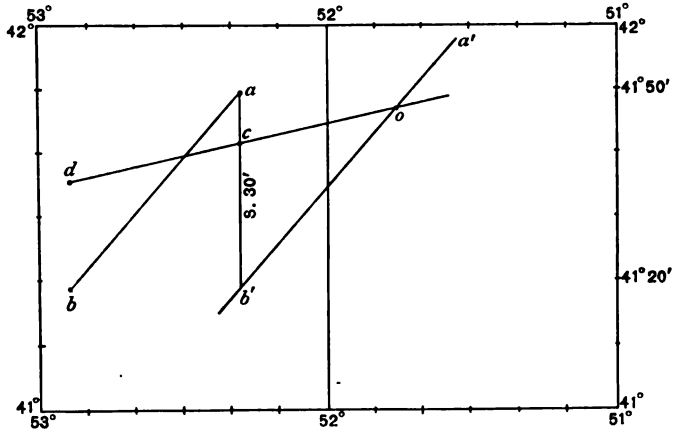


FIG. 106.

From (*a*) latitude 41° 50′ N, longitude 52° 18′ 15″, lay off position line *ab* S 41° 30′ W.

In accordance with § 135, paragraph 5, with estimated latitude 41° 50′, longitude 52° 18′ 15″, obtain a latitude as follows:

	h. m. s.
Chron., - -	1 3 18
error, - -	1 30 42
	2 34 0
Long., - -	3 29 13
S.M.T., - -	23 4 47
eqn. of time, -	10 47
S.A.T., - -	23 15 34

estd. lat.,	41° 50′	9·872208
dec., -	22 5	9·966910
S.A.T., -	23 h. 15 m. 34 s.	7·971632
		7·810750

θ = 9° 13′ 30″

vers zenith dist.,	0571109
vers θ, - -	0012933
	0558176
	7972
	204
Mer. zenith dist.,	63° 46′ 47″
dec., - - -	22 5 0
Lat. (*c*),	41 41 47

Through *c* latitude 41° 42′, longitude 52° 18′, draw the position line *dco* S 78° 30′ W. Project the line *ab* S 30′ to *a′b′*, and *dco* cuts *a′b′* at the position latitude 41° 47′ N, longitude 51° 44′ W.

N.B.—If the altitude at the second observation is greater than 60°, the true bearing cannot be obtained from the ordinary azimuth tables.

In this case Captain Weir's azimuth diagram is of great utility, or the true bearing may be obtained by a method communicated by Mr. W. D. Niven, C.B., Director of Studies at the R.N. College, Greenwich, which is to be found in the latest edition of the Brent-Williams tables:

"Use the altitude as if a meridian altitude; mark the spot so determined *A* on a meridian of a Mercator's chart. Take out or calculate the correction for reduction to the meridian, θ. At a distance from *A*'s meridian equal to one-half the hour angle measured in arc lay off θ from *A*'s parallel. Mark this spot *B*. Join *AB*. *AB* will be a line of position."

N.B.—If the half-hour angle measured in arc be turned into "departure," this and the reduction to the meridian can be laid off on a plan to any convenient scale, thus obviating the necessity of drawing the lines on a chart.

This method must not, however, be used with hour angles which are outside the limits of ex-meridian tables.

Failing one of the above methods, the true bearing must be calculated, or two longitudes assumed and position line found in accordance with § 134, paragraph 3.

Examples.—(1) April 30th, in latitude by account 45° N, longitude by account 140° 30′ W, the following observations were taken to find the ship's position:

App. time nearly.	Chron. time.	Obs. alt. sun's L.L.	True bearing.
h. m.	h. m. s.		
2 11 P.M.	2 5 7	48° 50′ 30″	S 56° W
4 48 P.M.	4 41 25	23 2 40	S 87° W

The error of the chronometer on G.M.T. was 2 h. 35 m. 5 s. fast; the run of the ship in the interval was south (true) 6 miles per hour; index error, 3′ 20″+; height of eye, 20 feet.

(2) December 9th, in latitude by account 37° 15′ N, longitude by account 10° W, the following observations were taken to find the ship's position:

App. time nearly.	Chron. time.	Obs. alt. sun's L.L.	True bearing.
h. m.	h. m. s.		
8 30 A.M.	6 15 27	12° 10′ 20″	S 48° 30′ E
11 30 A.M.	9 15 18	29 26 20	S 8° E

The error of the chronometer on G.M.T. was 2 h. 43 m. 11 s. slow, the run of the ship in the interval was N 12° W (true) 10 miles per hour; index error, 2′ 10″+; height of eye, 23 feet.

(3) April 26th, in latitude by account 54° 0′ N, longitude by account 7° 0′ E, the following observations were taken to obtain the ship's position:

App. time nearly.	Chron. time.	Obs. alt. sun's L.L.	True bearing.
h. m.	h. m. s.		
7 0 A.M.	8 46 19	19° 52′ 30″	S 85° 30′ E
10 30 A.M.	0 17 5	46 6 40	S 36° E

The error of the chronometer on G.M.T was 2 h. 11 m. 3 s. fast; the run of the ship in the interval was NEbN (true) 12 miles per hour; index error, 2′ 50″ −; height of eye, 20 feet.

(4) June 5th, in latitude by account 54° 40′ N, longitude by account 5° 20′ W, the following observations were taken to obtain the ship's position:

App. time nearly.	Chron. time.	Obs. alt. α Lyræ	
h. m.	h. m. s.	E of meridian.	True bearing.
10 15 P.M.	0 0 48	52° 39′ 30″	S 86° E

10 h. 45 m. P.M. a lighthouse (latitude 50° 41′ N, longitude 5° 31′ W) bore S 69° W (true). The speed of the ship was 10 knots; course, N 71° W (true); error of chronometer on G.M.T., 1 h. 26 m. 3 s. fast; index error, 2′ 40″ +; height of eye, 22 ft.

(5) April 27th, in latitude by account 53° 18′ N, longitude by account 5° 35′ W, the following observations were taken to obtain the ship's position:

App. time nearly.	Chron. time.	Obs. alt.	
h. m.	h. m. s.	sun's L.L.	True bearing.
7 30 A.M.	7 23 23	24° 15′ 30″	S 80° E

8 h. 30 m. A.M. a lightship in latitude 53° 4′ 30″ N, longitude 5° 44′ W bore S 60° E (true). The ship ran in the interval S 19° W (true) 10 miles The chronometer was 0 h. 26 m. 47 s. slow on G.M.T.; index error, 2′ 50″ −; height of eye, 20 feet.

(6) The compass bearing of the sun was S 2° 31′ E, the variation being 23° W. Two points A and B on a line of position were determined as follows:

Lat. A,	40° 20′ N	Long. A,	47° 15′ W
„ B,	40 40 N	„ B,	46 10 W

Find the deviation.

(7) From the following data find the ship's true position:

Latitude, 49° 35′ N; sun's declination, 12° 10′ N; true altitude sun's centre, 36° 56′ 30″; sun's true bearing, S 62° E; chronometer showed 10 h. 26 m. 31 s.; error fast on G.M.T., 1 h. 10 m. 1 s.; equation of time (+ to M.T.), 1 m. 29 s.; ship ran N 67° W 36′, when Bishop's Rock (49° 52′ N, 6° 27′ W) bore N 34° W.

N.B.—In the foregoing account of Sumner's method, the Sun alone has been considered, but the principle applies equally well to observations of any other heavenly body, or to simultaneous observations of any two bodies whose azimuths differ considerably.

CHAPTER XV.

DAY'S WORK.

The Day's Work.

§ 141. The various methods of finding a ship's latitude and longitude by observation and dead reckoning, course and distance to or from any point, etc., having been explained, that combination of these methods which is called the Day's Work or Daily Reckoning may be considered.

The ship's position is registered in the log for each day at noon, the latitude being determined, if possible, by meridian altitude of the sun, or by observations near the meridian. But the longitude is obtained by observations taken some hours before noon; and, as the latitude by dead reckoning at the time is probably incorrect, having been worked up from the previous noon, some more correct value of it must be found, or some means must be employed of deducing the correct longitude at time of observation from that found by means of the incorrect latitude. When this has been done, the longitude at noon is deduced by applying the difference of longitude made good between the time of observation and noon.

N.B.—It is advisable to call attention to the fact that the knots entered against any hour in the log are those for the hour which has just ended, *e.g.* the knots entered at 9 o'clock represent the distance run between 8 and 9, and *not* between 9 and 10—a frequent cause of error in the working of examples.

§ 142. *To find the correct latitude at the time the observations for longitude were taken.*

This is called "Working Back."

A Traverse is worked with the courses and distances run by the ship in the interval, and the diff. lat. and diff. long.

found. If the ship has run to the northward during the interval, she must have been further to the southward at the time of observation than she was at noon. Hence the difference of latitude must be applied as south.

If the ship has run to the southward, the opposite will be the case.

§ 142A. Until recent years the correct latitude at the time of the A.M. observation was usually found by "working back" from the latitude obtained by mer. alt. or observation near the meridian, and the longitude at the time of observation was then calculated.

This method has the great disadvantage of compelling a navigator to allow his A.M. observation to remain unused for three or four hours until he has obtained his latitude at noon; it is more suitable for the examination room than for a ship at sea, and may be discarded as unpractical, for those who have the charge of the navigation of a ship seldom if ever wait until after noon to work out their observation for longitude, as they wish to know, as soon as possible, the approximate position of the ship.

Another method has therefore been adopted in H.M.S. Britannia and for the use of Junior Naval Officers afloat which has the merit of enabling a navigator to obtain without delay all the information which his A.M. observation is capable of affording him. It must be borne in mind that *one* observation is not sufficient to fix the position of a ship, but it enables the navigator to lay down a line on which he knows the ship must be (*vide* Chapter XIV. §§ 130-135).

The dead reckoning latitude at the time of the observation for longitude having been found, the observation is worked with this latitude and a longitude obtained.

This position is marked on a *plan* drawn to any convenient scale; and the "position line" is drawn (the sun's true bearing having been obtained by calculation or from the Azimuth Tables).

If one course only is steered between the time of the observation and noon, lay off, from the position obtained, the true course and distance, and through the point thus determined draw a line parallel to the position line; *or*, if more than one course is steered, work a traverse, lay off the diff. lat. and departure thus obtained, and draw a new position line as before.

The point in which this position line intersects the parallel of the latitude at noon is to be marked, the departure made good between observations and noon measured off and turned into difference of longitude.

Hence the latitude and longitude at noon are determined.

If the latitude is obtained from an ex-meridian observation, its position line is to be used in place of the parallel of latitude.

From the point of intersection of the two position lines the further run to noon must be laid off, the latitude of the point thus obtained and the departure taken off the plan, departure turned into difference of longitude and the noon position obtained.

Examples.—(1) At 8 h. 30 m. A.M. latitude by D.R. 48° 28′ N, longitude calculated with this latitude 12° 12′ W. Sun's true bearing S 72° 30′ E; latitude by observation at noon 49° 10′ N, run between 8 h. 30 m. and noon N 31½° W, 32′.

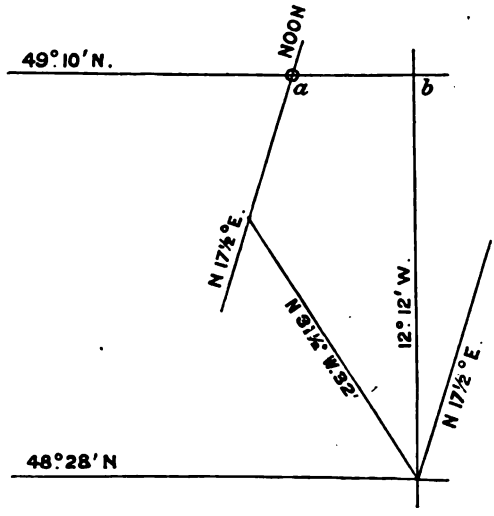


FIG. 107 A.

a marks the position of the ship at noon. *ab* measures the departure 12·5′, which gives 19′ W as the difference of longitude. The ship's position at noon is therefore latitude 49° 10′ N, longitude 12° 31′ W.

(2).—Latitude and longitude at 8 h. 30 m. A.M. as in Ex. (1); latitude 49° 5′ N obtained by ex-meridian observation at 11 h. 30 m. A.M. in estimated longitude 12° 34′ W; sun's true bearing S 13½° E; run to 11 h. 30 m. A.M. N 31½° W, 28′; from 11 h. 30 m. A.M. to noon N 31½° W, 4′.

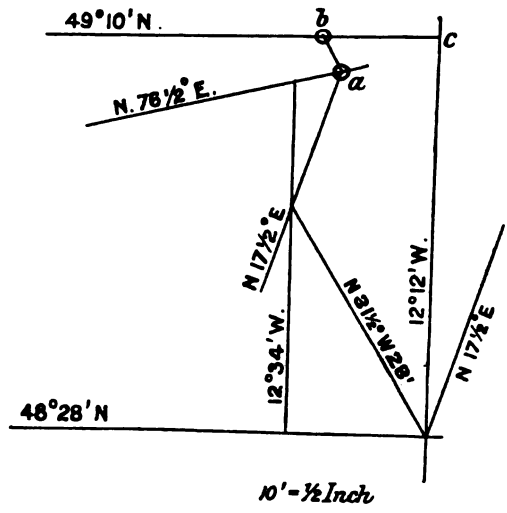


FIG. 107 B.

a marks the position of the ship at 11 h. 30 m. A.M.; *b* the position at noon; *bc* measures the departure 12·5′, and the ship's position at noon is latitude 49° 10′ N, longitude 12°·31′ W, as in Ex. (1).

N.B.—A scale of 10′ to an inch will be found sufficiently large for use in practice.

§ 143. When the true position at noon has been found, the course and distance made good since the previous noon, the current (considered as course and distance from the D.R. to the observed position), the true bearing and distance of a port, a point of land, a danger, or a rendezvous, are to be calculated and entered in the log, as in the worked example.

§ 144. In order to find the compass error and the deviation for some position of the ship's head, an azimuth or amplitude may be worked, the latitude and longitude at time of observation obtained by dead reckoning being sufficiently accurate for practical purposes. The true bearing is, however, usually obtained from azimuth tables.

§ 145. Other methods are available which do not depend on protraction for their accuracy.

The longitude observations are worked out with the latitude by dead reckoning worked up from the previous noon. When the correct latitude has been afterwards obtained, by working back from the correct latitude at noon, a correction is applied to the longitude which has already been obtained, arising from the difference between the true and assumed latitudes at the time of observation.

This correction may be found (1) by working with two latitudes ten miles or so apart on opposite sides of the D.R. latitude (instead of working with the D.R. latitude) and thus obtaining the change of longitude corresponding to a change of a mile of latitude in that part of the earth.

(2) From the formula

change in hour angle $= \frac{1}{15}$ change in lat. $\times$ sec lat. $\times$ cot azimuth, the bearing of the sun being taken at the time of observation or obtained from the Azimuth Tables. (See § 175.)

(3) From Table 25, *Inman's Tables*, new edition, and many other tables in which the results of the above formula are tabulated for different latitudes and bearings.

It is to be remembered that the error in hour angle is of different name to the error in latitude, or of the same name, according as the sun is on the polar or equatorial side of the prime vertical in the forenoon, and *vice versa* in the afternoon. This follows from the sign of cot azimuth.

In the case of ships proceeding at a high rate of speed, it is necessary to obtain the position more often than once a day, but the same principles apply, modified to suit the circumstances.

When using any of the methods of this section it is advisable to construct a rough figure as a check on the application of the difference of longitude.

Example of day's work.—June 1st a ship was in latitude 27° 20′ S, longitude 104° 10′ E, and afterwards sailed as by the following log account. Work up the reckoning to June 2nd at noon.

Hours.	Knots.	Tenths.	Standard Compass Course.	Deviation.	Wind.	Leeway.	Remarks.
						Points.	
1	7	5	S ½W	2° W	WbS	½	
2	8	8					
3	8	2					3 h. 10 m., obs.
4	7	2					for deviation.
5	7	0	NW	2° 15′ W	WbS ½S	1	
6	7	5					
7	6	5					
8	5	8					
9	5	5					
10	4	5					
11	2	2					
12	4	2	N ¾E	3° 20′ E	NWbW	1¼	Midnight.
1	4	8					
2	6	2					
3	6	0					
4	7	8					
5	9	5	SWbW ¾W	0° 40′ W	NNW	0	
6	10	5					
7	11	0					
8	12	0					8 h. 40 m., obs.
9	9	0	SW ¾S	0	WNW	¼	for Long.
10	8	5					Sun's true bearing, N 49° 30′ E.
11	8	5					
12	7	0					Noon, obs. for Lat.

Course and Distance.	Latitude.	Latitude.	Variation allowed.	True Bearing and Distance, Port Louis, Mauritius, { 20° 8′ S 57 29 E
S 79° W 82′	D.R., 27° 46·3′ S	D.R., 102° 57′ E	5° W	N 80° W. 2517′
Current. N 57° W 19′	Obs., 27° 36′ S	Chron., 102° 39′ E		

Observation for latitude.

Obs. mer. alt. sun's L.L., - - - 40° 0′ 40″

Observation for longitude.

Chronometer times.

h.	m.	s.	Obs. alt. sun's L.L.
8	56	22	20° 38′ 50″
	56	46	43 30
	57	4	46 40

On May 22nd at G.M. noon, the chronometer was 4 h. 48 m. 41 s. slow on G.M.T.; daily rate, 3·2 s. losing.

Observation for deviation.

Obs. alt. sun's L.L., - - - 22° 5′ 40″
Sun bore by compass, - - N 40 20 0 W

The index error for all observations was 3′ 20″+, and the height of the eye was 20 feet.

Observation for Longitude.

Working the Dead Reckoning up to 8 h. 40 m. A.M., we obtain latitude 27° 22·3′ S, longitude 103° 12′ E.

	h. m.
June 1st, - - -	20 40
Long. in time, approx., -	6 53
Greenwich app. time, -	13 47
equation of time, approx.,	2 −
Greenwich date, - -	13 45

May 22nd to June 1st at $13\frac{3}{4}$ hours.
$= 10\frac{55}{88}$ days.
$10\frac{55}{88} \times 3{\cdot}2 = 34$ s.
∴ accumulated rate = 34 s. slow.

	Chronometer. h. m. s.
	8 56 22
	46
	64
	132
chron. showed, - -	8 56 44
error, - - -	4 48 41 slow.
accumulated rate, -	34 slow.
G.M.T., - - -	13 45 59

Obs. alt. sun's L.L.	Declination.	Change.	Eq. of time. m. s.	Change.
20° 38′ 50″	22° 11′ 53″ N	19·9″	2 17·5	·38
43 30	3 24	$10\frac{1}{4}$	3·9	$10\frac{1}{4}$
46 40	22 8 29	204	2 21·4	3·9
129 0	90 0 0	3′ 24″	− app. time.	
20 43 0	112 8 29			
3 20				
20 46 20				
4 24				
20 41 56				
2 25				
20 39 31				
15 48				
20 55 19				
90 0 0				
69 4 41 zen. dist.				

			h. m. s.	
co-lat., -	62° 37′ 42″	10·051563	20 40 35·8	ship app. time.
pol. dist., -	112 8 29	10·033270	2 21·4	equation of time.
	49 30 47	4·934404	20 38 14·4	S.M.T.
zen. dist., -	69 4 41	4·230234	13 45 59	G.M.T.
	118 35 28	9·249471	6 52 15·4	Long. in time.
	19 33 54		Longitude 103° 3′ 45″ E.	

The Sun's True Bearing is found by calculation or from the Azimuth Tables to be N 49° 30′ E.

The ship is therefore somewhere on the line *ab* drawn S 40° 30′ E from the position, latitude 27° 22·3′ S, longitude 103° 3′ 45″ E.

The run of the ship from 8 h. 40 m. to noon is S 29° W, 27′. Project the line *ab* for this run into the position *cd*.

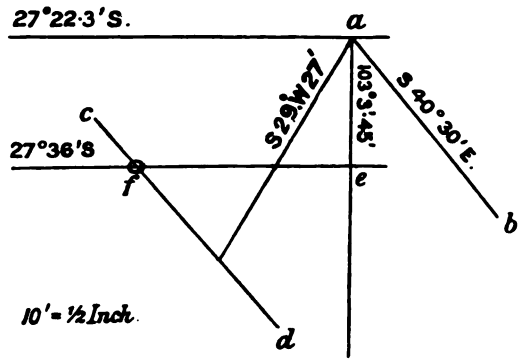


FIG. 108.

Observation for Latitude.

By correcting the latitude and longitude by dead reckoning at 8 h. 40 m. A.M. for the run of the ship to noon we obtain latitude and longitude by dead reckoning at noon 27° 46′ S, 102° 57′ E.

The sun's declination obtained with the Greenwich date corresponding to this longitude is 22° 9′ 39″ N, or it may be found by considering that noon is 3 h. 20 m. later than 8 h. 40 m. A.M., multiplying the hourly variation by 3⅓ and applying the result to the declination already obtained at that time, the small change of longitude in the interval causing no practical difference.

40°	0′	40″	obs. alt.
	3	20	
40	4	0	
	4	24	
39	59	36	
	1	3	
39	58	33	
	15	48	
40	14	21	
90	0	0	
49	45	39	S
22	9	39	N
27	36	0	S

Latitude observed at noon, 27° 36′ S.

Draw *ef* on the figure to represent the parallel of 27° 36′ S, *i.e.* draw it 13·7′ south of *a*. The point *f* in which it cuts *cd* is the position of the ship at noon. The distance *ef* or the departure=21·8′, which represents 24·7′ difference of longitude.

Longitude obtained at 8 h. 40 m. A.M., -	103°	3′	45″ E
Difference of longitude, from plan, -		24	42 W
Longitude at noon, - - - -	103	39	3 E

Ship's position at noon { Latitude, - 27° 36′ S
Longitude, - 103 39 E

To find the course and distance made good.

Lat.	Long.
27° 20′ S	104° 10′ E
27 36 S	102 39 E
16 S diff. lat.	1 31 W
	60
	91 diff. long.
	80·7 departure W
S 79° W	82′ (by inspection).

To find the current.

Lat.	Long.
27° 46′ S D.R.	102° 57′ E D.R.
27 36 S obs.	102 39 E chron.
10 N diff. lat.	18 W diff. long.
	16 W dep.
N 57° W	19′ (by inspection).

To find the true bearing and distance.

Lat.	Mer. parts.	Long.
27° 36′ S	1724·03	102° 39′ E
20 8 S	1233·66	57 29 E
7 28 N	490·37 mer. diff. lat.	45 10 W
60		60
448 N diff. lat.		2710 W diff. long.

13·432969	10·749590
2·690524	2·651278
10·742445	3·400868
N 79° 44′ 45″ W	2517′

True bearing of Port Louis, - - - N 80° W
Distance „ „ - - - 2517′

To find the position at 3 h. 10 m. p.m., June 1st.

		Diff. lat.	Dep.	Diff. long.
S 7° E	25′	24·8 S	3 E	3·4 E

Lat., 27° 20′ S
25 S
27 45 S

Long., 104° 10′ E
3 E
104 13 E

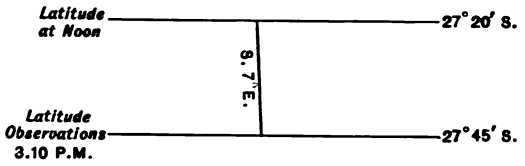


FIG. 109.

Observation for deviation.

h.	m.		Declination at app. noon, June 1st.	Change
3	10	June 1st.	22° 3′ 54·8″	20·5
6	57	Long.	1 18	3·8
20	13	Gr. ap. time, May 31st.	22 2 37 N	77·9
			90 0 0	
3	47		112 2 37 polar dist.	

Obs. alt.			
22° 5′ 40″	co-lat., -	62° 15′	10·053063
3 20	zen. dist., -	67 42	10·033760
22 9 0		5 27	4·931921
4 24	polar dist., -	112 3	4·904053
22 4 36		117 30	9·922797
2 15		106 35	
22 2 21			
15 48			
22 18 9			
90 0 0			
67 41 51	zen. dist.,		

True bearing, - - -	S 132° 24′ W	
	180 0	
	N 47 36 W	
Compass bearing, - -	N 40 20 W	
Compass error, - - -	7 16 W	
Variation, - - -	5 0 W	
Deviation, - - -	2 16 W	

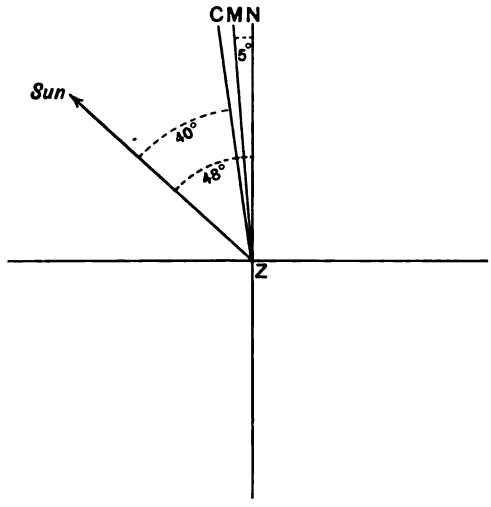


FIG. 110.

$NZS = 47°\ 36'$,
$CZS = 40\ \ 20$;
$\therefore$ $NZC = 7\ \ 16$, and is W, because C falls to the *left* of N.
$NZM = 5$;
$\therefore$ MZC = deviation = $2°\ 16'$, and is W, because C falls to the *left* of M.

§146. If the longitude observations are worked without waiting to obtain the latitude by meridian altitude, we may proceed thus:

(1) *Working with two latitudes, 10 miles (or any other convenient number) apart.*

Latitude by D.R. at 8 h. 40 m. is 27° 22·3′ S. Assume two latitudes, 27° 25′ S and 27° 15′ S.

62°	35′	0″	10·051743	62°	45′	0″	10·051090
112	8	30	10·033270	112	8	30	10·033270
			4·934505				4·934129
49	33	30	4·229218	49	23	30	4·232876
69	4	41		69	4	41	
			9·248736				9·251365
118	38	11		118	28	11	
19	31	11		19	41	11	

With lat. 27° 25′.				With lat. 27° 15′.			
h.	m.	s.		h.	m.	s.	
20	40	46·4	ship app. time.	20	40	8	
	2	21·4			2	21·4	
20	38	25		20	37	46·6	
13	45	59		13	45	59	
6	52	26		6	51	48	

Long.,	- 103° 6′ 30″	102° 57′ 0″
	102 57 0	
	9 30	

∴ error of longitude = ·95′ for 1′ error of latitude, and the greater the latitude the greater the longitude.

The correct latitude at time of observation (cf. § 142) is found to be 27° 12′.

Hence the latitude 27° 15′ being 3′ too great, the resulting longitude is 3′ × ·95 too great.

∴ the correct longitude at 8 h. 40 m. A.M. is

$$102° 57′ - 2′ 51″ = 102° 54′ 9″,$$

which agrees very nearly with the longitude obtained with lat. 27° 12′.

(2) *Working with the D.R. latitude, and applying the correction error in long. = error in lat. × sec lat. × cot azimuth.*

		h. m. s.
62° 37′ 42″	10·051563	20 40 35·5
112 8 30	10·033270	2 21·4
	4·934404	
49 30 48	4·230234	20 38 14
69 4 41		13 45 59
	9·249471	
118 35 29		6 52 15
19 33 54		Long., 103 3 45 E

Error in latitude =	0° 10·2′	1·008600
Latitude, - -	27 12	10·050895
Azimuth, -	N 49 30 E	9·931499
		0·990994
		9·8 = 9′ 48″
Longitude, - -	103° 3′ 45″	
Correction, - -	9 48 −	
	102 53 57	

The correction from table 25, page 110 of *Inman's Tables*, new edition, gives ·95′ as the error in longitude per mile of latitude—practically the same result as that just obtained by calculation.

Hence it is seen that the longitudes obtained by the various methods agree within a few seconds of arc.

N.B.—If the current is large, it may be further necessary to correct the longitude for the effect of the current between the time when the observations for longitude were taken and noon.

This is, however, rarely necessary in the ordinary work of navigation.

The expression "true bearing" is a technical expression for the true course on a Mercator's chart, from the observed position of the ship to the position of the port, etc.

Examples.—(1) April 2nd, at noon, a ship was in latitude 22° 21′ N, longitude 122° 3′ E; afterwards sailed as by following log account. Work up the reckoning to April 3rd at noon.

Hours.	Knots.	Tenths.	Standard Compass Course.	Deviation.	Wind.	Leeway.	Remarks.
						Points.	
1	6	8	NbW ¾W	3° 10′ W	NE	¾	
2	7	2					
3	7	8					3 h. 30 m., obs.
4	8	0					for deviation.
5	7	2					
6	6	8	SEbE ¾E	2° 50′ W	NE	½	
7	7	2					
8	7	8					
9	7	5					
10	6	5					
11	6	2					
12	6	8	NbE	1° E	EbN	½	Midnight.
1	7	2					
2	7	5					
3	7	5					
4	8	2					
5	7	8					
6	7	5	SEbS	3° 10′ W	ENE	¼	
7	7	2					7 h. 30 m., obs.
8	6	8					for Long.
9	6	5					Sun's true bear-
10	7	8	NNE	3° E	East	½	ing, S 86° E.
11	8	0					
12	9	2					Noon, obs. for Lat.

Course and Distance made good.	Latitude.	Longitude.	Variation allowed.	True bearing and distance of Cape Sagarni, { Lat. 35° 8′ N / Long. 139 41 E
Current.	D.R., Obs.,	D.R., Chron.,	1° E	

Observation for latitude.

Obs. mer. alt. sun's L.L., - - 71° 32′ 20″ (zenith N)

Observation for longitude.

Chronometer times. h. m. s.	Obs. alt. sun's L.L.
9 18 13	22° 27′ 0″
18 29	31 20
18 46	35 10

On March 21st, at G.M. noon, the chronometer was 2 h. 2 m. 59 s. slow on G.M.T., daily rate 3·3 s. losing.

Observation for deviation.

Obs. alt. sun's L.L., - - - 36° 11′ 40″

Sun bore by compass, - - - S 81° W

The index error for all observations was 2′ 50″ -, and the height of the eye was 22 feet.

(2) April 26th, at noon, a ship was in latitude 23° 50′ S, longitude 36° 10′ E, and afterwards sailed as by the following log account. Work up the reckoning to next day at noon.

Hours.	Knots.	Tenths.	Standard Compass Course.	Deviation.	Wind.	Leeway.	Remarks.
						Points.	
1	8	0	SW ½W	3° W	SbE	1¼	
2	7	5					3 h. 30 m., obs.
3	6	8					for deviation.
4	6	8					
5	6	2					
6	6	2					
7	6	5					
8	6	8	EbS ½S	3° E	South	½	
9	7	5					
10	8	2					
11	8	5					
12	9	0					Midnight.
1	8	8	SEbE	2° E	SbW	¾	
2	8	5					
3	8	2					
4	7	5					
5	6	0	W ½N	4° W	NbW	¾	
6	6	8					
7	6	2					
8	6	8					8 h. 30 m., obs.
9	7	2					for Long.
10	7	2	NEbE	2° 20′ E	ESE	½	Sun's true bear-
11	7	8					ing, N 59° E.
12	8	0					Noon, obs. for Lat.

Course and Distance.	Latitude.	Longitude.	Variation allowed.	True bearing and distance of Sunda Strait, { Lat. 6° 9′ S / Long. 105 27 E
Current.	D.R., Obs.,	D.R., Chron.,	27° W	

Observation for latitude.

Obs. alt. sun's L.L., - - 51° 30′ 30″ (zenith S)

Observation for longitude.

Chronometer times. h.	m.	s.	Obs. alt. sun's L.L.		
8	47	42	25°	40′	20″
	47	57		43	20
	48	18		47	50

April 14th, at G.M. noon, the chronometer was 2 h. 46 m. 35 s. fast on G.M.T., rate 3·2 s. gaining.

Observation for deviation.

The sun bore by compass N 29° 30′ W when the chronometer showed 3 h. 51 m. 3 s.

The index error for all observations was 1′ 50″−, and the height of the eye was 22 feet.

(3) June 3rd, at noon, a ship was in latitude 40° 40′ N, longitude 47° 15′ W, and afterwards sailed as by the following log account. Work up the reckoning to next day at noon.

Hours.	Knots.	Tenths.	Standard Compass Course.	Deviation.	Wind.	Leeway.	Remarks.
						Points.	
1	7	5	NE ¾E	2° 30′ E	SEbE	¼	
2	7	8					
3	8	2					
4	8	0					
5	7	5					
6	6	5	SbE ¼E	1° W	EbS	1¼	
7	6	2					
8	6	0					
9	5	8					
10	5	5					
11	6	0	E ¼S	1° 30′ E	NNE	¾	
12	7	0					Midnight.
1	7	5					
2	7	8					
3	7	2					
4	7	5					
5	6	5	NNE	3° E	East	½	
6	6	5					7 h. 30 m., obs. for Longitude and deviation.
7	6	2					
8	6	8					
9	7	5	NWbN	2° 30′ W	NEbN	½	Sun's true bearing, N 86° E.
10	7	8					
11	8	2					
12	8	5					Noon, obs. for Lat.

Course and Distance.	Latitude.	Longitude.	Variation allowed.	True bearing and distance of Tory Island, { Lat. 55° 16′ N / Long. 8 15 W
Current.	D.R., Obs.,	D.R., Chron.,	23° W	

Observation for latitude.

Obs. mer. alt. sun's L.L., - - 70° 38′ 0″ (zenith N)

Observation for longitude.

Chronometer times. h.	m.	s.	Obs. alt. sun's L.L.		
6	55	36	30°	58′	10″
	55	53	31	1	20
	56	16	31	6	0

May 20th, at G.M. noon, the chronometer was 3 h. 32 m. 5 s. slow on G.M.T., rate 4·8 s. losing.

Observation for deviation.

Sun bore by compass S 73° 40′ E.

The index error for all observations was 2′ 30″ −, and the height of the eye was 21 feet.

(4) December 3rd, at noon, a ship was in latitude 18° 45′ S, longitude 99° 18′ E, and afterwards sailed as by the following log account. Work up the reckoning to next day at noon.

Hours.	Knots.	Tenths.	Standard Compass Course.	Deviation.	Wind.	Leeway.	Remarks.
						Points.	
1	8	8	NE ¼N	3° 30′ E	ESE	¼	
2	9	2					4 h. 30 m., obs. for deviation.
3	9	5					
4	8	5					
5	8	0					
6	7	8	SbE	1° 15′ W	EbS	¾	
7	7	5					
8	7	2					
9	7	5					
10	8	0					
11	9	2	EbE ¼N	2° E	SEbS	¼	
12	9	8					Midnight.
1	10	2					
2	10	5					
3	9	8					
4	9	5					
5	7	2	N ¼E	2° 20′ E	EbN	1	
6	7	2					
7	6	8					8 h., obs. for Long.
8	6	0					
9	5	8					Sun's true bearing, S 74½° E.
10	6	2	NNE	2° 25′ E	East	½	
11	6	8					
12	7	0					Noon, obs. for Lat.

Course and Distance.	Latitude.	Longitude.	Variation allowed.	True bearing and distance of Java Head, { Lat. 6° 47′ S / Long. 105 11 E
Current.	D.R., Obs.,	D.R., Chron.,	30° W	

Observation for latitude.

Obs. mer. alt. sun's L.L., - - 84° 28′ 30″ (zenith N)

Observation for longitude.

Chronometer times. h.	m.	s.	Obs. alt. sun's L.L.		
4	27	35	33°	18′	50″
	27	51		22	30
	28	13		27	50

November 15th, at G.M. noon, the chronometer was 3 h. 17 m. 33 s. fast on G.M.T., rate 4 s. gaining.

Observation for deviation.

Sun bore by compass N 79° 40′ W ; chronometer showed 1 h. 0 m. 57 s.

The index error for all observations was 1′ 50″+, and the height of the eye was 24 feet.

CHAPTER XVI.

LONGITUDE BY LUNAR.

Longitude by Lunar Observations.

§ 147. As the moon revolves round the earth, it is seen to be continually changing its distance from the sun and certain of the bright stars which lie near its path. The angular distance of the moon from such of these bodies as are conveniently situated has been calculated and inserted in the *Nautical Almanac* for intervals of three hours of Greenwich time, viz. at noon, 3 h., 6 h., etc.

These distances are such as would be seen from the centre of the earth. Hence, the distance which an observer measures, or the *apparent* distance, must be corrected for the effects of parallax and refraction so as to obtain the *true* distance at the time of observation as referred to the centre of the earth. This operation is called "clearing the distance." When this true distance has been obtained, if it exactly agrees with one of the true distances in the *Nautical Almanac*, the Greenwich time of the observation is found; if it does not agree exactly, the Greenwich time may be found by a simple proportion, with a correction in certain cases, arising from the fact that the moon's distance is changing irregularly. This is called the correction for "second differences," a rule for its application being given at the end of the *Nautical Almanac*. The Ship Mean Time may be found from the observed altitude of a heavenly body, and then the difference between this S.M.T. and the G.M.T. gives the longitude in time.

Whatever method may be employed the calculation is always laborious, but the great advantage is that the longitude may be obtained independently of the error of chronometer on Greenwich Mean Time. If it should be *necessary* to make

use of this lunar method, observations should, of course, be taken of every heavenly body whose lunar distance is recorded in the *Nautical Almanac* for the required day, and which is suitable for observation; the mean of all the results being, of course, much more likely to be correct than the result obtained from one body.

To clear the distance.

§ 148. Given the apparent distance, and the apparent and true altitudes of the moon and some other heavenly body, to find the true distance. Let M, M'; S, S' be the true and apparent places of the moon and other heavenly body. Then M will be above M', since the moon is more depressed by parallax than it is raised by refraction; while S will be below S' for the opposite reason. Z being the zenith, ZM' and ZS are practically circles of altitude, as the effect of parallax may be supposed to take place in a vertical circle, which it does very nearly.

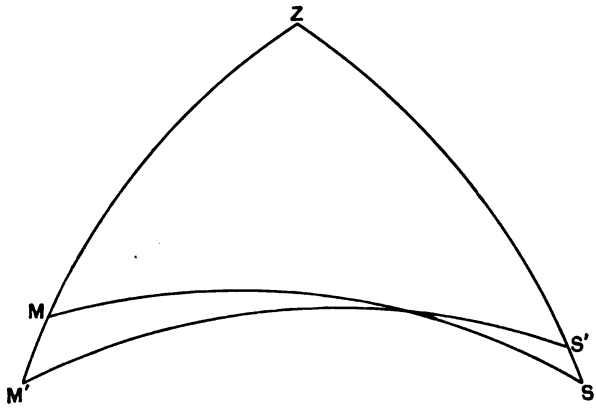


FIG. 111.

Let $ZM = z$, the true zen. dist. of the moon.

$ZS = z_1$, the true zen. dist. of the sun or star.

$ZM' = 90° - a$, a being the apparent altitude of the moon.

$ZS' = 90° - a_1$, a_1 being the apparent altitude of the sun or star.

$S'M' = d$, the apparent distance of the centres of the two bodies.

$SM = D$, the true distance of the centres of the two bodies.

In the triangle ZMS

$$\cos Z = \frac{\cos D - \cos z \,.\, \cos z_1}{\sin z \,.\, \sin z_1}.$$

In the triangle $ZM'S'$

$$\cos Z = \frac{\cos d - \sin a \,.\, \sin a_1}{\cos a \,.\, \cos a_1},$$

$$\therefore \quad \frac{\cos D - \cos z \,.\, \cos z_1}{\sin z \,.\, \sin z_1} = \frac{\cos d - \sin a \,.\, \sin a_1}{\cos a \,.\, \cos a_1},$$

$$\therefore \quad \frac{\cos D - \cos z \,.\, \cos z_1}{\sin z \,.\, \sin z_1} + 1 = \frac{\cos d - \sin a \,.\, \sin a_1}{\cos a \,.\, \cos a_1} + 1,$$

$$\therefore \quad \frac{\cos D - (\cos z \,.\, \cos z_1 - \sin z \,.\, \sin z_1)}{\sin z \,.\, \sin z_1} = \frac{\cos d + \cos a \,.\, \cos a_1 - \sin a \,.\, \sin a_1}{\cos a \,.\, \cos a_1},$$

$$\therefore \quad \frac{\cos D - \cos(z + z_1)}{\sin z \,.\, \sin z_1} = \frac{\cos d + \cos(a + a_1)}{\cos a \,.\, \cos a_1},$$

$$\therefore \quad \cos D - \cos(z + z_1) = \frac{\sin z \,.\, \sin z_1}{\cos a \,.\, \cos a_1}\Big\{\cos d + \cos(a + a_1)\Big\}.$$

In order to simplify this expression, assume

$$2 \cos A = \frac{\sin z \,.\, \sin z_1}{\cos a \,.\, \cos a_1}, \; A \text{ being an auxiliary angle,}$$

$$\begin{aligned}\therefore \quad \cos D - \cos(z + z_1) &= 2 \cos A \cos d + 2 \cos A \cos(a + a_1) \\ &= \cos(d + A) + \cos(d \sim A) + \cos(\overline{a + a_1} + A) \\ &\quad + \cos(\overline{a + a_1} \sim A),\end{aligned}$$

$$\begin{aligned}\therefore \quad 1 - \cos D &= 1 - \cos(z + z_1) + 1 - \cos(d + A) + 1 - \cos(d \sim A) \\ &\quad + 1 - \cos(\overline{a + a_1} + A) + 1 - \cos(\overline{a + a_1} \sim A) - 4,\end{aligned}$$

$$\begin{aligned}\therefore \quad \operatorname{vers} D &= \operatorname{vers}(z + z_1) + \operatorname{vers}(d + A) + \operatorname{vers}(d \sim A) \\ &\quad + \operatorname{vers}(a + \overline{a_1} + A) + \operatorname{vers}(\overline{a + a_1} \sim A) - 4,\end{aligned}$$

or $$\begin{aligned}\operatorname{vers} D &= \operatorname{vers}(z + z_1) + \operatorname{vers}(d + A) + \operatorname{vers}(d \sim A) \\ &\quad + \operatorname{vers}(a + \overline{a_1} + A) + \operatorname{vers}(\overline{a + a_1} \sim A) - 4{,}000{,}000,\end{aligned}$$

where the versines are understood to be *tabular* versines.

Hence the *true distance* D may be found.

§ 149. Next, *to find the corresponding Greenwich mean time.*

Let d = difference of the distances in the *Nautical Almanac*, between which the computed true distance lies.

d' = difference between the former of these distances and D.

t = time elapsed since the hour corresponding to the former distance.

Then, if the moon's change of distance is considered to be uniform throughout the three hours,

$$d : d' :: 3 \text{ hours} : t,$$

$$\therefore \quad t = \frac{3d'}{d}.$$

§ 150. But as the calculation might be somewhat laborious, it is shortened by the use of what are called "proportional logarithms."

[The difference between the log of any number and that of a constant number larger than itself is called the "proportional logarithm" of the number.

In the ordinary tables of proportional logarithms used in navigation, the log of any number of hours minutes and seconds less than 3 hours, or of degrees minutes and seconds less than 3 degrees, has been subtracted from the log of 3 hours or 3 degrees, and the result formed into a table.]

We have $$t = \frac{3d'}{d},$$

$$\therefore \quad \log t = \log 3 \text{ h.} + \log d' - \log d,$$

$$\therefore \quad \log 3 \text{ h.} - \log t = \log 3^\circ - \log d' - (\log 3^\circ - \log d),$$

$$\therefore \quad \text{prop. log } t = \text{prop. log } d' - \text{prop. log } d;$$

prop. log d is found in the column "P.L. of diff." in the *Nautical Almanac*.

Hence t is found, and being added to the hour corresponding to the first distance, the result is G.M.T., the difference between which and the S.M.T. gives the longitude in time.

§ 151. It is seldom or never, however, the case that the moon's distance from a heavenly body increases or decreases regularly, and the change of distance cannot therefore be considered as increasing or decreasing uniformly during a period of three hours.

The correction for second differences is therefore to be applied according to the rule in the *Nautical Almanac* (explanation of pages xiii. to xviii. of each month), to obtain a more correct result. Its effect, however, is not of much importance unless the "prop. log of difference" changes rapidly.

NOTE.—The calculation of the table for finding the auxiliary angle A will be found in § 191.

§ 152. The "P.L. of diff." column points out which are the most suitable bodies for observation. From the definition of proportional logarithms it appears that the greater the number the smaller will be its proportional logarithm.

The moon is, therefore, changing its distance most rapidly from those bodies which have the smallest "P.L. of diff.," and these bodies are most suitable; for the more rapidly the distance is changing, the smaller will be the error in time produced by an error in distance.

§ 153. The bodies whose distances from the moon are inserted in the *Nautical Almanac* are those whose declinations do not lie outside the limits of the moon's declination, as the change of its distance from such bodies will proceed more regularly and rapidly than it would in the case of bodies whose declinations differed much from that of the moon. Fomalhaut is the only one whose declination exceeds that of the moon, but only slightly, and it is useful as a star of south declination with no other bright star in its neighbourhood.

Example.—Sun Lunar. April 16th, about 8 h. A.M. apparent time, in latitude 32° 23′ S, longitude by account 74° 45′ E, the following lunar was observed:

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. distance.
	18° 41′ 40″	58° 14′ 40″	99° 11′ 30″
Index error,	1 20 +	2 50 −	2 10 +

Height of eye, 22 feet.

	h.	m.		Sun's declination.			Equation of time. s.	
S.A.T., - -	20	0	15th.	10° 7′ 57″		53·2	13·1	·61
eqn. of time, -	0	0	−	7 59	−	9	5·5	9
S.M.T., - -	20	0		9 59 58	N	478·8	7·6	5·49
Long., - -	4	59					Sub. from app. time.	
G.D., - -	15	1		Sun's S.D., 15′ 58″				

Moon's S.D.	Moon's hor. par.	Obs. dist.
14′ 59″	54′ 54″	99° 11′ 30″
12		2 10
15 11		99 13 40
		15 58
		15 11
		99 44 49

obs. alt., sun,	18° 41′ 40″		obs. alt., moon,	58° 14′ 40″	
	1 20			2 50	
	18 43 0			58 11 50	
	4 37			4 37	
	18 38 23			58 7 13	
	15 58			15 11	
app. alt.,	18 54 21		app. alt.,	58 22 24	
	2 43			28 11	Auxiliary angle A
	18 51 38			58 50 35	60° 25′ 27″
	90 0 0			90 0 0	27
					2
zen. dist.,	71 8 22		zen. dist.,	31 9 25	60 25 56

To calculate the true distance.

Moon's zen. dist.,		31° 9′ 25″		
Sun's „		71 8 22		
			Versines.	
	sum,	102 17 47 vers.	1212746	223
Moon's app. alt.,		58° 22′ 24″		
Sun's „		18 54 21		
	sum,	77 16 45		
Auxiliary angle A,		60 25 56		
	sum,	137 42 41 vers.	1739631	134
	diff.,	16 50 49 vers.	0042849	68
distance,		99° 44′ 49″		
Angle A,		60 25 56		
	sum,	160 10 45 vers.	1940684	75
	diff.,	39 18 53 vers.	0226160	162
			662	662
			5162732	
			4000000	
			1162732	
			465	
			267	
true distance,		99° 21′ 56″	Prop. log. of diff.	
distance at XV. h.,		99 22 43	·33410	
		47	P.L., 2·36133	

prop. log. of elapsed time, 2·02723

	h.	m.	s.
elapsed time, - - -		1	41
	15	0	0
G.M.T., - - - -	15	1	41

(*N.B.*—In practice it is sufficient to take the last five figures of the versines and of their sum. The required distance will then be found in the column headed by the degrees in the observed distance, or in one of the columns on either side of it. The above versines will then be arranged thus:

12746	223
39631	134
42849	68
40684	75
26160	162
662	662
62732	

The nearest corresponding five figures are 62465, in the column headed 99°, and the distance is 99° 21′ 56″ as before.)

To find S.M.T. and the longitude.

			h.	m.	s.
57° 37′ 0″	cosec 10·073409	Sun's hour angle, - -	20	0	8
100 0 0	cosec 10·006649	eqn. of time, - - -			8
42 23 0		S.M.T., - - - -	20	0	0
71 8 22		G.M.T., - - - -	15	1	41
113 31 22	½ hav 4·922416	Long. in time, -	4	58	19
28 45 22	½ hav 4·395043				
	9·397517	Longitude, 74° 34′ 45″ E.			

§ 154. If the lunar distance of a star or planet is observed, the method of calculation is similar. It is, however, necessary to notice whether the distance of the star is observed from the nearer or farther limb of the moon; the moon's semi-diameter being added to the observed distance in the former case and subtracted from it in the latter, in order to obtain the distance of the centres.

There is no distinction of cases with regard to the sun, the bright limb of the moon being always turned towards it, and the observed lunar distance being always that of the nearer limbs.

Examples.—(1) April 1st, about 3 h. P.M. apparent time, in latitude 47° 15′ N, longitude by account 67° 30′ W, the following lunar was observed:

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. dist. nearer limbs.
	32° 26′ 40″	58° 14′ 20″	75° 56′ 30″
Index error,	2 20−	1 30+	2 40+

Height of eye, 20 feet.

(2) June 3rd, about 7 h. 30 m. P.M. apparent time, in latitude 17° 10′ N, longitude by account 70° 10′ E, the following lunar was observed:

	Obs. alt. Pollux (W of meridian).	Obs. alt. moon's L.L.	Obs. dist. nearer limbs.
	26° 22′ 30″	54° 35′ 20″	93° 33′ 50″
Index error,	0 50+	1 20−	2 30+

Height of eye, 18 feet.

(3) April 15th, about 7 h. 30 m. A.M. apparent time, in latitude 18° 45′ N, longitude by account 136° E, the following lunar was observed:

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. dist. nearer limbs.
	23° 58′ 40″	26° 9′ 50″	112° 51′ 30″
Index error,	2 0−	1 30−	1 30−

Height of eye, 22 feet.

(4) December 24th, about 4 h. P.M. apparent time, in latitude 37° 42′ S, longitude by account 93° E, the following lunar was observed:

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. dist. nearer limbs.
	37° 9′ 10″	36° 54′ 30″	91° 59′ 40″
Index error,	1 30+	2 10−	1 40+

Height of eye, 23 feet.

(5) December 2nd, about 9 h. 30 m. P.M. apparent time, in latitude 13° 5′ N, longitude by account 60° W, the following lunar was observed:

	Obs. alt. α Pegasi (W of meridian).	Obs. alt. moon's L.L.	Obs. dist. F.L.
	45° 1′ 30″	42° 25′ 0″	87° 35′ 20″
Index error,	3 10−	2 20+	1 50+

Height of eye, 25 feet.

(6) December 3rd, about 1 h. 30 m. A.M. apparent time, in latitude 32° 47′ N, longitude by account 15° 50′ W, the following lunar was observed:

	Obs. alt. Jupiter's *centre* (E of meridian).	Obs. alt. moon's L.L.	Obs. dist. N.L.
	51° 49′ 40″	78° 1′ 50″	48° 13′ 10″
Index error,	1 20−	1 50+	2 10−

Height of eye, 18 feet.

(7) December 24th, about 1 h. 30 m. P.M. apparent time, in latitude 35° 12′ S, longitude by account 105° E, the following lunar was observed:

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L. (E of meridian).	Obs. dist. N.L.
	66° 58′ 0″	16° 4′ 10″	90° 48′ 20″
Index error,	1 30 −	1 50 +	1 40 −

Height of eye, 21 feet.

§ 155. If from any cause, such as an indistinct horizon, the altitudes of the moon and other heavenly body cannot be conveniently observed, they may be calculated, provided that the error of the chronometer on ship mean time is known with some accuracy.

This error may be obtained by an observation of some convenient heavenly body taken shortly before the lunar observation, any change of longitude in the interval being allowed for. The estimated longitude will be sufficiently accurate for obtaining a Greenwich date with which to correct the necessary astronomical elements.

The hour angles of the moon and other heavenly body being obtained from the expression

hour angle of body

= ship mean time + R.A. mean sun − R.A. of body,

or (in the case of the sun)

= ship mean time ± equation of time;

the zenith distances may then be calculated by means of the ordinary rule for finding the third side when two sides and the included angle are known.

Thus the true altitudes are known, and the apparent altitudes are obtained by *adding* the correction in altitude in the case of the sun or a star, and *subtracting* it in the case of the moon. It must be here noticed that the correction for the moon's altitude, taken out for the true altitude, is only an approximation, as the correction is calculated for the apparent altitude, the true correction being that taken out for the approximate apparent altitude. The same principle holds in the case of the sun or a star, but the corrections are so small in all probable cases that a second approximation is usually unnecessary.

The apparent altitudes having been obtained, the Greenwich mean time is found as before, and, the ship mean time being known, the longitude can be obtained.

§ 156. In the example which follows, which is practically the same as that worked out for the first method, the S.M.T. is taken 28 seconds different from that which is there obtained, in order to show that a somewhat considerable error in the S.M.T. does not cause a large error in the resulting longitude, provided that the lunar distance is correctly observed.

Example.—April 16th, about 8 h. A.M. apparent time, in latitude 32° 23′ S, longitude by account 74° 45′ E, the obs. dist. of the nearer limbs of the sun and moon was 99° 11′ 30″, when a chronometer, whose error on S.M.T. was 2 h. 15 m. 17 s. fast, showed 10 h. 15 m. 45 s. The index error was 2′ 10″+.

	h.	m.	s.		Sun's declination.		Equation of time. s.	
chron., -	10	15	45		10° 7′ 57″	53·2	13·1	·61
error fast, -	2	15	17		7 59	9	5·5	9
	8	0	28		9 59 58 N	478·8	7·6	5·49
	12	0	0				+ to M.T.	
S.M.T., -	20	0	28		Sun's S.D., 15′ 58″			
Long., E, -	4	59	0		Moon's S.D. 14′ 59″		Moon's hor. par. 54′ 54″	
Gr. date, -	15	1	28	15th.	12			
					15 11			

Moon's R.A. h.	m.	s.		Moon's declination.		R.A. mean sun. h.	m.	s.
19	13	58	22·4	27° 7′ 24″ S	50	1	33	34·2
		3	·15	7	·15		2	27·8
19	14	1	1120	27 7 17	7·5	1	36	2
			224					
			3·360					

To find sun's hour angle.

	h.	m.	s.
S.M.T., - - -	20	0	28
eqn. of time, - -			8
Sun's hour angle, - -	20	0	36

To find moon's hour angle.

	h.	m.	s.
S.M.T., - - -	20	0	28
R.A.M.S., - - -	1	36	2
	21	36	30
Moon's R.A., - -	19	14	1
Moon's hour angle, -	2	22	29

To calculate sun's altitude.

co-lat., - -	57° 37′	sin 9·926591
P.D., - -	100 0	sin 9·993351
	42 23	

To calculate moon's altitude.

57° 37′ 0″	sin 9·926591
62 52 43	sin 9·949413
5 15 43	

h. m. s.		h. m. s.	
20 0 36	hav 9·395968	2 22 29	hav 8·971069
	hav θ 9·315910		hav θ 8·847073
	$\theta = 54° 7' 15''$		$\theta = 30° 45' 15''$
			37
	vers θ 0413863		vers θ 0140594
	59		vers 5° 15′ 43″ 0004195
	vers 42° 23′ 0261348		18
	0675270		0144844
	57		786
	13		58
zen. dist.,	71° 3′ 3″	zen. dist.,	31° 13′ 23″
	90 0 0		90 0 0
true alt.,	18 56 57	true alt.,	58 46 37
	2 39	approx. cor.,	27 52
app. alt.,	18 59 36	approx. app. alt.,	58 18 45
		true cor.,	28 14
Angle *A*.		correct app. alt.,	58 18 23
60° 25′ 26″		obs. dist.,	99° 11′ 30″
27		Moon's S.D.,	15 11
2		Sun's S.D.,	15 58
60 25 55		Index error,	2 10
			99 44 49

To calculate the true distance.

		Versines.	
Moon's zen. dist.,	31° 13′ 23″	12462	123
Sun's ,,	71 3 3	39827	177
	102 16 26 v.	43017	6
		40684	73
Moon's app. alt.,	58° 18′ 23″	26160	165
Sun's ,,	18 59 36	544	544
	77 17 59	62694	
A.,	60 25 55	465	
	137 43 54 v.	229	
	16 52 4 v.		
dist.,	99° 44′ 49″	True dist.,	99° 21′ 48″
A.,	60 25 55	at XV. h.,	99 22 43 ·33410
	160 10 44 v.		55 2·29306
	39 18 54 v.	prop. log. of elapsed time,	1·95896
		elapsed time = 1 m. 59 s.	

	h.	m.	s.
∴ G.M.T. =	15	1	59
and S.M.T. =	20	0	28
	4	58	29

∴ Longitude = 74° 37′ 15″ E,

which differs by 2′ 30″ from the previous result, although the error on S.M.T. was 28 s. in error.

§ 157. An error as large as this is not likely to occur in actual practice, so that this method would appear to be preferable to the former, unless the altitudes can be observed with great accuracy, which is seldom the case with a star except in the dusk. If the lunar distance is *accurately* observed with a well-adjusted instrument, other *small* errors of observation or time do not appear to affect the result to any great degree, but an error in the distance (which includes incorrect index error) appears to produce an error about thirty times as great in the resulting longitude. The index error should therefore be *carefully* ascertained before a lunar distance is observed.

Examples.—(1) June 3rd, about 10 h. 2 m. P.M apparent time, in latitude 35° 15′ S, longitude by account 76° 15′ E, the observed distance of Antares (E of meridian) from the further limb of the moon was 42° 44′ 30″, index error 2′ 50″+, when a chronometer, whose error on S.M.T. was 3 h. 18 m. 47 s. slow, showed 6 h. 42 m. 3 s.

(2) June 3rd, about 7 h. 30 m. P.M. apparent time, in latitude 17° 10′ N, longitude by account 70° 10′ E, the observed distance of Pollux (W of meridian) from the nearer limb of the moon was 93° 33′ 50″, index error 2′ 30″+, when a chronometer, whose error on S.M.T. was 1 h. 48 m. 29 s. fast, showed 9 h. 15 m. 45 s.

(3) December 3rd, about 1 h. 30 m. A.M. apparent time, in latitude 32° 47′ N, longitude by account 15° 50′ W, the observed distance of Jupiter's *centre* (E of meridian) from the nearer limb of the moon was 48° 13′ 10″, index error 2′ 10″−, when a chronometer, whose error on S.M.T. was 3 h. 29 m. 16 s. slow, showed 9 h. 51 m. 23 s.

(4) December 2nd, about 9 h. 30 m. P.M. apparent time, in latitude 13° 5′ N, longitude by account 60° W, the observed distance of α Pegasi (W of meridian) from the further limb of the moon was 87° 35′ 20″, index error 1′50″+, when a chronometer, whose error on S.M.T. was 2 h. 38 m. 21 s. fast, showed 11 h. 58 m. 33 s.

CHAPTER XVII.

INVESTIGATION OF CORRECTIONS (DIP, PARALLAX, REFRACTION, ETC.). INVESTIGATION OF ERRORS IN LATITUDE, HOUR ANGLE, COMPASS, ETC., ARISING FROM SMALL ERRORS IN LATITUDE, ALTITUDE, ETC.

To calculate the correction for "dip" for a height of h *feet above the level of the sea.*

§ 158. Let the heavenly body be in the direction of S, then an observer at P, h feet above the sea, will measure the angle SPD when he brings the body down to his horizon.

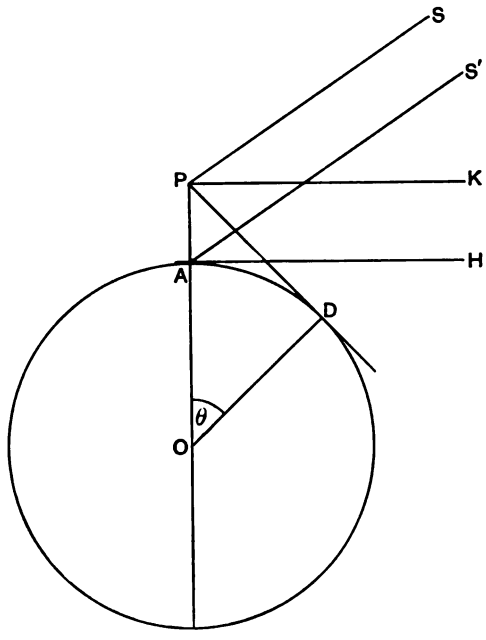


FIG. 112.

The body's altitude, if observed at A, at sea level, vertically below P, would be measured by $S'AH$, where AH is in the

plane of the sensible horizon. And, since PS and AS' are parallel (as the distance of the body is infinitely great compared with PA), $SPK = S'AH$, where PK is parallel to AH.

Hence KPD is the correction to be subtracted from the observed altitude to reduce it to what it would have been if observed at A.

This correction is named the correction for "the dip of the sea horizon," or briefly, the correction for "dip."

It may be calculated thus:

Let the earth's radius $= a$, then $OP = a + h$.

The angle $KPD = 90° - OPD = POD = \theta$ suppose.

$$\cos\theta = \frac{a}{a+h},$$

$$1 - \cos\theta = 1 - \frac{a}{a+h} = \frac{h}{a+h} = \frac{h}{a} \text{ very nearly;}$$

$$\therefore \quad 2\sin^2\frac{\theta}{2} = \frac{h}{a}.$$

But as θ is very small, we may write $\theta \sin 1'$ for $\sin\theta$;

$$\therefore \quad \theta^2 \sin^2 1' = \frac{2h}{a};$$

$$\therefore \quad \theta \text{ in minutes of arc} = \frac{\sqrt{h}}{\sin 1'} \cdot \frac{1}{\sqrt{\frac{a}{2}}}$$

$$a = 3960 \text{ miles nearly;}$$

$$\frac{a}{2} = 1980.$$

1980	3·296665
5280	3·722634
	2)7·019299
	3·509650
sin 1′	$\bar{4}$·463726
	$\bar{1}$·973376
1·063	·026624

$$\therefore \quad \theta = 1{\cdot}063\sqrt{h}.$$

The dip is diminished by the terrestrial refraction (which raises the horizon), the amount being variously estimated at from $\frac{1}{10}$ to $\frac{1}{15}$ of itself.

Taking $\frac{3}{40}$ as an average value

$$\text{dip} = {\cdot}984\sqrt{h}.$$

Starting from $\sin\frac{\theta}{2}=\sqrt{\frac{h}{2a}}$, by writing the circular measure of $\frac{\theta}{2}$ for $\sin\frac{\theta}{2}$ we obtain

$$\frac{\pi}{180\times 60}\cdot\frac{\theta}{2}=\sqrt{\frac{h}{2a}};$$

$$\therefore\quad \theta=\frac{180\times 0}{\pi}\sqrt{\frac{2h}{3960\times 5280}},$$

which produces the same result as before, viz. $\theta=1{\cdot}063\sqrt{h}$.

To find the distance at which an object can be seen at sea if its height above sea level is h *feet.*

§ 159. As in the calculation of the dip it may be shown that the circular measure of the angle $O=\sqrt{\frac{2h}{a}}$.

But the arc AD and the distance PD, which are practically equal,

$$=\text{circ. meas. of } O\times a$$
$$=\sqrt{2ah}.$$

This may also be obtained from the fact that $DP^2=(d+h)h$, where d is the earth's diameter.

$DP^2=2ah+h^2=2ah$ since h^2 is so small compared with a;

$$\therefore\quad DP=\sqrt{2ah}.$$

If refraction is taken into account, it will be found that this distance is increased by about the same amount that the dip is decreased.

In the formation of the table in *Inman's Tables*, the distance given $=1\frac{3}{17}\sqrt{2ah}$.

Refraction.

§ 160. When a ray of light passes through a transparent medium of uniform density it moves in a straight line. But if it passes obliquely from one such medium to another its path will be bent at the point of incidence, in accordance with the two conditions:

(1) The directions before and after incidence and the perpendicular to the surface at that point will lie in the same plane.

(2) The angles which these directions make with the perpendicular will have their sines in a constant ratio, so long as the media remain the same. When the ray passes from a

vacuum into a medium the ratio of the sines is called "the coefficient of refraction" for that medium; and when the ray passes from one medium to another the sines of the angles will be *inversely* as the coefficients of refraction.

The earth is surrounded by an atmosphere of variable density, which may, however, be considered as consisting of an infinite number of concentric layers, each of uniform density throughout the layer, while the densities decrease rapidly the further we recede from the earth.

Hence a ray of light from a heavenly body *S*, which is not in the zenith, will not travel directly to the eye of an observer at *E*, but will travel along the path *S*... *ABCDE*.

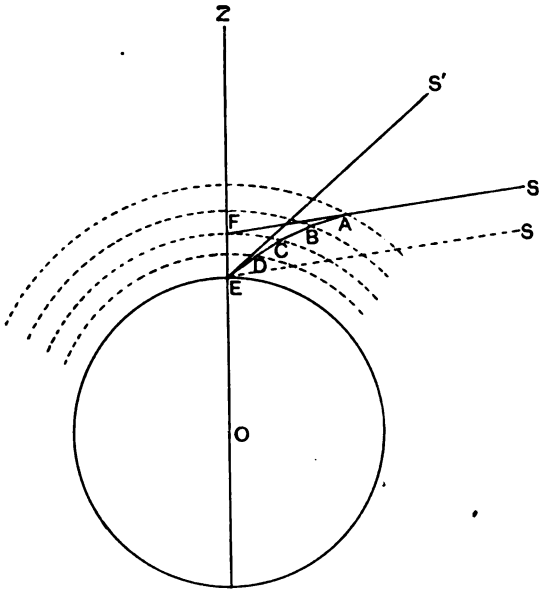


FIG. 113.

The body will therefore appear in the direction *ES'*, where *ES'* is the tangent to the path at *E*.

Let the radius *OE* be produced to meet the celestial concave at *Z*. Produce *SA* to meet *OZ* at *F*.

Then *ZFS* or *ZES* (as *ES* is parallel to *FS* owing to the great distance of the star), will be very nearly the true zenith distance, while *ZES'* is very nearly the apparent zenith distance. Hence refraction causes heavenly bodies to appear above their true places, and the correction must be subtracted from the observed altitude.

§ 161. The correction may be calculated approximately as follows:

Let r be the mean refraction in seconds, and c the constant ratio between the sines of incidence and refraction for the first and last layers, and let $ZES' = z$. Then

$$\sin(z+r) = c \sin z$$

$$\sin z \,.\, \cos r + \cos z \,.\, \sin r = c \sin z.$$

But since r is always small, its greatest value being little over 30′, we may write

$$\cos r = 1,$$

$$\sin r = r \sin 1'';$$

$$\therefore \quad \sin z + \cos z \,.\, r \sin 1'' = c \sin z.$$

$$\therefore \quad r = \frac{c-1}{\sin 1''} \,.\, \tan z = a \tan z, \quad \text{if } a = \frac{c-1}{\sin 1''}.$$

Hence the correction for refraction decreases as the zenith distance decreases, as might have been expected, the rays of light from a body with a low altitude having to pass through a larger amount of atmosphere of greater density than those from a body of greater altitude.

A more exact formula is found to be

$$r = \frac{c-1}{\sin 1''} \tan(z - 3r).$$

By repeated observations of circumpolar stars, Bradley found 57·54″ as the value of $\frac{c-1}{\sin 1''}$.

Hence the formula for the mean refraction from which the tables are calculated becomes

$$r = 57{\cdot}54'' \tan(z - 3r).$$

This is calculated for a barometric height of 29·6 inches and a temperature of 50° F.

Hence refraction corrected for a barometric height $h = \frac{h}{29{\cdot}6}\, r$.

Taking into account the increase of volume of air for increase of temperature, a final formula is arrived at:

$$r = \frac{491^\circ}{441^\circ + t^\circ} \times \frac{h}{29{\cdot}6} \times 57{\cdot}54'' \tan(z - 3r),$$

where h is the height of the barometer in inches and t is the temperature Fahrenheit.

To show how to determine the value of a in the expression $r = a \tan z$ *by observations of circumpolar stars.*

§ 162. Let z and z' be the apparent zenith distances of a circumpolar star at inferior and superior transits.

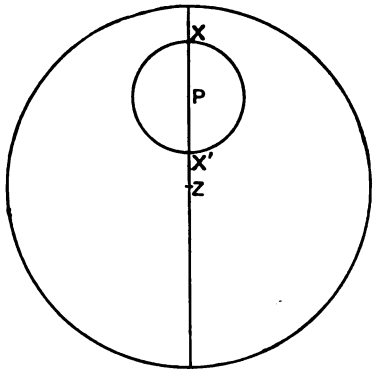


FIG. 114.

Then $PZ = ZX - PX$

$= z + a \tan z - p.$

Similarly

$PZ = z' + a \tan z' + p;$

$\therefore\ 2PZ = z + z' + a(\tan z + \tan z').$

In the same manner, if $\bar{z}$ and $\bar{z}'$ are the apparent zenith distances of another circumpolar star,

$$2PZ = \bar{z} + \bar{z}' + a(\tan \bar{z} + \tan \bar{z}');$$

$$\therefore\ z + \bar{z} + a(\tan z + \tan z') = \bar{z} + \bar{z}' + a(\tan \bar{z} + \tan \bar{z}');$$

$$\therefore\ a = \frac{z + z' \sim (\bar{z} + \bar{z}')}{\tan \bar{z} + \tan \bar{z}' \sim (\tan z + \tan z')}.$$

To show the effect of the terrestrial refraction (1) *on the distance at which an object can be seen at sea,* (2) *on the dip.*

§ 163. If A and B be two places on the earth's surface, O the angle between the perpendiculars at A and B, ADB the curved path of the rays by which A and B are visible to

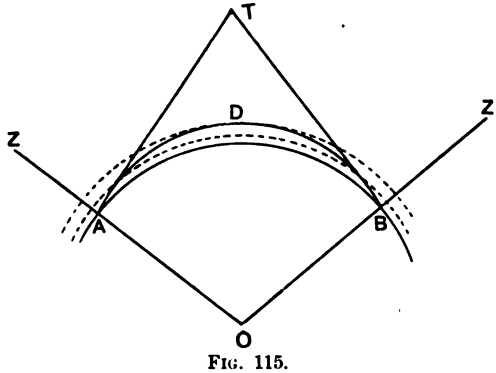


FIG. 115.

each other, it is found that, if r be the terrestrial refraction, $r = aC$, where a has a mean value = ·078, and C is the circular measure of the angle AOB.

(1) Let $PA=h$, and let PDB be the curved ray by which P is visible from B; PT the tangent to the ray at P. Join PB.

Then $Z'BP=90°+r$ (since PDB touches the earth at B).

$$\therefore\ ZPB=180°-(90°+r-C)$$
$$=90°-r+C.$$

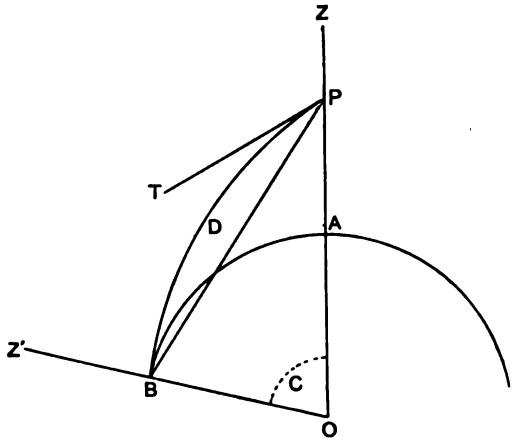


FIG. 116.

$$\frac{a+h}{a}=\frac{\sin(90°+r)}{\sin(90°-r+C)}=\frac{\cos r}{\cos(C-r)};$$

$$\therefore\ \frac{h}{a}=\frac{\cos r-\cos(C-r)}{\cos(C-r)}=\frac{2\sin\frac{C}{2}.\sin\left(\frac{C}{2}-r\right)}{1-2\sin^2\frac{C-r}{2}};$$

$$\therefore\ \frac{2h}{a}=\frac{C(C-2r)}{1-\frac{(C-r)^2}{2}},\quad C \text{ and } r \text{ being expressed in circular measure.}$$

$$\therefore\ \frac{2h}{a}=C^2\left(1-\frac{2r}{C}\right) \text{ nearly};$$

$$\therefore\ \sqrt{\frac{2h}{a}}=C\left(1-\frac{r}{C}\right)=C(1-a).$$

But if there is no refraction, $\sqrt{\frac{2h}{a}}$ is the value of C. Writing C' for this value,

$$C'=C(1-a);$$
$$\therefore\ C=C'(1+a) \text{ very nearly}$$
$$=C'(1+\cdot078)$$
$$=\frac{14}{13}C'.$$

Hence the distance at which an object can be seen is increased by about $\frac{1}{13}$ of itself on account of refraction. In many tables $\frac{1}{12}$ is considered the correct amount.

(2) If D and D' be corresponding values of the dip,

$$D = ZPT - 90° = ZPB - r - 90°$$
$$= C - 2r = C(1-2a).$$

But $D' = C' = C(1-a)$;

$$\therefore \quad D = D'\frac{1-2a}{1-a} = D'(1-a)$$
$$= D'(1-\cdot 078)$$
$$= \frac{12}{13}D' = \frac{37}{40}D' \text{ very nearly.}$$

Hence the dip is decreased on account of refraction.

To calculate the Dip of a Shore Horizon.

§ 164. Let d = true dip for the given height h, a = the angle subtended at the centre of the earth by the distance of the

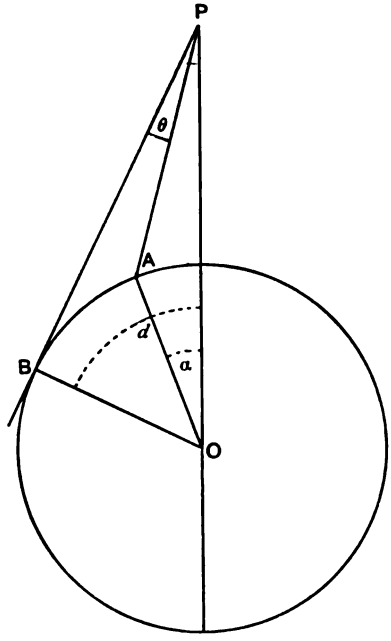


Fig. 117.

shore horizon, θ = correction to be applied to the apparent dip for the given height in order to obtain the dip required.

Then the angle $PAO = 90° + \overline{d+\theta-a}$,

the angle $APO = 90° - \overline{d+\theta}$;

$$\therefore \quad \frac{\cos \overline{d+\theta-a}}{\cos \overline{d+\theta}} = \frac{1}{\cos d};$$

$$\therefore \quad \left(1 - 2\sin^2 \frac{d+\theta-a}{2}\right)\left(1 - 2\sin^2 \frac{d}{2}\right) = 1 - 2\sin^2 \frac{d+\theta}{2};$$

$$\therefore \quad \sin^2 \frac{d+\theta-a}{2} + \sin^2 \frac{d}{2} = \sin^2 \frac{d+\theta}{2}$$

(powers above the second being neglected). As all the angles are small, circular measure may be substituted for sine, and the expression becomes

$$(d+\theta-a)^2 + d^2 = (d+\theta)^2;$$

$$\therefore \quad a^2 - 2a(d+\theta) + d^2 = 0,$$

$$\therefore \quad \theta = \frac{(d-a)^2}{2a}.$$

Examples.—(1) Find the dip for 20 feet, the distance of the shore horizon being 1 mile.

$$d = 1\tfrac{3}{12} \times 4' 24'' = 4' 46'';$$

$$\therefore \quad \theta = \frac{(3' 46'')^2}{2} = 7' 6'';$$

$$\therefore \text{ dip required} = 4' 24'' + 7' 6'' = 11' 30''.$$

(2) Find the dip for 30 feet, the distance of the shore horizon being 3 miles.

$$d = 1\tfrac{3}{12} \times 5' 24'' = 5' 51'';$$

$$\therefore \quad \theta = \frac{(2' 51'')^2}{6} = 1' 20'';$$

$$\therefore \text{ dip required} = 5' 24'' + 1' 20'' = 6' 44''.$$

To find the Reduced Latitude.

§ 165. In certain problems it is necessary to know the reduced latitude. The calculation of the correction to be applied to the true latitude, in order to obtain the reduced latitude, requires a knowledge of the properties of the ellipse. The result, therefore, is merely stated.

If l be the true latitude, the reduced latitude $= l - 11 \sin 2l$ very nearly.

To find the Reduced Zenith Distance when the True Zenith Distance and the Azimuth are given.

Let Z be the true zenith, Z' the reduced zenith, XZQ the azimuth.

Then ZX and $Z'X$ are the true and reduced zenith distances, and, if a circle be described with centre X and distance XZ' to cut XZ in D, ZD will be the correction to be applied to the true, to obtain the reduced, zenith distance.

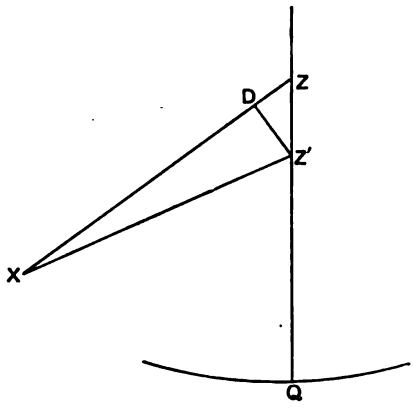


FIG. 118.

$ZZ'D$ may be considered as a plane right-angled triangle; and ZZ' is the reduction of the latitude $= 11 \sin 2l$.

Now $$ZD = ZZ' \cos DZZ'$$
$$= 11 \sin 2l \,.\, \cos \text{azimuth},$$

where the sign of ZD depends on that of cos azimuth.

Parallax.

§ 166. Let A be the position of the observer, Z the zenith, Z' the reduced zenith, X a heavenly body, as the sun, moon, or a planet (owing to their immense distance the fixed stars have no appreciable parallax).

Draw AXm', CXm to meet the celestial concave. Then $m'm$ produced will pass through Z'.

mm' or the angle AXC is called the "diurnal parallax," and if X' be the position of the body when on the horizon, $AX'C$ is called the "horizontal parallax."

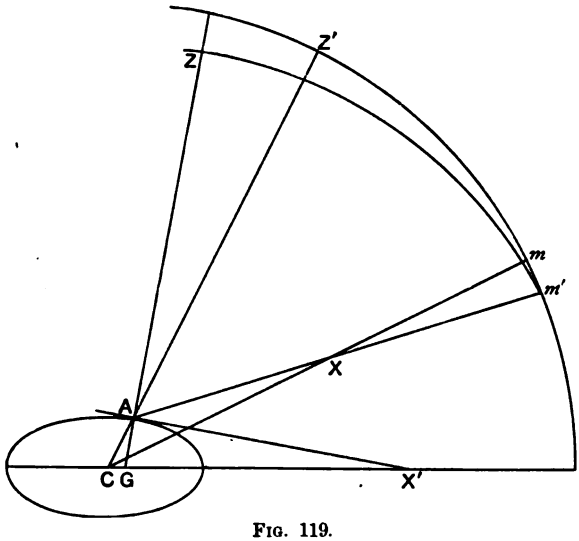


FIG. 119.

Since the earth is very nearly a sphere, $m'Z'$ may be practically considered to coincide with $m'Z$, and to be a vertical circle.

To find the Horizontal Parallax in any latitude.

§ 167. The moon's horizontal parallax tabulated in the *Nautical Almanac* is the *equatorial* horizontal parallax, and as this is equal to $\frac{\text{rad. of the earth}}{\text{distance of moon}}$, the horizontal parallax will be smaller as we approach the pole, on account of the spheroidal form of the earth.

Hence, if P be the equatorial horizontal parallax, P' its corresponding value in latitude l,

$$P' = P(1 - \tfrac{1}{300}\sin^2 l)$$

very nearly, the calculation requiring a knowledge of the properties of the ellipse. The principle is the same for any heavenly body, but the effect is appreciable only in the case of the moon.

Given the apparent reduced zenith distance and the horizontal parallax, to calculate the diurnal parallax, and thence the parallax in altitude.

§ 168. Let AC, the radius of the earth $= r$,

$CX = CX'$, the distance of the body $= d$,

$Z'Am'$, the apparent reduced zenith distance $= z'$,

$AXC = mm'$, the diurnal parallax $= p$,

$AX'C$, the horizontal parallax $= P$.

In the triangle AXC,

$$\frac{r}{d} = \frac{\sin AXC}{\sin CAX}$$

$$= \frac{\sin AXC}{\sin Z'Am'}$$

$$= \frac{\sin p}{\sin z'}.$$

In the right-angled triangle $AX'C$

$$\frac{r}{d} = \sin AX'C$$

$$= \sin P.$$

Equating the two values of $\frac{r}{d}$,

$$\frac{\sin p}{\sin z'} = \sin P;$$

$$\therefore \quad \sin p = \sin P \,.\, \sin z'.$$

But since P is small even for the moon, we may write $p \sin 1''$ for $\sin p$, and $P \sin 1''$ for $\sin P$.

$$\therefore \quad p = P \sin z'$$

or diurnal parallax = horizontal parallax × sin apparent reduced zenith distance (corrected for refraction).

If the true reduced zenith distance is given, an expression for the diurnal parallax may be found thus.

Let z' be the apparent reduced zen. dist.,

z be the true reduced zen. dist.,

p be the diurnal parallax.

Then $z' = z + p$.

Now $p = P \sin z'$ (P being the horizontal parallax)

$= P \sin (z + p)$

$= P(\sin z \,.\, \cos p + \cos z \,.\, \sin p)$,

$= P(\sin z + \cos z \,.\, p \sin 1'')$ very nearly.

$\therefore \quad p(1 - P \cos z \,.\, \sin 1'') = P \sin z$;

$$\therefore \quad p = \frac{P \sin z}{1 - P \cos z \,.\, \sin 1''}$$
$$= P \sin z (1 + P \cos z \,.\, \sin 1'' + \ldots)$$
$$= P \sin z + \tfrac{1}{2} P^2 \sin 2z \,.\, \sin 1'' + \ldots .$$

§ 169. In ordinary Nautical Astronomy problems, the earth is considered as a sphere. Hence z and z' coincide, the apparent reduced zenith distance becomes the apparent zenith distance, and the diurnal parallax becomes the parallax in altitude.

Hence parallax in altitude
= horizontal parallax × sin apparent zenith distance (corrected for refraction)
= hor. par. × cos app. alt.

Given the meridian zenith distances of a heavenly body at two distant places on the same meridian, to calculate the horizontal parallax.

§ 170. Let A and B be two places on the same meridian, X a heavenly body on the meridian of A and B.

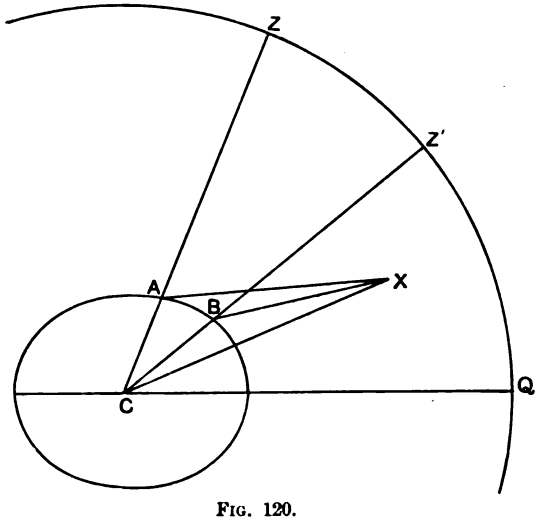


FIG. 120.

Let A and B be on the same side of the equator.

ZQ = reduced latitude of $A = l$,
$Z'Q$ = „ „ $B = l'$,
ZAX = reduced zen. dist. at $A = z$,
$Z'BX$ = „ „ $B = z'$,

Horizontal parallax at $A = P$,

" " $B = P'$,

$AC = r$, $BC = r'$ $CX = d$.

Then diurnal parallax at A

$$= P \sin z = AXC,$$

and diurnal parallax at B

$$= P' \sin z' = BXC.$$

Also, $$AXC = ZAX - ACX = z - ACX,$$

$$BXC = Z'BX - BCX = z' - BCX;$$

$$\therefore \quad P \sin z - P' \sin z' = z - z' - (ACX = BCX)$$

$$= z - z' - (l - l').$$

But $$P = \frac{r}{d}, \quad P' = \frac{r'}{d} \text{ nearly (cf. § 168)};$$

$$\therefore \quad P' = P\frac{r'}{r} = P, \text{ since } \frac{r'}{r} = 1 \text{ very nearly};$$

$$\therefore \quad P(\sin z - \sin z') = z - z' - (l - l');$$

$$\therefore \quad P = \frac{z - z' - (l - l')}{\sin z - \sin z'}$$

$$= \frac{z - z' - (l - l')}{2 \cos \frac{z + z'}{2} \sin \frac{z - z'}{2}}.$$

If the places are on opposite sides of the equator we shall find similarly

$$P = \frac{z + z' - (l + l')}{2 \sin \frac{z + z'}{2} \cos \frac{z - z'}{2}}.$$

Augmentation of the Moon's Horizontal Semi-diameter.

§ 171. Let O be the earth's centre, M the moon in the horizon of a place A, M_2 the position of the moon in the zenith, and M_1 any other position. The moon's parallax being small, AM and OM are very nearly equal. Hence AM_2 must be less by nearly the earth's radius than AM. And at any intermediate position of the moon, AM_1 will be less than AM. Hence, as the moon's altitude increases, the apparent diameter will also increase, and, in order to find the altitude of the

centre, above or below the point on the limb which has been observed, it will be necessary to apply a larger semi-diameter than the one tabulated in the *Nautical Almanac*, which is

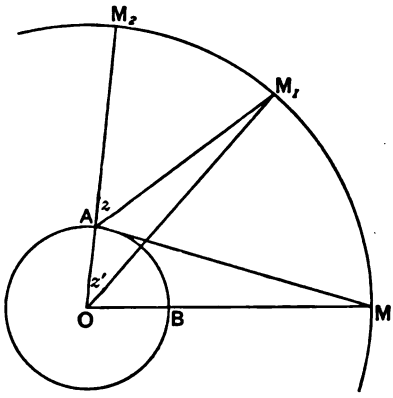


FIG. 121.

calculated for the position of the moon on the horizon, and is therefore called the *horizontal* semi-diameter.

To find the Augmentation of the Moon's Horizontal Semi-diameter.

§ 172. Let E be the centre of the earth, M that of the moon, A the position of the observer. Draw EC and AB to touch the moon, and join AM, EM, BM, CM.

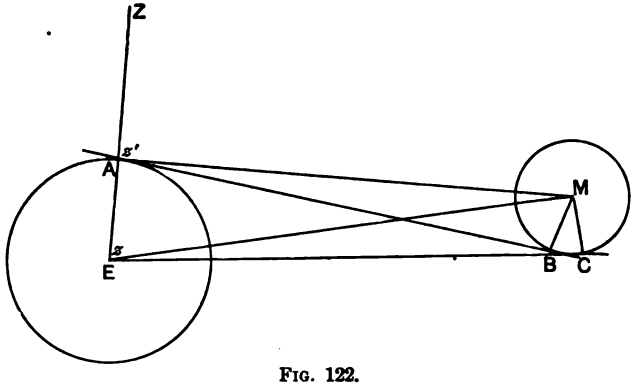


FIG. 122.

Then the angles MEC, MAB measure the true and apparent semi-diameters of the moon, when its zenith distance is z.

Let $MEC = R$, $MAB = R'$. Then $R' - R$ is the correction to be determined.

In the right-angled triangles MEC, MAB, we have

$$\sin R = \frac{MC}{ME},\ \sin R' = \frac{MB}{MA};$$

$$\therefore\ \frac{\sin R'}{\sin R} = \frac{ME}{MA},\ \text{since } MB = MC.$$

In the triangle MAE, if z and z' be the true and apparent zenith distances,

$$\frac{\sin z'}{\sin z} = \frac{ME}{MA};$$

$$= \frac{\sin R'}{\sin R}$$

$$= \frac{R'\sin 1''}{R\sin 1''},\ \text{since } R \text{ and } R' \text{ are small,}$$

$$= \frac{R'}{R};$$

$$\therefore\ \frac{R'-R}{R} = \frac{\sin z'-\sin z}{\sin z} = \frac{2\cos\frac{1}{2}(z'+z)\sin\frac{1}{2}(z'-z)}{\sin z}.$$

Let AME, the parallax in altitude $= p$.

Then $$p = z'-z;$$

$$\therefore\ z = z'-p.$$

Substituting this value of z we obtain

$$R'-R = 2R\cos\left(z'-\frac{p}{2}\right)\sin\frac{p}{2}\,.\,\mathrm{cosec}(z'-p);$$

from which $R'-R$ may be found.

§ 173. Considering that, as an average value, the horizontal parallax $= 3{\cdot}6697$ times the horizontal semi-diameter, and that $p =$ hor. par. $\times \sin z'$, the above expression may be put into a form without p.

If these substitutions are made and small terms neglected, it takes the form,

augmentation in seconds

$$= AR^2\cos z' + \frac{A^2R^3}{2} + \frac{A^2R^3}{2}\cos^2 z',$$

where $$A = 3{\cdot}6697 \times \sin 1'',$$

from which formula the table is constructed.

To find the error in hour angle arising from a small error in altitude.

§174. Let S and S' be the true and erroneous places of the heavenly body.

With centre Z and distance ZS' describe the arc $S'D$; then $SD = dz$ = error in altitude.

With centre P and distance PS or PS' describe the arc SS'; then $SPS' = dh$ = error in hour angle.

Now the triangle SDS' is practically a plane right-angled triangle;

$$\therefore \quad SD = SS' \cos DSS'$$
$$= SS' \sin PSZ, \text{ since } PSS' = 90^\circ.$$

But

$$SS' = SPS' \sin PS$$
$$= dh \sin p;$$

$$\therefore \quad dz = dh \sin p \,.\, \sin PSZ$$
$$= dh \sin PZ \,.\, \sin PZS$$
$$= dh \cos \text{lat.} \sin \text{azimuth.}$$

$$dh = \frac{dz}{\cos \text{lat.} \sin \text{azimuth}} \text{ (in arc)}$$

$$= \frac{\frac{1}{15} dz}{\cos \text{lat.} \sin \text{azimuth}} \text{ (in time).}$$

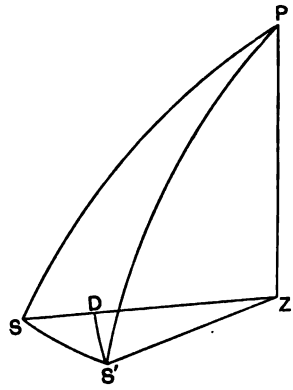


FIG. 123.

The latitude being considered constant, the error in hour angle is evidently least when sin azimuth is greatest, *i.e.* when $PZS = 90^\circ$, or the heavenly body is on the prime vertical.

Observations for longitude should therefore be taken when the body observed is as nearly as possible on the prime vertical, provided, of course, that the altitude is sufficiently great (7° to 10°) to avoid errors arising from irregularity in the effects of refraction.

By writing the formula arrived at in the form

$$dz = dh \cos l \,.\, \sin \text{azimuth},$$

it appears that heavenly bodies, whatever be their declinations, change their altitude most rapidly when on the prime vertical, with the same result as before—that longitude observations should be made near the prime vertical, as a small error in altitude will then produce a comparatively smaller error in time.

N.B.—In the case of stars the altitude can be obtained with the greatest accuracy just after sunset or just before sunrise, as the horizon is then clearly visible, which is not often the case in the dark.

Longitudes may therefore be obtained with *great* accuracy from stars which are near the prime vertical under such conditions, and if, at or about the same time another star is on or near the meridian, the position of the ship can be accurately determined.

With the aid of the ordinary Azimuth Tables, and (especially) Azimuth Tables for the Higher Declinations (Goodwin) the principles of Sumner's method may be very satisfactorily applied in finding the position by observations taken in the twilight.

Observations of stars in the dark are not much to be depended on, as a rule.

To find the error in hour angle arising from a small error in latitude.

§ 175. Let Z and Z' be the correct and erroneous zeniths. With centre P and distance PZ describe an arc ZD, and with centre S and distance SZ the arc ZZ'.

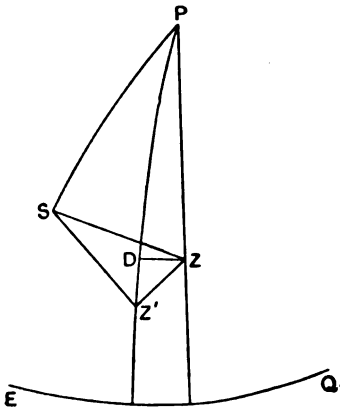


FIG. 124.

Then

$Z'D$ = error in latitude = dl,

and ZPZ' = error in hour angle = dh.

Then

$$ZD = dh \cos \text{lat.}$$

In triangle DZZ', which is practically a plane right-angled triangle,

$$DZ' = DZ \tan DZZ'$$
$$= DZ \tan PZS$$

($\because$ PZD and SZZ' each = 90°);

$\therefore$ $dl = dh \cos l$. tan azimuth;

$\therefore$ $dh = dl \sec l$. cot azimuth (in arc)

$= \frac{1}{15} dl \sec l$. cot azimuth (in time).

Hence the sign of the error in hour angle is the same as or opposite to that of the error in latitude according as the azimuth is less or greater than 90° in the forenoon, and *vice versa* in the afternoon.

To find the error in hour angle corresponding to an error in the declination.

§ 176. With the usual notation and construction

$$SD = dh \cos d \text{ (fig. 125)}$$

$$S'D = SD \tan S'SD$$
$$= SD \tan PSZ \,(PSD \text{ and } ZSS' \text{ being right angles});$$
$$\therefore \quad dp = dh \cos d \,.\, \tan PZS,$$
$$\text{and } dh = \frac{dp}{\cos d \,.\, \tan PSZ} \text{ (in arc)}$$
$$= \frac{\frac{1}{15}dp}{\cos d \,.\, \tan PSZ} \text{ (in time).}$$

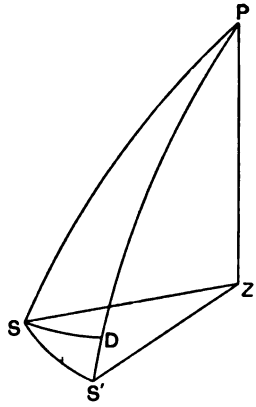


FIG. 125.

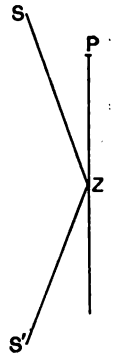


FIG. 126.

To find the error in latitude arising from a small error in hour angle.

§ 177. By a preceding proof

$$dl = dh \cos \text{lat} \,.\, \tan \text{azimuth}$$
$$= dh \cos l \,.\, \tan PZS \text{ if the azimuth is} < 90° \text{ (fig. 126),}$$
$$\text{or } = dh \cos l \,.\, \tan PZS' \text{ if the azimuth is} > 90°.$$

Now the tangent of an angle $< 90°$ increases as the angle increases, and of an angle $> 90°$ decreases as the angle increases. Hence the nearer ZS or ZS' is to the meridian the less is the error in latitude arising from error in the hour angle; and it is for this reason that observations for latitude, out of the meridian, must be made when the body is not far from the meridian in azimuth.

To find the error in latitude arising from an error in altitude.

§ 178. Let Z and Z' be the true and erroneous zeniths, then $ZZ' =$ the error in latitude $= dl$, and if ZD be drawn at right angles to $Z'X$, $Z'D =$ the error in altitude $= dz$. Fig. 127.

Then $ZZ' = Z'D \sec ZZ'D$,

$dl = dz \sec \text{azimuth}$;

$\therefore$ observations for latitude should be taken near the meridian in azimuth.

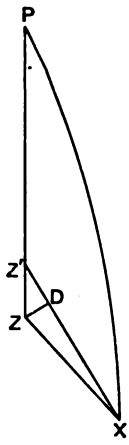


Fig. 127.

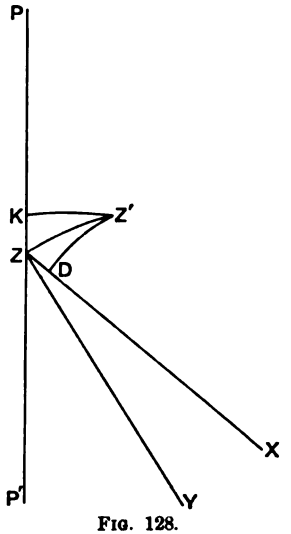


Fig. 128.

To find the error in the latitude in a double altitude or Sumner observation, occasioned by an error in the altitude at either of the observations.

§ 179. Let X and Y represent the positions of the bodies, or two positions of the same body. Fig. 128.

Let $PZX = a$,

$PZY = b$,

XZY or the difference of the bearings $= d$.

Suppose that there is an error in the altitude of X, while that of Y is correct.

Let Z and Z' be the true and erroneous zeniths, then $YZZ' = 90°$ as $Z'Y = ZY$. With centre X and dist. XZ' describe the arc $Z'D$, and let $Z'K$ be a parallel of latitude. Then

$ZD = \text{error in altitude} = dz$,

and $ZK = \text{error in latitude} = dl$;

$$ZZ' = KZ \sec KZZ' = dl \operatorname{cosec} P'ZY = \frac{dl}{\sin b},$$

$$ZZ' = DZ \sec Z'ZD = dz \operatorname{cosec} XZY = \frac{dz}{\sin d};$$

$$\therefore \quad dl = dz \frac{\sin b}{\sin d}.$$

Similarly, it may be shown that if there be an error dz' in the altitude of Y

$$dl' = dz' \frac{\sin a}{\sin d}.$$

Hence the error in the latitude resulting from an error in the altitude of either body = error in alt. $\times \frac{\text{sine bearing of other body}}{\text{sine difference of bearings}}$.

Thus the nearer the difference of the bearings is to 90° the less will be the resulting error.

§ 180. If we wish to consider the most favourable position with regard to the meridian, etc., of the bodies observed or the two positions of one body, we may suppose, for the sake of the argument, that personal and instrumental errors will probably be in the same direction at each observation, and probably also of the same amount. If the bodies are on the opposite sides of the meridian the errors will tend to increase each other's effect.

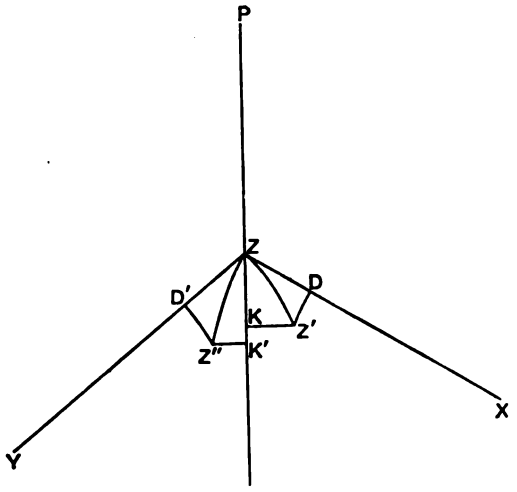


FIG. 129.

Let X and Y be the positions of the body on opposite sides of the meridian; and suppose that the altitudes are both too great, so that XD is the zenith distance of X instead of XZ. Considering the altitude of Y as correct, Z', the zenith obtained with the erroneous altitude of X, will be at the intersection of circles with X and Y as centres and XD and YZ as radii, ZK being the resulting error in latitude.

Similarly Z'' would be the zenith obtained if the altitude of Y is considered erroneous and that of X as correct, ZK' being the corresponding error in the latitude, which is in the *same* direction as ZK.

But if the bodies are on the same side of the meridian, ZK and ZK', determined in a similar manner, are in *opposite* directions, and the errors tend to neutralize each other.

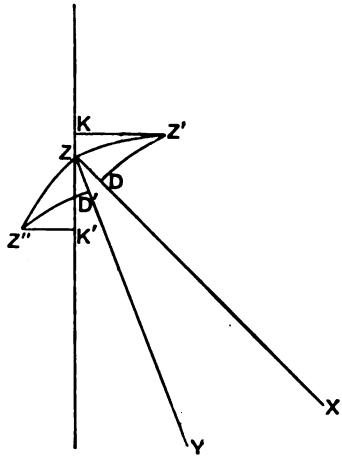


FIG. 130.

If then the errors in altitude at the two observations are considered equal, the total error in this latter case will depend on $\sin a \sim \sin b$, or, as the difference between a and b should be 90° in the most favourable case, on

$$\sin \overline{90° + a} - \sin a = 2 \cos \overline{45° + a} \sin 45°.$$

This vanishes when $a = 45°$ and then b will $= 135°$.

Hence, on the whole, we see that the most favourable condition of observation is when the two bodies are on the same side of the meridian with their circles of altitude equally inclined to the prime vertical, the azimuths differing by as nearly 90° as possible. (For a simple geometrical proof of this result, see Godfray's *Astronomy*, 4th edition, pp. 122, 123.)

To find the error in latitude arising from a small error in the polar angle in a double altitude observation.

§ 181. Let XPY be the correct and $X'PY$ the erroneous polar angle arising from error in the altitude of X. $PX = PX'$, $XD =$ error in altitude, $XPX' =$ error in polar angle $= dh$.

Using erroneous zenith distance ZX' and erroneous polar angle $X'PY$, the correct co-latitude PZ would be obtained.

Hence the error in altitude just counteracts the error in polar angle.

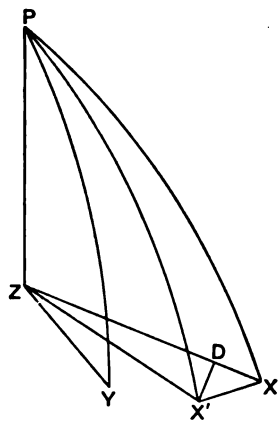


FIG. 131.

$$dl = XD\frac{\sin b}{\sin d}$$

$$= XX'\cos DXX'\frac{\sin b}{\sin d}$$

$$= XX'\sin PXZ\frac{\sin b}{\sin d}$$

$$= dh\sin PX\frac{\sin PZX\,.\,\sin PZ}{\sin PX}\cdot\frac{\sin b}{\sin d}$$

$$= dh\cos\text{lat.}\frac{\sin a\,.\,\sin b}{\sin d}.$$

N.B.—The error in the polar angle may be considered as arising from an error in one of the altitudes, and the above then becomes a special case of an error in altitude, but it has been thought worth while to insert the calculation, as errors are frequently made in obtaining the polar angle in the *practical* working of examples.

To find the errors in azimuth corresponding to errors in altitude, latitude, and hour angle respectively.

§182. (1) Error in *altitude.*

With centre Z and distance ZS describe the arc SD. Then $S'D$ = error in altitude $= dz$, and $SS'D$ = a small plane right-angled triangle. Let dA = corresponding error in azimuth. With centre P and distance PS describe the arc SS'. Then PSS' and ZSD each $= 90°$;

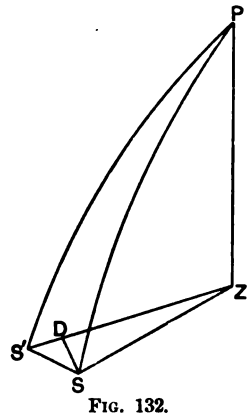


FIG. 132.

$$\therefore\quad \angle S'SD = \angle PSZ,$$

$$SD = SZS'\sin ZS$$

$$= dA\sin z,$$

$$S'D = dz = SD\tan DSS'$$

$$= dA\sin z\,.\tan PSZ;$$

$$\therefore\quad dA = dz\frac{\cot PSZ}{\sin z} \qquad \text{.....(1).}$$

(2) Error in *latitude.*

Let PZ = the true, and $P'Z$ = the erroneous co-latitude. Draw the arcs PK, PP' in the usual manner. Then

$$KP' = dl,$$
$$PZP' = dA,$$
$$PK = dA \cos \text{lat.},$$
$$PK = P'K \cot P'PK$$
$$= dl \cot SPZ$$
$$= dl \cot \text{hour angle};$$
$$\therefore \quad dA = dl \sec \text{lat.} \cot \text{hour angle} \ldots\ldots\ldots\ldots\ldots (2).$$

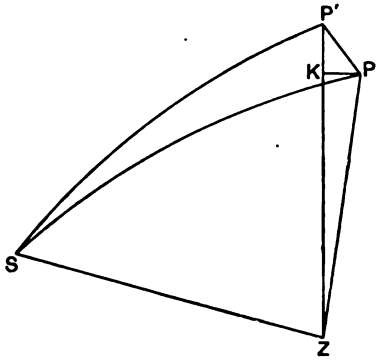


FIG. 133.

(3) Error in *hour angle.*

In fig. 132, $SD = dA \sin z,$

$$SD = SS' \cos DSS'$$
$$= SS' \cos PSZ,$$

and $SS' = dh \sin p;$

$$\therefore \quad dA = \frac{\sin p \,.\, \cos PSZ}{\sin z} dh \ldots\ldots\ldots\ldots\ldots (3),$$

which last equation may be written

$$dA = \frac{\sin \text{azimuth} \,.\, \cos PSZ}{\sin \text{hour angle}} dh \ldots\ldots\ldots\ldots (4).$$

The angle PSZ has a value 90° at the time of the maximum azimuth. When the latitude and declination are of the same name, and the latitude is the greater, it can be proved that PSZ has its greatest value when S is on the prime vertical.

When the latitude and declination are of different names, PSZ will have its greatest value when S is on the horizon.

Considering equations (1) and (3), we see that the error in azimuth is least for an error in altitude or hour angle when *PSZ* is 90°, or at the time of the maximum azimuth. When there is no maximum azimuth, the larger the values of *PSZ* and of the zenith distance the smaller is the resulting error in azimuth.

Considering equations (2) and (4), the error in azimuth is least for an error in latitude or hour angle when the hour angle is 90°, or the sun is on the 6 o'clock hour circle.

Combining these results, we obtain, the general rule, that there is less probability of error in the azimuth obtained from observation of a heavenly body, if it is observed before it has reached the prime vertical, with a moderate altitude.

When the latitude and declination are of different names, the observation for azimuth should be taken as soon as convenient after sunrise, or before sunset.

APPENDIX TO CHAPTER XVII.

The errors calculated in §§ 174 *seqq.* may be very simply obtained by the use of "lines of position" on a plane chart or plan.

The following are given as examples of the method:

§ 174A. Let *A*, fig. 123A, be the true position, *AB* the position line:

If *dz* be the error in altitude, the line *CD* drawn parallel to *AB* at a distance *dz* from it will be the position line due to the erroneous altitude.

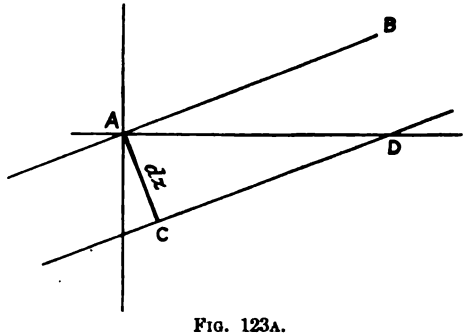


FIG. 123A.

If *CD* cuts the parallel through *A* at *D*, *AD* will be the error in departure, and is $= dh \,.\, \cos l$.

But $AD = AC \cdot \sec DAC$

$= dz \cdot \text{cosec azimuth}$;

$\therefore \quad dh = \dfrac{dz}{\cos l \cdot \sin az}$ (in arc).

§ 175A. In this case the erroneous position line must be drawn through C, where AC = error in latitude dl,

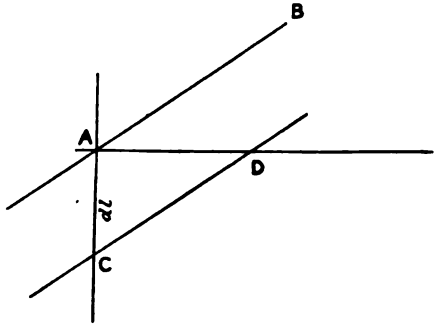


FIG. 124A.

$AD = dh \cos l$; and $AD = AC \cdot \cot ADC = dl \cdot \cot az$;

$\therefore \quad dh \cos l = dl \cdot \cot az$,

or $dh = dl \cdot \sec l \cdot \cot az$ (in arc).

§ 179A. Let A be the true position, AB the position line for the first observation, AC that for the second observation.

Let dz_1 and dz_2 be the errors in altitude,

dl_1 and dl_2 the corresponding errors in latitude.

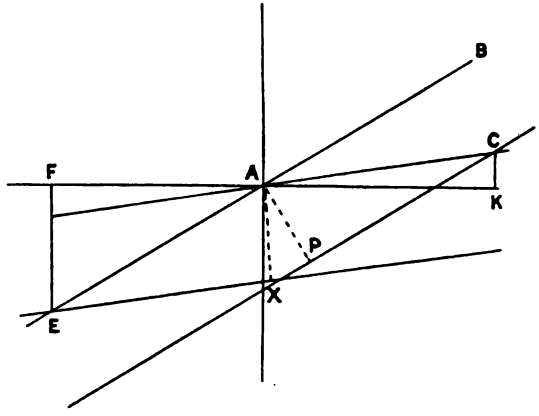


FIG. 128A.

Draw XPC parallel to AB at a distance $AP = dz$ cutting

AC at C. Draw CK perpendicular to the parallel through A. Then $CK = dl$, on account of the first error.

$$AC = AP \cdot \operatorname{cosec} ACP$$
$$= AP \cdot \operatorname{cosec}(A_1 \sim A_2) \text{ where } A_1, A_2 \text{ are the azimuths,}$$
and $$AC = CK \cdot \operatorname{cosec} CAK$$
$$= dl_1 \cdot \operatorname{cosec} A_2;$$
$$\therefore \quad dl_1 = dz_1 \cdot \frac{\sin A_2}{\sin(A_1 \sim A_2)}.$$

Next draw EX parallel to AC at a distance $= dz_2$ cutting AB in E, and XPC in X. Then EF drawn perpendicular to the parallel through $A_1 = dl_2$ on account of the second error, and X is the false position due to both errors.

dl_2 is found similarly to dl_1 and

$$= dz_2 \frac{\sin A_1}{\sin(A_1 \sim A_2)}$$

and whole error in latitude

$$= dl_2 \sim dl_1 = \frac{dz_1 \sin A_2 \sim dz_2 \sin A_1}{\sin(A_1 \sim A_2}.$$

If the bodies, or the two positions of the same body, were on opposite sides of the meridian the whole error would be

$$dl_2 + dl_1.$$

CHAPTER XVIII.

MISCELLANEOUS PROBLEMS, ETC.

POLE STAR. EQUAL ALTITUDES. TABULAR VALUE OF AUXILIARY ANGLE *A*. MINIMUM TWILIGHT. INTERPOLATIONS, ETC.

To find the latitude by observation of the Pole Star.

§ 183. Let x be the correction which must be applied to the corrected altitude in order to obtain the latitude; subtractive in (1), additive in (2).

Then $\cos ZX = \cos PZ . \cos PX + \sin PZ . \sin PX . \cos ZPX$;

$\therefore \quad \cos z = \sin l . \cos p + \cos l . \sin p . \cos h.$

But $\quad l \pm x = \text{altitude} = 90° - z$;

$\therefore \quad \sin(l \pm x) = \sin l . \cos p + \cos l . \sin p . \cos h$;

$\therefore \quad \sin l . \cos x \pm \cos l . \sin x$

$$= \sin l . \cos p + \cos l . \sin p . \cos h;$$

$\therefore \quad \pm \cos l . \sin x$

$$= -\sin l . (\cos x - \cos p) + \cos l . \sin p . \cos h$$

$$= -\sin l . 2 \sin \frac{p+x}{2} . \sin \frac{p-x}{2} + \cos l . \sin p . \cos h.$$

But since x and p are small, write $\frac{(p+x)}{2} \sin 1''$ for $\sin \frac{(p+x)}{2}$, $\frac{(p-x)}{2} \sin 1''$ for $\sin \frac{(p-x)}{2}$, and $p \sin 1''$ for $\sin p$; and dividing throughout by $\sin 1''$ we obtain

$$\pm x \cos l = -\sin l . \frac{p^2 - x^2}{2} \sin 1'' + \cos l . \cos h . p.$$

Draw XM at right angles to PZ, and, as a first approximation, let $PM = x$.

Considering PMX as a plane right-angled triangle,

$$x = p \cos h;$$

$$\therefore \quad p^2 - x^2 = p^2 - p^2\cos^2 h = p^2\sin^2 h;$$

$\therefore$ substituting and dividing throughout by $\cos l$,

$$\pm x = -\frac{p^2\sin^2 h}{2}\tan l \,.\, \sin 1'' + p\cos h;$$

$$\therefore \quad a - l = p\cos h - \frac{p^2\sin^2 h}{2}\tan l \,.\, \sin 1'';$$

$$\therefore \quad l = a - p\cos h + \frac{p^2\sin^2 h}{2}\tan l \,.\, \sin 1''.$$

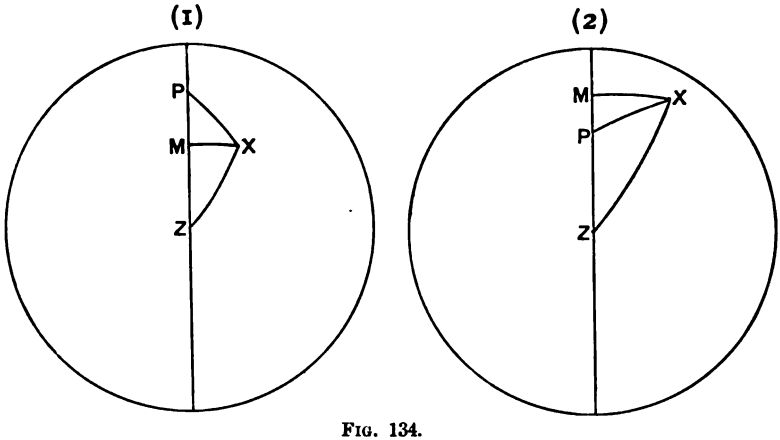


FIG. 134.

As l is always nearly equal to a, and, when h is greatest, is equal to a, no appreciable error is introduced by writing $\tan a$ for $\tan l$;

$$\therefore \quad l = a - p\cos h + \frac{p^2\sin^2 h}{2}\tan a \,.\, \sin 1''.$$

Correction I. in the *Nautical Almanac* is the value of $p\cos h$, the argument being the sidereal time on which h depends; the values of the star's polar distance and right ascension being their mean values for the year.

Correction II. is the value of $\frac{p^2\sin^2 h}{2}\tan a \,.\, \sin 1''$, with the sidereal time and the approximate altitude as arguments.

This is always to be added.

Correction III. is special for each month and depends on the difference between the true and assumed values of the polar distance and right ascension.

As this correction is sometimes positive and sometimes negative, in order to avoid confusion 1′ is added to each value; the correction is then always considered as additive, 1′ being

subtracted from the corrected altitude to compensate for the 1′ added in the table.

More exact methods obtain as a fourth term

$$-\tfrac{1}{3}\sin^2 1''(p\cos h)(p\sin h)^2;$$

but, as this may be shown never to exceed half a second, it may safely be neglected.

To obtain the error of the chronometer by "Equal Altitudes" of the sun.

§ 184. If the sun is observed when it has the same altitude on either side of the meridian, in order to obtain the error of chronometer, the mean of the chronometer times will not be accurately the chronometer time of the meridian passage, on account of the change in the declination in the interval. The correction to be applied to the mean chronometer time is called the "equation of equal altitudes."

The difference between the Greenwich time and the chronometer time of apparent noon thus found, is the error of the chronometer.

§ 185. To calculate the equation of equal altitudes we may proceed thus:

Let X and x be the positions of the sun at the two observations. Then $ZX = Zx$.

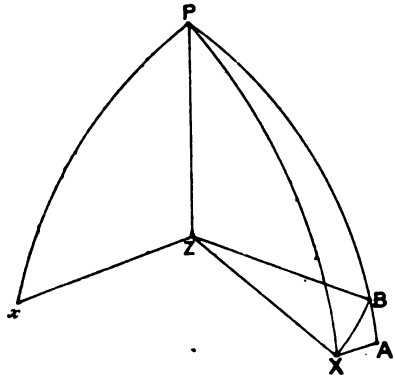


FIG. 135.

Suppose the declination to be increasing, and of the same name as the latitude, Px is less than PX. Hence ZPx is greater than ZPX, and, if ZPB be made equal to ZPx (B being a point on the circle described with centre Z and distance

ZX), PB will equal Px. And, if XA be the arc of a circle with centre P and distance PX, AB equals the change of declination in the interval $= dp$ suppose.

Let the angle XPx or the elapsed time $= 2t$, and the angle $XPB = 2E$.

The elapsed time $XPx = 2ZPB - 2E$;

$$\therefore \quad t = ZPB - E;$$

$$\therefore \quad t - E = ZPB - 2E$$

$$= ZPX,$$

or the time that must elapse before the sun reaches the meridian.

If the declination is decreasing, in the same way it may be shown that $t + E$ is the time that will elapse before the sun reaches the meridian.

Let $PX = p = 90° - d$,

$PZ = 90° - l =$ co-lat.,

$dp =$ change of declination in the interval.

In the small triangle XAB, which may be considered a plane right-angled triangle,

$$XA = AB \cot AXB$$

$$= AB \cot PXZ \text{ (since } PXA = 90° = ZXB).$$

Also $\quad XA \sin 1'' = \sin p \,.\, 2E \sin 1''.$

Substituting

$$2E \sin p = dp \cot PXZ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

But $\quad \sin PXZ = \dfrac{\sin PZ}{\sin ZX} \,.\, \sin ZPX$

$$= \cos \text{ lat. sec alt. } \sin \tfrac{1}{2} \text{ elapsed time very nearly.}$$

From this, or from the ordinary rule, "Given three sides, to find an angle," a value of PXZ can be obtained with sufficient accuracy, and, substituting the value thus obtained in (1) we have

$$2E = dp \sec d \,.\, \cot PXZ,$$

or $\quad E$ in time $= \dfrac{dp}{30} \sec d \,.\, \cot PXZ.$

It is evident from the figure that if the polar distance is *increasing*, E is *positive*, and if the polar distance is *decreasing*, E is *negative*.

§ 186. The foregoing method of finding E is more simple than those in which an expression is obtained by substituting a value for $\cot PXZ$ or by analytical calculation.

This other expression for E may be obtained as follows:

$$\cot PXZ \sin ZPX = \cot PZ \,.\, \sin PX - \cos PX \,.\, \cos ZPX;$$

or $\cot PXZ \,.\, \sin t$ (nearly) $= \tan l \,.\, \cos d - \sin d \,.\, \cos t$ (nearly);

$\therefore$ substituting this value of $\cot PXZ$ in (1),

$$2E = dp\ (\tan l \,.\, \operatorname{cosec} t - \tan d \,.\, \cot t);$$

$\therefore$ E (in time) to be *subtracted*

$$= \frac{dp}{30} \tan \text{lat.} \operatorname{cosec} \tfrac{1}{2} \text{ elapsed time}$$

$$- \frac{dp}{30} \tan \text{dec.} \cot \tfrac{1}{2} \text{ elapsed time};$$

or E (in time) to be *added*

$$= \frac{dp}{30} \tan \text{dec.} \cot \tfrac{1}{2} \text{ elapsed time}$$

$$- \frac{dp}{30} \tan \text{lat.} \operatorname{cosec} \tfrac{1}{2} \text{ elapsed time.}$$

In the above calculation dp is considered as positive when the sun is moving to the north, the latitude and declination being both north.

Hence mark dp + or − according as the sun is moving to the north or the south, $\tan l$ + or − according as l is N or S, $\tan d$ + or − as d is N or S.

N.B.—The above expression for E might have been obtained by taking the latitude and declination S and the sun moving to the south, and considering ap, $\tan l$, and $\tan d$ as positive under these conditions. It will be found, however, that the resulting signs of the two parts of the expression for E will remain the same, though the signs of the respective factors may vary, and the rule contained in the preceding paragraph will cover all cases.

E is then to be added, with its proper sign, *i.e.* subtracted if the sign is −.

This general rule covers every case, with the exception noticed in § 186*, and avoids a number of rules for the signs of the two parts of E in various cases.

If the elapsed time is over 12 hours, the negative sign must of course be given to $\cot \frac{1}{2}$ elapsed time.

§ 186*. The rules given in §§ 185, 186, for determining the sign of the equation of equal altitudes, hold good in almost every case.

But when the declination is greater than the latitude and of the same name, the reader will naturally observe that $\cot PXZ$ (§ 185) has a negative sign if the A.M. observation is

taken *after* the time of the maximum azimuth (at which time $PXZ = 90°$ and the equation $= 0$).

In such a case, therefore, the rule must be inverted, the *negative* sign being affixed to the equation when the polar distance is increasing, and *vice versa*.

A similar result may be obtained from the consideration of the expression of § 186, viz. that in such a case, the sign affixed to the equation is the opposite of that obtained by the given rule.

Such cases will only happen during a small portion of the year, on a small portion of the earth's surface, and are, moreover, very unlikely to occur in practice. It is, however, worthy of notice that the time of the maximum azimuth is a very suitable one for equal altitude observations (provided the altitude be not too great), for two reasons: (1) at that time the equation of equal altitudes vanishes; (2) the azimuth is greatest, and there is therefore less error in time arising from any small error in the altitude.

To find the error of the chronometer by "Equal Hour Angles" of the sun.

§ 187. When an A.M. altitude of the sun has been observed, a correction may be obtained to enable us to find a P.M. altitude which will give the same hour angle, and "equal altitudes"

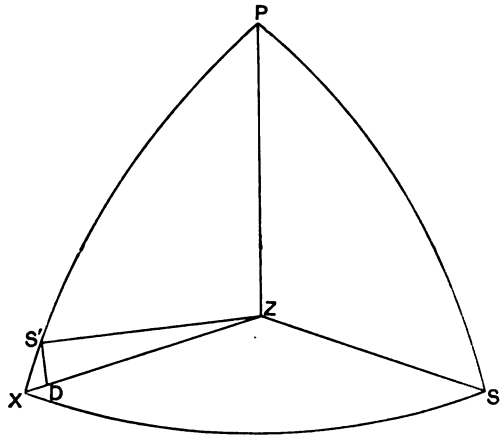


FIG. 136.

of the sun are thus made similar to equal altitudes of a star, *i.e.* the mean of the chronometer times will be the chronometer time of apparent noon.

Let S be the position of the sun A.M.,
S' „ „ „ P.M.,
consider the declination as increasing, and the hour angles SPZ, $S'PZ$ being equal, make

$$PX = PS,$$

then $$ZX = ZS.$$

Describe an arc $S'D$ with centre Z and distance ZS', then $S'DX$ is practically a plane right-angled triangle, and XD is the required correction in altitude $= dz$, while $S'X = dp =$ change of declination in the interval, which is, of course, twice the A.M. interval from the meridian.

Now $$dz = dp \cos PXZ \quad \text{(cf. §§ 174, 176)}$$
$$= dp \cos PSZ.$$

An approximate value of PSZ can easily be found, and thence ZS'.

The altitude thus obtained being placed on the sextant, and the time taken when the sun has this altitude, the mean of the chronometer times will be the time of apparent noon.

Example.—April 16th, about 8 h. 12 m. A.M. mean time, the true zenith distance of the sun was 56° 25′ 6″, when a chronometer showed 8 h. 46 m. 57 sec. (approximate error, 4 h. 25 m. slow on G.M.T.), latitude 30° N, longitude 75° W. Required the correction in altitude necessary to obtain the same P.M. hour angle.

The A.M. hour angle from noon will be found to be 3 h. 47 m. 46 s., declination 10° 9′ N, equation of time at app. noon 16 s. subtractive from app. time. The elapsed time being twice the hour angle, *i.e.* 7 h. 35 m. 22 s., and the change of declination per hour 52·96 s.,

$$dp = 52{\cdot}96 \times 7{\cdot}6 = 402{\cdot}5'' = 6'\ 42{\cdot}5''.$$

To find PSZ.			*To find dz.*	
PS	80°	·006649	402·5″	2·604766
ZS	56	·081426	60° 26′ cos	9·693231
	24			2·297997
PZ	60			60) 198·6
	84	4·825511		$dz = 3'\ 19''$
	36	4·489982		
		9·403568		

$PSZ = 60°\ 26'$

As the declination is increasing, dz is subtractive;

$\therefore$ P.M. zen. dist. $= 56°\ 25'\ 6'' - 3'\ 19'' = 56°\ 21'\ 47''$,

and the P.M. declination is 10° 15′ 43″. Calculation produces the same hour angle as before. The chronometer time of apparent noon will therefore be

20 h. 46 m. 57 s. + 3 h. 47 m. 46 s. = 0 h. 34 m. 43 s.

To find the error of chronometer.

h.	m.	s.	
0	0	0	apparent time of apparent noon.
		16	equation of time.
23	59	44	mean time of apparent noon.
5	0	0	
4	59	44	G.M.T. of apparent noon.
	34	43	chronometer time of apparent noon.
4	25	1	chronometer slow on G.M.T.

Worked as an equal altitude, the chronometer times are 8 h. 46 m. 57 s., 4 h. 22 m. 45 s., and the equation of equal altitudes is found to be 8 s. –

h.	m.	s.	
8	46	57	A.M. time.
3	47	54	½ elapsed time.
	34	51	
		8	– equation of equal altitudes.
	34	43	chronometer time of apparent noon as before.

To find the Moon's Hour Angle when at its maximum altitude.

§ 188. If the declination be constant, a heavenly body will attain its maximum altitude when on the meridian; but when the declination is increasing rapidly, the decrease of altitude immediately after meridian passage, due to the earth's revolution on its axis, will be more than counterbalanced by the rise due to the increase of declination; and a greater altitude than that on the meridian will be attained. Similarly, if the declination is decreasing rapidly, the maximum altitude will precede the meridian passage.

Hence the moon is at its maximum altitude when its rate of change of altitude arising from change in declination is equal and opposite to its rate of change of altitude arising from the diurnal motion.

Now, change of altitude = 15 cos lat. sin azimuth × interval in time;

$\therefore$ change in 1 sec. = 15 cos lat. sin azimuth in seconds.

If d'' = moon's change of declination in 10 minutes,

$$\frac{d''}{10 \times 60} = \text{change in } 1^s.$$

As the body is so near the meridian we may consider the change in declination and in altitude as practically equal;

$$\therefore \quad 15 \cos l \,.\, \sin az = \frac{d''}{600};$$

$$\therefore \quad 15 \cos l \frac{\sin h \,.\, \sin p}{\sin z} = \frac{d''}{600};$$

$$\therefore \quad \sin h = \frac{d''}{9000} \cdot \frac{\sin z}{\cos l \,.\, \cos d}$$

$$= \frac{d''}{9000} \cdot \frac{\sin (l \pm d)}{\cos l \,.\, \cos d}.$$

In the same manner the hour angle of any other heavenly body, when at its maximum altitude, may be obtained.

To find the time when the sun will dip, when the change of altitude is considerable, owing to the rapid motion of the ship.

§ 189. Consider, firstly, for simplicity, the case when the course is directly or almost directly towards or away from the sun.

As the sun's change of declination is practically in the meridian, it may be considered to have the same effect as a change of altitude, and it is evident that the sun will reach its greatest altitude when the change of altitude due to the combined effect of the ship's speed and the sun's change of declination is counterbalanced by the change of altitude in the opposite direction, due to the revolution of the earth.

Let c = the algebraic sum of the ship's speed in miles, and the sun's hourly change of declination.

$\frac{c \times 60}{60 \times 60} = \frac{c}{60}$ = corresponding change in seconds of arc per second of time.

If dz = change of altitude owing to the revolution of the earth in time dh,

$dz = dh$ cos lat. sin azimuth

= 15 cos lat. sin azimuth in seconds, per second of time.

The sun will have its maximum altitude when $dz = \frac{c}{60}$;

$$\therefore \quad 15 \cos l \frac{\sin h \,.\, \cos d}{\sin z} = \frac{c}{60};$$

$$\therefore \quad \sin h = c \frac{\sin (l \pm d)}{\cos l \,.\, \cos d} \times \frac{1}{60 \times 15'}$$

since z is very nearly equal to the meridian zenith distance.

h is the time, after meridian passage, of the sun's attaining its maximum altitude when the run is towards the sun, or before meridian passage when the run is away from it, l being the latitude by account, and d the declination for the G.M.T. of local apparent noon.

If the run be not directly towards or away from the sun, the change of altitude, due to the ship's motion, will be (very nearly) equal to the speed × cosine of the ship's course, which can be found from the Traverse Table.

Example.—November 6th, 1894, a ship was steaming due south in latitude by account 50° N, longitude 0°. Find when the sun dipped, the ship's speed being 20 knots. Declination, 16° 4′ 30″ S; hourly change, 45″ S.

$$\sin h = \frac{19\frac{1}{4}}{60 \times 15} \times \frac{\sin (l+d)}{\cos l \,.\, \cos d}$$

19·25	1·284431
$\sin (l+d)$	9·960983
$\sec l$	·191933
$\sec d$	·017322
	11·454669
900	2·954243
	8·500426

$\therefore$ $h = 7$ m. 15 s. past noon, as the altitude is *increased* by the motion of the ship.

Hence it is necessary under such conditions to calculate from the morning observations the time of the sun's meridian passage, and to take the altitude at that time as the meridian altitude; the altitude when the sun dips may be also observed, and the time taken, so as to obtain an ex-meridian altitude as a check on the latitude previously obtained.

Note.—For a complete discussion of this problem, with tables, see *The Ex-Meridian treated as a problem in Dynamics*, by H. B. Goodwin. (Philip & Son.)

If a ship has moderate steam power, so that she can steam much faster off the wind with the aid of fore and aft sail, it is possible to find how far she must steer under fore and aft sail, and then furl sails and steer directly for her port, so as on the whole to effect a saving in time.

§ 190. Let A be the position of the ship and B that of the port.

Suppose that the ship can steam m knots head or nearly head to wind, and n knots with fore and aft sail.

Let O be the point at which she must alter course so as to get to B in the shortest time.

Then the time will be practically unaffected, whether the ship goes on to P or turns at O and goes on to Q, where OP and OQ are both very small distances.

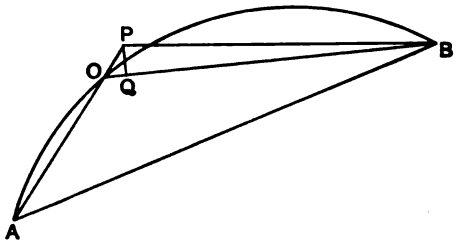


FIG. 137.

Hence $BQ=BP$, and the angle at Q is practically a right angle.

Then $$\cos QOP=\frac{OQ}{OP}=\frac{m}{n}=\frac{\text{slower speed}}{\text{faster speed}}.$$

Hence the point O is on the segment of a circle containing an angle $=180°-\cos^{-1}\frac{m}{n}$. Its position may be determined in any particular case by describing the segment on AB (AB bring measured to any desired scale), and drawing AO to make with AB the number of degrees kept away.

By measuring AO and BO it can be found in any case whether a saving in time can be effected.

Investigation of the construction of the table to find the Auxiliary Angle A.

§ 191. We have $2\cos A=\dfrac{\sin z\,.\,\sin z_1}{\cos a\,.\,\cos a_1}$.

Let $A=A'+\theta$ where A' is the main correction dependent upon the altitude of the moon only, θ that part of it dependent on the altitude of the sun or star.

Thus $2\cos A'=\dfrac{\sin z}{\cos a}$, z and a being the true zenith distance and apparent altitude of the moon's centre.

Again $2\cos(A'+\theta)=\dfrac{\sin z\,.\,\sin z_1}{\cos a\,.\,\cos a_1}$, z_1 and a_1 being similar quantities in the case of the sun or star.

It is required to find the value of θ.

Let p, p_1 be the corrections in altitude for the moon and sun or star respectively.

Then $$2\cos(A'+\theta)=\frac{\cos(a+p)\cos(a_1-p_1)}{\cos a\,.\cos a_1},$$

$$2\cos A'=\frac{\cos(a+p)}{\cos a};$$

$\therefore$ dividing we obtain

$$\frac{\cos(A'+\theta)}{\cos A'}=\frac{\cos(a_1-p_1)}{\cos a_1};$$

$$\therefore\quad \cos\theta-\tan A'.\sin\theta=\cos p_1+\tan a_1\,.\sin p_1.$$

Now, as θ and p_1 are both very small, $\cos\theta$ and $\cos p_1$ may be taken as equal to 1, and $\theta\sin 1''$, $p_1\sin 1''$ may be put for $\sin\theta$ and $\sin p_1$.

The expression then becomes

$$\theta=-p_1\frac{\tan a_1}{\tan A'}.$$

This correction is always negative, and lies between $-26''$ and $-34''$. In order, therefore, to avoid the negative sign in the small table, $34''$ is added to the value of θ and subtracted from the value of A'; thus

$$A=(\overline{A'-34''}+\overline{34''+\theta}).$$

To find the error of Collimation.

§ 192. In (1) QI and HR are the normals to the index and horizon glasses.

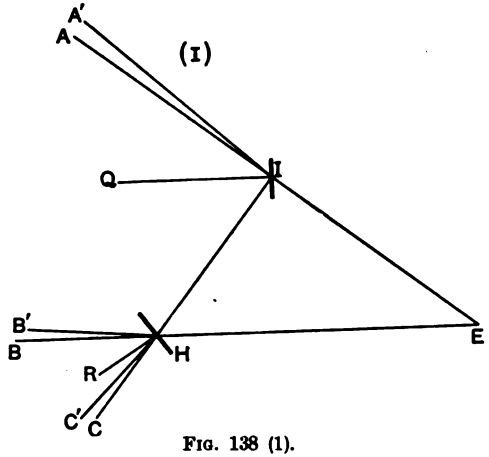


FIG. 138 (1).

Let the plane of the sextant meet a sphere of large radius in the great circle ABC, of centre E and pole P, the directions of QI and HR meeting the great circle in Q and R.

Let IH produced meet the great circle in C,
EH „ „ „ „ B,
EI „ „ „ „ A;
$\therefore$ $RC = RB$ and $QA = QC$.

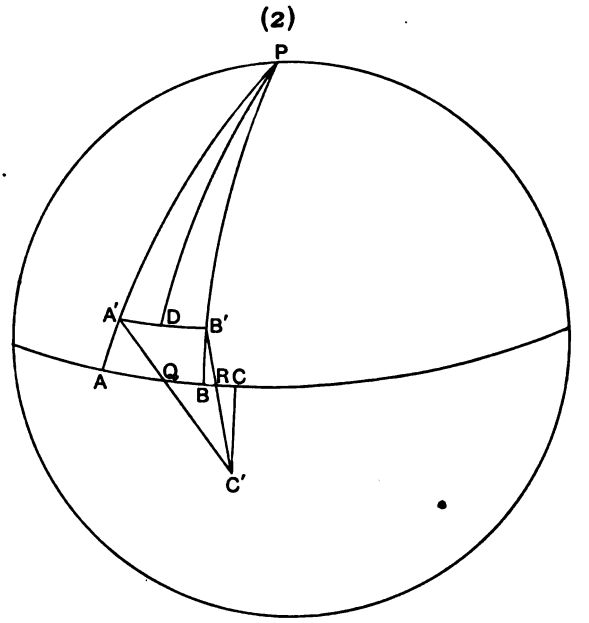


FIG. 138 (2).

In (2) QR = angle between planes of mirrors,
AB = angle between objects when there is no collimation error
$= 2RQ$.

Now suppose that the line of collimation is inclined at an angle θ to the plane of the instrument, A', B', C' being the corresponding positions of A, B, C in (1).

Let IH produced meet the sphere in C', and make $RB' = RC'$, and $QA' = QC'$. A' and B' will be the points in which EH and EI produced meet the sphere, and $A'EB'$ or the arc $A'B'$ will be the true angle between the objects.

But the angle read off on the sextant will be the arc AB.

From P drop a perpendicular PD on $A'B'$.

Let $A'B'$ = true angular distance $= \alpha$,
AB = distance read off $= \beta$,
$AA' = BB'$ = inclination of line of collimation $= \theta$.

In the right-angled triangle $PA'D$

$$\sin\frac{a}{2}=\sin\frac{\beta}{2}\cos\theta.$$

Let error of collimation $=\beta-a=e$.

$$\sin^2\frac{a}{2}=\sin^2\frac{\beta}{2}(1-\sin^2\theta);$$

$$\therefore\quad \sin^2\frac{\beta}{2}-\sin^2\frac{a}{2}=\sin^2\frac{\beta}{2}\sin^2\theta;$$

$$\therefore\quad \sin\frac{\beta+a}{2}\sin\frac{\beta-a}{2}=\sin^2\frac{\beta}{2}\sin^2\theta.$$

But $$\frac{\beta+a}{2}=\beta\text{ (very nearly)};$$

$$\therefore\quad \sin\beta\,.\sin\frac{e}{2}=\sin^2\frac{\beta}{2}\sin^2\theta;$$

$$\therefore\quad 2\sin\frac{\beta}{2}\cos\frac{\beta}{2}\cdot\frac{e}{2}\sin 1''=\sin^2\frac{\beta}{2}\theta^2\sin^2 1'';$$

$$\therefore\quad e=\theta^2\sin 1''\,.\tan\frac{\beta}{2}.$$

To find the time of year when the amount of Twilight is a minimum at any place.

§ 193. Let h_1 and h_2 be the hour angles at sunset and the end of evening twilight respectively.

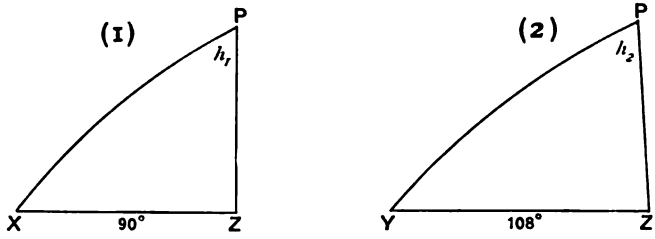


FIG. 139.

Then $$dh_1=\frac{1}{\tan PXZ\,.\cos d}\,dp \quad (\S\ 176),$$

and $$dh_2=\frac{1}{\tan PYZ\,.\cos d}\,dp \quad ,,$$

At the time of minimum twilight the corresponding changes in the hour angles of sunset and end of twilight for a given change of declination must be the same.

Hence $dh_1 = dh_2$, and therefore, from the above equations, $PXZ = PYZ$.

In (1) $$\sin l = \sin p \,.\, \cos X;$$

„ (2) $$\cos Y = \frac{\sin l - \cos p \,.\, \cos 108^\circ}{\sin p \,.\, \sin 108^\circ};$$

$$\therefore \quad \sin l = \frac{\sin l - \cos p \,.\, \cos 108^\circ}{\sin 108^\circ};$$

$$\therefore \quad \sin l(1 - \sin 108^\circ) = \cos p \,.\, \cos 108^\circ;$$

$$\therefore \quad \sin l(1 - \cos 18^\circ) = -\cos p \,.\, \sin 18^\circ;$$

$$\therefore \quad 2 \sin l \,.\, \sin^2 9^\circ = -2 \cos p \,.\, \sin 9^\circ \,.\, \cos 9^\circ;$$

$$\therefore \quad \sin d = \cos p$$

$$= -\sin l \,.\, \tan 9^\circ.$$

The value of d determines the time of year required.

To find the duration of Twilight when it is a minimum.

§ 194. Let S and S' be the positions of the sun at sunset and at the end of evening twilight. Then $SPS' =$ the duration.

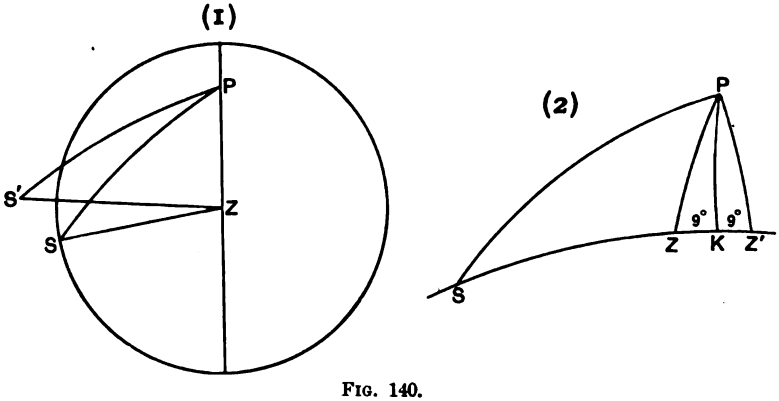


FIG. 140.

In (2) let the triangle PZS represent the same triangle as in (1), and let the triangle PZS represent the triangle PZS'; then, since the angles PSZ and $PS'Z$ are equal, the two triangles may be combined as in (2), where $SZ = 90^\circ$, $SZ' = 108^\circ$, and $ZPZ' = SPS' =$ duration required $= t$ suppose.

Let fall PK perpendicular to ZZ'; then

$$ZK = KZ' = 9^\circ$$

and $$ZPK = \frac{t}{2};$$

$$\therefore \quad \sin 9^\circ = \sin \frac{t}{2} \cos l;$$

$$\therefore \quad \sin \frac{t}{2} = \frac{\sin 9^\circ}{\cos l}.$$

§ 195. Since
$$\left.\begin{aligned} PZS &= 180^\circ - PZZ' \\ &= 180^\circ - PZ'S \end{aligned}\right\} (2);$$
$$\therefore \quad PZS = 180^\circ - PZS' \quad (1),$$

or the azimuths are supplementary.

§ 196. If PZ is less than 9°, ZZ' could not be equal to 18°, hence 81° is the limiting value of the latitude in which it is possible to have the twilight a minimum.

Here
$$\sin d = -\tan 9^\circ . \cos 9^\circ$$
$$= -\sin 9^\circ;$$
$$\therefore \quad d = 9^\circ \text{ of opposite name to the latitude,}$$

also
$$\sin \frac{t}{2} = \frac{\sin 9^\circ}{\cos 9^\circ} = 1;$$
$$\therefore \quad \frac{t}{2} = 6 \text{ hours,}$$

and
$$t = 12 \text{ hours;}$$

$\therefore$ the sun comes to the horizon at noon, and reaches the twilight parallel at midnight.

Interpolation.

§ 197. In most tables with which the reader is familiar, the tabulated numbers at any part of the tables increase or diminish uniformly with the numbers opposite to them.

In that case the principle of proportional parts, as it is called, may be correctly used. This principle may be put into analytical expression thus:—

If d_1 represent the increase of the tabulated number for an increase of n seconds of time, and d represent that corresponding to a time t seconds less than n seconds, then

$$d = \frac{t}{n} d_1.$$

If d_2 be the difference in the next n seconds, then d_2 ought to be very nearly equal to d_1. Should this not be so, we must examine whether $d_2 - d_1$ is approximately constant in the neighbourhood of that part of the tables which is being considered, that is, we must take a third interval, find d_3 corresponding to it, and compare $d_3 - d_2$ with $d_2 - d_1$.

If d_3-d_2 is then nearly equal to d_2-d_1, as will generally be found to be the case, we may say that the rate of change of the tabulations increases uniformly, or in other words, this rate is uniformly accelerated.

The character of the increase is thus similar to that of a point's motion in a line with an initial velocity u, which receives a uniform acceleration f.

In that case, as is well known, the whole increase at any time t from the beginning of the motion is given by

$$d=ut+\tfrac{1}{2}ft^2;$$

The same formula would apply to the tables, but it is usual to put it into a somewhat different form, in order to adapt it for purposes of calculation.

We may provisionally write it thus:—

$$d=(u+\tfrac{1}{2}fn)\,t+\tfrac{1}{2}ft(t-n);$$

and, assuming that this represents the law of increase over d_1+d_2, proceed to determine the coefficients in the two terms.

If t be put $=n$, we know that d becomes d_1;

$$\therefore\quad u+\tfrac{1}{2}fn=\frac{d_1}{n}.$$

If t be put $=2n$, we know that d becomes d_1+d_2.

$$\therefore\quad d_1+d_2=\frac{d_1}{n}2n+fn^2;$$

$$\therefore\quad f=\frac{d_2-d_1}{n^2},$$

usually written $\dfrac{\Delta_2}{n_2}$.

Hence
$$d=d_1\frac{t}{n}+\frac{1}{2}\Delta_2\frac{t}{n}\left(\frac{t}{n}-1\right).$$

In the event of the tabulations being such that d_2-d_1 is not sufficiently close to d_3-d_2 to justify the supposition that the rate of change increases uniformly, we must then suppose that a closer approximation is to be found from a formula obtained by taking three intervals instead of two; and adding a term in t^3, or following the analogy presented by the above result,

$$d=d_1\frac{t}{n}+\frac{1}{2}\Delta_2\frac{t}{n}\left(\frac{t}{n}-1\right)+\frac{1}{6}\Delta_3\frac{t}{n}\left(\frac{t}{n}-1\right)\left(\frac{t}{n}-2\right),$$

in which Δ_2, as before, stands for d_2-d_1, and the new symbol Δ_3, as can be proved, stands for $d_3-d_2-(d_2-d_1)$.

It is rare, however, that the tables will require third differences for their use. We proceed to illustrate the foregoing by working a numerical example, in which second differences suffice:

Example.—Given the lunar distance of Fomalhaut,

At midnight, - - - -	29°	44′	16″
„ XV. h., - - - -	28	40	38
„ XVIII. h., - - - -	27	39	21

find the correct distance at 14 h.

Here $d_1 = -1^\circ\ 3'\ 38''$,

$d_2 = -1\ 1\ 17$.

$d_2 - d_1 = +2'\ 21''$,

$\therefore$ distance required

$$= 29^\circ\ 44'\ 16'' + \tfrac{2}{3}(-1^\circ\ 3'\ 38'') + \frac{\frac{2}{3}(\frac{2}{3} - 1)}{2}(+2'\ 21'')$$

$$= 29^\circ\ 44'\ 16'' - 42'\ 25{\cdot}3'' - 15{\cdot}7''$$

$$= 29^\circ\ 1'\ 35^\circ.$$

MISCELLANEOUS EXAMPLES.

(1) Two ships left Cape St. Mary and Port Philip respectively, each sailing on the great circle which touches the parallel of the place of departure, and fell in with one another in $12\frac{1}{2}$ days. Find their rates of sailing.

Cape St. Mary { Lat. 25° 39′ S. / Long. 45 7 E. } Port Philip { Lat. 38° 18′ S. / Long. 144 43 E.

(2) A ship sails on a great circle from A (lat. $2l°$ N) to B (lat. $l°$ N). The distance from A to the vertex is double the distance from B to the point in which the great circle would cut the equator, and the difference of longitude between B and this point is 15°. Find the latitude of the vertex without first finding l.

(3) Two ships steam at the same speed from latitude 40° N, between south and west, on complementary courses. After a time it is found that one ship's change of latitude and longitude are respectively $\frac{1}{3}$ and $1\frac{1}{4}$ of the other ship's change of latitude. Find the courses and the latitudes arrived at.

(4) If a ship left the equator, steaming 10 knots on a great circle, and reached the tropic of Cancer 150 hours afterwards, find when she would reach the Arctic circle at the same rate.

(5) The latitude by account was 49° 46′ N. True altitude of sun's centre 34° 25′. Declination 20° N. True bearing of sun S 80° E. G.M.T. 8 h. 12 m. 41 s. A.M. Eq. T. 3 m. 40 s. (−from A.T.). Ship then steamed N 22·5′, when the Eddystone (lat. 50° 11′ N, long. 4° 16′ W) bore N 38° W. Determine the true position of the ship.

(6) A ship steaming through a current running WSW 3 knots, has to steer N 55° E in order to reach her port. She arrives in 8 hours, while, without the current, she would have arrived in $6\frac{1}{2}$ hours. Find her speed and the bearing and distance of the port at starting.

(7) The sun's meridian altitude at noon was 35°, and at midnight 5° 9′, the hourly change of declination being 45″ increasing. Find the latitude of the observer.

(8) The declination of a star is 60° N, and the latitude of an observer 48° N. Find the interval in mean time between the two periods at which the star bears NbE ½E above and below the pole respectively.

(9) June 3rd, 1895, in lat. 53° 20′ N, long. 6° 40′ W, find approximately the time of rising of the moon's centre.

(10) If the mean times of sunrise and sunset be 6 h. 22 m. and 5 h. 12 m. respectively, find the equation of time, giving your reason for saying whether it is additive to, or subtractive from, mean time.

(11) Determine the time at which the centre of the planet Jupiter appeared on the eastern horizon of a spectator in lat. 42° N, long. 55° 30′ W, on December 2nd, 1895.

(12) On December 19th, 1895, in long. 114° E, the time shown by a sidereal clock was 19 h. 17 m. 8 s., the error of the clock on sidereal time at the place being 0 h. 49 m. 54 s. fast. Required the correct local mean time at the instant, having given R.A.M.S. at G.M. noon, December 18th, 17 h. 47 m. 23·67 s.

(13) From the following data calculate the sun's *apparent* right ascension at mean midnight, June 1st:

Sidereal time, mean noon, June 1st,	4 h. 40 m. 46·3 s.
Equation of time, mean noon, June 1st, + to M.T.	0 h. 2 m. 22·96 s.
Equation of time, hourly change,	− ·378 s.

(14) In latitude 20° N, a star (dec. 26° 12′ S, R.A. 16 h. 22 m. 54 s.) had true altitude (E of meridian) 22° 45′, when a sidereal clock showed 11 h. 18 m. 29 s. What is the error of the clock on local sidereal time?

(15) Show that the length of the longest and shortest days at a given place are together equal to 24 hours. What is the latitude of a place in which the longest day is twice as long as the shortest?

(16) The moon passes the meridian of Greenwich at 7 h. 0 m. 6s. mean time, and its variation in R.A. is 2 m. 15·4 s. per hour. What will be the G.M.T. when the moon's hour angle is 1 h. 47 m. 20 s. at a place in longitude 33° 10′ W?

(17) The mean time of transit of the first point of Aries was 14 h. 29 m. 59 s. The equation of time at the preceding mean noon was 4 m. 40·73 s. subtractive from mean time. What was the sun's apparent right ascension at the preceding mean noon?

(18) At a certain place south of the equator the sun is always on the prime vertical at the same local sidereal time. What inference may be drawn as to the latitude of such a place? Find the mean time at which this occurred on December 22nd, 1895, at a place in this latitude upon the prime meridian.

(19) On December 4th, 1895, at sunrise, a ship left a point in latitude 40° N, longitude 5° W, sailing along the parallel. Sunrise next day occurred at the same Greenwich sidereal time as on the preceding day. Find the course and distance made good.

(20) Given that the time of a complete revolution of the moon with regard to the fixed stars is $27\frac{1}{3}$ days, show that the time of its revolution with respect to the sun is $29\frac{1}{2}$ days nearly.

(21) Given sidereal time 4 h. 30 m. 25 s., and the mean time of transit of the first point of Aries for the day 16 h. 32 m. 49 s., find the corresponding mean time. Longitude 115° E.

(22) Mars' period of revolution round the sun is 687 of our days, and, on his axis, $24\frac{2}{3}$ of our hours. What is the difference in length between his mean solar and sidereal day?

(23) If the R.A. of the sun at sunrise be 12 h. 35 m. 12 s., the sidereal time of sunrise 6 h. 53 m. 19 s., and the mean time of sunrise 18 h. 7 m. 20 s., find the equation of time at the following apparent noon, having given the equation of time at the previous apparent noon 10 m. 33 s. subtractive from apparent time.

(24) On June 3rd, 1895, determine the mean times when the poles of the ecliptic passed the meridian of Greenwich.

(25) Show that if $\cos(p-h) = \tan l \,.\, \sin p$, the equation of equal altitudes is equal to the change of polar distance for (h) one-half the interval between the observations.

(26) Show that the equation of equal altitudes vanishes and changes its sign at the time of the maximum azimuth.

(27) On May 4th, at 10 h. 15 m. 26 s. by chronometer, and on May 12th at 9 h. 43 m. 38 s., a star had equal altitudes on the same side of the meridian. Required the rate of the chronometer.

(28) A light is marked on a chart as visible 21 miles. What is its height and its distance when just visible from a height of 20 feet above the water?

(29) The sun rose at 4 h. 31 m. S.M.T., bearing by compass E 11° 30′ N, and set at 7 h. 21 m. S.M.T., bearing W 42° 50′ N. Find (1) the compass error, (2) the latitude of the place.

(30) At A, longitude 120° 16′ E, chronometer was slow on M.T. at place 9 h. 6 m. 12·61 s., daily rate 2·23 s. losing; at B, after an interval of 4 days, the chronometer was slow on M.T. at place 8 h. 57 m. 24·55 s., daily rate 0·65 s. losing. Required the longitude of B.

(31) In running a meridian distance between two places A and B, it was found that:

				h.	m.	s.
May 1st,	chronometer	fast	on S.M.T. at A,	6	45	7·5
„ 8th,	„	„	„ B,	6	52	29·2
„ 14th,	„	„	„ B,	6	52	49·0
„ 21st,	„	„	„ A,	6	46	10·7

What is the difference of longitude between A and B, and which place is further to the eastward?

(32) In measuring a base line by sound the time with the wind was 10 seconds, and against the wind was 12·5 seconds, the temperature being 64° F. Find the length of the base, having given that the velocity of sound is 1090 feet per second at 32° F., and increases 10 feet per second for an increase of 9° F. of temperature.

(33) Find the duration of morning twilight in latitude 48° 50′ N, the sun's declination being 21° N.

What is the lowest latitude in which twilight will last all night with the given declination?

(34) Show that the duration of twilight at the equator is

$$\tfrac{1}{15}\sin^{-1}(\sin 18^\circ \sec\delta) \text{ hours},$$

where δ is the declination of the sun.

(35) At a certain place in north latitude the sun rose at 5 A.M. on the longest day. Find the latitude of the place and the duration of morning twilight.

(36) Find the time in the afternoon when, at a place in latitude 8° N, the sun will appear stationary in azimuth, and the time during which the shadow will move backward, the declination being 20° 40′ N.

(37) In what parts of the earth can the ecliptic be perpendicular to the horizon?

At what time will this happen, in latitude 15° N in the month of May, when the sun's declination is 21° N, and what will be the sun's bearing at this time?

(38) The extremity of the shadow of a steeple was 175 feet from its base at noon at midwinter and 70 feet at the equinox. Find the latitude of the place.

(39) Given the sun's declination 22° 8′ S, and R.A. 16 h. 38 m. 25 s., find his longitude and the obliquity of the ecliptic.

(40) Given the moon's latitude 4° 3′ 30″ N, longitude 137° 50′, and the sun's longitude 51° 13′ 45″, find the angular distance of the moon from the sun.

(41) The sun's longitude was 50° 40′, the moon's R.A. 17 h. 45 m. 36 s., and its declination 15° 40′ S. Find the sun's lunar distance.

(42) If D be the true, and d the apparent lunar distance, A_1, A_2, Z_1, Z_2 the true altitudes and apparent zenith distances of the moon and the heavenly body respectively, prove that

$$\begin{aligned}\operatorname{vers} d = {} & \operatorname{vers}(Z_1 + Z_2) + \operatorname{vers}(B + D) + \operatorname{vers}(B - D) \\ & + \operatorname{vers}(A_1 + A_2 + B) + \operatorname{vers}(A_1 + A_2 - B) - 4,\end{aligned}$$

where B is an auxiliary angle determined by the relation

$$2\cos B = \frac{\sin Z_1 \sin Z_2}{\cos A_1 \cos A_2}.$$

Find an equation connecting B and A, the usual auxiliary angle.

(43) Calculate the apparent distance of Regulus from the farther limb of the moon, having given:

Obs. Alt. Moon's L.L.	Obs. Alt. Regulus.	True Central Distance.
40° 8′ 50″	40° 48′ 00″	76° 47′ 50″

In. cor. −1 40 In. cor. +2 00

Moon's hor. par. 54′ 27″. Moon's horizontal semi-diameter, 15′ 00″. Height of eye, 18 feet.

(44) If θ is the inclination of the line of collimation to the plane of a sextant, A the true angle between two stars, B the angle shown by the sextant, prove that

$$\sin \tfrac{1}{2} A = \cos \theta \times \sin \tfrac{1}{2} B,$$

and hence deduce the formula

$$\text{error} = \theta^2 \sin 1'' \times \tan \tfrac{1}{2} B.$$

(45) In latitude 33° N find approximately the time in the forenoon when a small error in the latitude used in working out the sun's hour angle produces an equal error in the longitude, the declination being 17° 43′ S.

(46) In working a sun double altitude in north latitude, the sun's semi-diameter, 16′, was omitted in the correction of the first altitude. What was the error in the resulting latitude, the bearings of the sun at the two observations being EbN and SSE respectively?

(47) If h be the sun's hour angle, in latitude l, when a small change in the latitude produces an equal change in the azimuth, show that $\cot h = \cos l$.

(48) If the sun (declination d) rises at a place in latitude l, with the same hour angle as it has next day at rising at another place, show that the difference of latitude between the places

$$= -\frac{\sin 2l}{\sin 2d} \times \text{sun's change of declination (nearly).}$$

(49) A ship leaves latitude l on a day when the sun's declination is d. It is found that the sun rises on the same bearing next morning. Show that

$$\text{change of latitude} = -\cot l \,.\, \cot d \,.\, \text{change of declination.}$$

(50) When the sun's declination is d, show that the latitude l, in which a small change in the latitude produces an equal change in the amplitude, is given by the equation

$$\cos^2 l = \sin d.$$

(51) Show that at a given place all bodies situated upon the same vertical circle are changing their altitude at the same rate.

Find the most advantageous altitude for determination of longitude under the following conditions:

(1) Latitude of place, 5° N; declination, 20° N.

(2) Latitude of place, 20° N; declination, 15° N.

(52) At a place in latitude l an observation of the sun is taken near the prime vertical, his altitude being a. Had the sun been truly upon the prime vertical his altitude would have been a', and hour angle h'.

Prove that the actual hour angle h is given approximately by

$$h = h' \pm (a - a') \sec l.$$

(53) Two stars have declinations d N and d S respectively, and differ by 12 hours in right ascension.

Show that at any instant at a given place the rates of change in the zenith distances of the stars are numerically equal.

(54) Prove that, in latitude 40° N, when the sun's declination is 15° N, the amplitude of the sun is about half a degree greater than it would be if there were no refraction.

(55) Prove that, at a given place, all heavenly bodies, at rising, change their azimuth at the same rate.

(56) Calculate the augmentation of the moon's horizontal semi-diameter when the apparent altitude of the moon's centre is 49°, the horizontal parallax 54′ 26″, and the horizontal semi-diameter 14′ 51″.

(57) Calculate the augmentation when the apparent altitude of the moon's centre is 51°, the horizontal parallax being 54′ 26″, and the horizontal semi-diameter 14′ 54″.

(58) If the true reduced zenith distance of the moon is 60°, and the diurnal parallax 52′ 20″, find the horizontal parallax.

(59) At a place in latitude 45° N (nearly), longitude 75° W, on December 23rd, 1895, the moon was observed at its maximum altitude instead of upon the meridian. Find the error in the latitude so occasioned, the declination used being that due to the Greenwich date of the meridian passage.

(60) On April 1st, 1895, a ship is steaming north at the rate of 16 knots, in latitude 37° N, longitude 10° W. At what time will the sun dip?

(61) On December 2nd, 1895, the moon and the star Capella were observed to be on the meridian at the same instant. Find the longitude of the place.

(62) Investigate an expression for finding the time of sunrise at a place m feet above the level of the sea.

Prove that an observer, whose height above the sea is $\frac{1}{n}$th of the earth's radius, will observe the time of sunrise

$$\frac{12}{\pi}\frac{\sec\delta}{\sin\theta}\sqrt{\frac{2}{n}}\text{ hours}$$

earlier than an observer on the sea level, where θ is the angle between the path of the rising sun and the horizon, and δ the declination of the sun.

(63) In latitude 64° 40′, at what height above the earth's surface will the sun's rising be accelerated by 4 minutes, on March 21st? (Neglecting the effects of refraction.)

(64) Find the time occupied by the sun in rising at Greenwich on April 9th, 1895.

(65) Two vertical rods of given heights are, on a given day, placed at a given distance apart. During the forenoon it is observed that the extremity of the shadow of the shorter rod just reaches the base of the longer, and, in the afternoon, the extremity of the shadow of the longer just reaches the base of the shorter.

Show how to find the latitude of the place; show also that, if the heights are 6 and 8 feet and the distance apart 20 feet, the problem is impossible in any latitude if the declination exceeds a certain value, and find that value.

(66) Supposing the sun's motion in longitude to be uniform, show that for a place within the Arctic Circle the period of continuous daylight in summer is given by the expression

$$\frac{365\frac{1}{4}}{\pi}\cos^{-1}(\cos l \,.\, \operatorname{cosec}\omega) \text{ days},$$

where l is the latitude of the place and ω the obliquity of the ecliptic.

In the particular cases where $l = 90°$ and $90° - \omega$, show that the corresponding periods are, respectively, half the year, and zero.

(67) If two stars rise together at the same place in latitude l, and afterwards have a common azimuth Z, prove that $\tan l = \sin Z$.

(68) At a certain place two stars rise at the same time. They also bear west at the same time. From these data find the latitude.

(69) Show that the time when the sun bears SW is determined by the expression

$$\tfrac{1}{15}\{\theta - \sin^{-1}(\cos\theta \,.\, \cos l \,.\, \tan d)\},$$

where l = lat. (N), d = sun's declination (N), and $\tan\theta = \sin l$.

(70) If the angle between the circles of altitude and declination of a heavenly body be supplementary to the azimuth, prove that

$$\cos\tfrac{1}{2} \text{ zen. dist.} = \cos\tfrac{1}{2} \text{ hour angle} \times \sin \text{ polar dist.}$$

(71) If a is a small change in the sun's longitude, b, b' the corresponding changes in the right ascension at the equinox and solstice respectively—

Show that $$bb' = a^2,$$

and $$\frac{b'}{b} = \cos^2\omega,$$

where ω is the obliquity of the ecliptic.

(72) In latitude l N, the sum of the sun's declination and altitude h hours before noon is s. Show how to find the declination and altitude.

(73) Show that the time when the sun's azimuth, reckoned from the elevated pole, is supplementary to his hour angle, is given by the formula

$$\cos h = -\cot p \,.\, \cot\frac{c}{2},$$

where p is polar distance and c is co-latitude.

Show also that in order that this phenomenon may occur before sunset the declination and latitude must be of contrary names.

(74) At a place in north latitude two stars A and B, whose right ascensions and declinations were α, α'; δ, δ' respectively were simultaneously on the same vertical circle in the NE quadrant. And a

great circle, let fall from the pole, at right angles to the vertical circle, intersected it at a point between A and B. Show that if ϕ be the angle between this perpendicular and the declination circle of the higher star, whose hour angle from the meridian is h,

$$\tan\left(\phi - \frac{a' - a}{2}\right) = \frac{\sin(\delta' - \delta)}{\sin(\delta' + \delta)} \cot\frac{a' - a}{2},$$

and

$$\cos(h + \phi) = \tan \text{ lat} \,.\, \cos\phi \,.\, \cos\delta.$$

(75) In latitude 3° N, the sun's hour angle is the same at the beginning of morning twilight on two days when his declination differs by 15°. Find the declinations.

(76) What will be the difference between the length of the morning and afternoon on April 10th, 1895, in latitude 40° N, when the change in the sun's declination is taken into account?

(77) At two places, A and B, the sun had the altitude 15° at the same instant of time, being to the eastward of the meridian at both places. Find the declination.

Latitude A 20° N.	Longitude A 10° W.
„ B 55° N.	„ B 5° W.

(78) Given three altitudes of a heavenly body and the intervals between the observations, show how to find the hour angle, declination, and latitude.

(79) Given the moon's declination—

At 12 h.	22°	8′	3·3″
13 h.	22	18	24·1
14 h.	22	28	37·5

find the exact declination at 13 h. 18 m.

(80) Given the moon's declination—

At 9 h.	21°	41′	55·1″
10 h.	21	51	39·4
11 h.	22	1	17·6

find accurately the declination at 9 h. 50 m.

(81) Having given Moon's R.A.—

	h.	m.	s.
At noon	15	14	58·4
2 h.	15	19	1·6
4 h.	15	23	16·4

find accurately the R.A. at 3 h. 15 m.

(82) Having given the following elements from the *Nautical Almanac*:

Moon's declination at	6 h.,	June 11th,	27° 14′ 24·5″ S
„	12 h.,	„	27 15 43·1 S
„	18 h.,	„	27 11 31·3 S

calculate the declination at 15 h. 30 m.

(83) Having given the lunar distances of α *Aquilæ* as follows:

Midnight	49° 7′ 16″
XV. h.	48 8 56
XVIII. h.	47 11 46

find, by interpolation, the lunar distance at XIV. h.

(84) Given lunar distance of *Aldebaran*—

Midnight	29° 51′ 24″
XV. h.	31 30 34
XVIII. h.	33 10 33

find the exact distance at XVI. h.

(85) Given the declination of *Venus*—

August 3 - - -	N 0° 6′ 25·4″
„ 4 - - -	S 0 18 0·4
„ 5 - - -	S 0 42 9·7

find accurately the declination at 8 P.M., August 3rd.

(86) Given the following approximate values of the longitude of the sun and moon, find the time of new moon:

		Sun's longitude.	Moon's longitude.
Oct. 28th	Noon	215° 4′ 30″	212° 0′ 0″
„	Midnight	215 34 30	218 41 15
„	Noon	216 4 30	225 18 30

(87) An observation of the sun, in latitude by account 53° 40′ N, places a ship in longitude 4° 53′ W, the sun's true bearing at the time being S 70° E. After the ship has sailed S 55° W (*true*) 40 miles, a light-ship in latitude 53° 20′ 30″ N, longitude 5° 53′ W, bears S 67° W (*true*). Find the ship's position.

(88) At a place in latitude 49° N, the amplitude of the sun was equal to its meridian altitude. Find the sun's declination.

(89) Find the distance between the sun and the moon, having given—

Sun's longitude - - -	38° 44′ 30″
Moon's longitude - - -	317 4 0
Moon's latitude - - -	4 34 0 S

(90) The angular distance of *Aldebaran* from the moon's centre at 3 h. 50 m. 22 s. S.M.T. was 66° 20′ 10″. From the *Nautical Almanac*, at noon and at 3 h., the distances of the star were 65° 10′ 40″ and 66° 43′ 20″. Determine the longitude of the place.

(91) Given the following approximate values of the longitudes of the sun and moon, required the time of full moon:

		Sun.	Moon.
Dec. 31st,	Noon	279° 36′ 30	95° 0′ 30″
„	Midnight	280 7 0	102 1 30
Jan. 1st,	Noon	280 37 30	109 6 0

(92) In latitude 51° 30′ N, what is the maximum rate of change in the sun's altitude per minute of time, and in what latitude is the maximum rate greater by one-half than this maximum?

(93) April 20th, 1895, in latitude 36° 5′ N, longitude 14° 31′ 30″ E, the sun had equal altitudes (about 37°) at the following times by chronometer:

A.M.	P.M.
11 h. 14 m. 45 s.	6 h. 13 m. 15 s.

Find the error of the chronometer on G.M.T.

(94) Show how to find the distance of a target by observing the angle subtended, at a given height, between the target and the horizon.

Ex.: Height of observer 50 feet; observed angle 1° 15′.

(95) In a certain latitude the sun rises one hour earlier when the declination is 18° N than it does when the declination is 7° N. Find the latitude.

(96) A sailing ship, which can sail within six points of the wind, leaves 51° 53′ N, 55° 22′ W, bound for 55° 26′ N, 7° 14′ W, the wind being E ½N (true). On which tack should she first proceed, and why?

(97) Construct a chart on the gnomonic projection, extending from 60° S to 30° S, and from 70° W to 170° E.

Lay down the great circle tract from Otago 45° 47′ S, 170° 45′ E, to Valparaiso 33° 2′ S, 71° 40′ W, and find the latitude and longitude of the vertex.

Lay down also the composite track, not going higher than 50° S, and find the longitude of the points at which the track touches the parallel.

(98) At a place in north latitude the sun rose bearing N $x°$ E. After an interval of 10 h. 57 m. the bearing was S $x°$ W. Having given the declination 10° N, find x and the latitude of the place.

(99) Find the latitude in which, on midsummer day, the sun's centre reaches the prime vertical 3 h. 40 m. 20 s. after rising, neglecting the effects of refraction.

(100) Given meridian passage of *Jupiter*, March 16th, h. 8 m. 26 s. 24
" " " " 17th, 8 22 18
" transit of first point of *Aries*, " 16th, 0 21 39
find the R.A. of *Jupiter* at G.M. noon, March 17th.

(101) April 17th, 1895, at 4 h. P.M. mean time at Greenwich, find the inclination of the ecliptic to the horizon.

(102) When the right ascension of the meridian is 18 hours, what is the altitude of the North Pole of the ecliptic, in latitude 37° N?

(103) In latitude 33° N find the time in the forenoon when a small error in the declination produces an equal error in the longitude, the declination being 13° N.

(104) Find the time occupied by the sun in rising, as seen by a spectator on board a ship steaming due east 12 knots at the equator, at the time of the equinox, the sun's diameter being 32′.

(105) If a person set out at sunrise to travel round the earth's equator and returned to his starting point at sunset, the sun having crossed his meridian n times, show that his daily speed was $\frac{4\pi r}{2n-3}$ if he travelled east, or $\frac{4\pi r}{2n+1}$ if he travelled west, r being the earth's radius.

(106) The latitudes of two places are complementary to each other, and the sun rises 3 hours later at one place than at the other when the declination is 22° N. Find the latitudes.

(107) Given that the sun's declination is 10° 3′ N and 10° 24′ 15″ N on successive days, the change of the sun's R.A. in the interval being 3 m. 42·25 s., find the sun's R.A. and the obliquity of the ecliptic.

(108) A vessel, steaming at a uniform rate, is observed to bear N 25° E; after an interval of 15 minutes, she bears N 10° W; and after a further interval of 20 minutes, she bears N 80° W. Find her course. (Projection may be used.)

(109) On April 29th, 1895, at noon, a ship's clock was set to local apparent time in lat. 47° N, long. 63° W. If the ship steams N 69° E (true) 12 knots, what time, to the nearest minute, will the clock show when the star α *Virginis* is next on the meridian, and what will be the star's altitude?

(110) Given the sun's declination 20° N, and that sunset takes place 2 h. 52 m. 47 s. after the sun is on the prime vertical, find the latitude.

(111) April 16th, 1895, in latitude 42° N, longitude 63° W, the sun dipped, to an observer on board a ship steaming on the meridian, 3 m. 21 s. before noon. Was the ship's course north or south (give a reason for your answer), and what was her speed?

(112) Two stars (R.A. 6 h., dec. 12° N; and R.A. 5 h. 10 m., dec. 3° S) rise together at a certain place, and when the south declination star is setting, the north declination star is on the prime vertical. Find the latitude.

(113) n divisions on the vernier of a sextant coincide with $n+1$ divisions on the arc, which is cut to 10′. Find n if the sextant is to read off to 15″.

If the reading on the sextant is 46° 12′, which division on the arc will coincide with a division on the vernier?

(114) In the month of May the sun has the same altitude, 46° 30′, at two places 5145 miles apart, on the same meridian, on opposite sides of the equator, the ship apparent time being 11 h. 26 m. A.M. Find the latitudes.

(115) Two stars, R.A. 8 h. 52 m. 5 s. and R.A. 7 h. 38 m. 9 s., rise together at a certain place in north latitude, their true bearings being N 49° 30′ E and N 73° E respectively. Find the latitude of the place and the declinations of the stars.

(116) A ship A, sailing on a known course (N x° E) at a known rate, observes another ship B, bearing from her (N y° E) at a distance of d knots. At the end of two equal intervals of time, the bearing of B from A is observed to be N α° E and N β° E respectively: show how to find, by geometrical construction, the course and rate of sailing of B (supposed uniform). If a be the distance sailed by A in each of the two intervals, show that θ, the course of B, may be found from the formula

$$\tan\left(\theta - \frac{\alpha+\beta}{2}\right) = \tan\frac{\alpha-\beta}{2} \cdot \tan(45° + \phi),$$

where

$$\tan\phi = \frac{2a\sin(x-\alpha) - 2d\sin(y-\alpha)}{2a\sin(x-\beta) - d\sin(y-\beta)}.$$

(117) At 8 h. 15 m. 37 s. A.M. apparent time the sun's true bearing was N 99° 30′ E, the declination being 16° N. Find the latitude.

(118) A squadron is steaming N 58° E at 10 knots; a cruiser, which bears from a flagship S 32° E, 5′, is ordered to take up a

position 4 cables on the starboard beam of the flagship. Supposing that the cruiser can steam 16 knots, what course should she steer and how long will it take her to reached the required position?

(119) Two ships *A* and *B* start at the same time from an anchorage. *A* steers N 20° E, and *B* N 60° E, each at a speed of 12 knots. *B* experiences a current setting north at a rate of 2 knots, and *A* a current also setting north. At the end of an hour *B* bears S 42° E from *A*. Required the rate of the current experienced by *A*.

(120) From a cruiser *A* steaming N 20° W at 10 knots, another cruiser *B*, steering a steady course, is observed bearing N 80° E distant 4′. After an interval of half an hour *B* bears N 65° E distant 5·5′. Find, by a graphical construction, the course and speed of *B*.

(121) Two ships *A* and *B* are anchored 4 cables apart, *A* having the western billet, in a line with a lighthouse which bears W b S from *A* distant 14 cables. They weigh together, and *A* steers N 26° E at 10 knots, while *B* steers N 49° E. At the end of half an hour *A* finds that *B* is still on the same bearing E b N, and that the lighthouse now bears S 50° W distant 6·2 miles. Find by protraction the set and rate of the current experienced, and the course and distance made good by *B*.

(122) From a cruiser a lighthouse bears due north distant 12 miles. The cruiser is ordered to steam in a north-easterly direction till the lighthouse bears due west, and thereafter to proceed to a rendezvous which bears from the lighthouse S 50° E, 44 miles. Find the shortest route by which the cruiser can execute the above orders; and obtain approximately the course she should steer, and her distance from the lighthouse when she alters course.

(123) Required the distance of a ship from Mount St. Peter, Ascension, having given

Observed altitude Mount St. Peter { on the arc 1° 44′, off the arc 1° 38′.

Height of eye, 24 feet.

Log cosine dip of Mount St. Peter = 9·999954.

(124) Required the distance of a ship from Pico Ruivo, Madeira, having given

Observed altitude Pico Ruivo { on the arc 3° 2′, off the arc 3° 7′.

Height of Pico Ruivo 6,188 feet.

Height of eye, 24 feet.

(125) At a place in latitude l N, at h hours after noon, the shadow of a vertical flagstaff on the top of a wall made an angle ϕ with the edge of the shadow of the wall. Given that the direction of the wall is N $\alpha°$ E and declination of the sun is d N, prove that

$$\phi = \alpha - \beta,$$

$$\text{where } \beta = \cot^{-1}\left(\frac{\sin l \cos h - \cos l \tan d}{\sin h}\right).$$

(126) From a boat a short distance from the foot of a cliff, the maximum angle subtended by a lighthouse on the top of the cliff is found to be 15° 50′. The cliff is known to be 120 feet high; find the height of the lighthouse and the distance of the boat from the foot of the cliff at the time the maximum angle was taken.

(127) Two ships x and y are bound for a port A in latitude l N, longitude λ_1 E; x starts from B latitude l S, longitude λ_2 E; at the same time y starts from C latitude l S, longitude λ_3 E, and they steam on arcs of great circles BA and CA. In order that they may arrive at A at the same time, show that the ratio of the speed of x to that of y must be

$$\frac{\cos^{-1}\left(\cos l \,.\, \cos\frac{\lambda_1 - \lambda_2}{2}\right)}{\cos^{-1}\left(\cos l \,.\, \cos\frac{\lambda_1 - \lambda_3}{2}\right)}.$$

(128) If ϕ be the latitude of the vertex of the great circle passing through two places whose latitudes are θ_1, θ_2, respectively, and whose difference of longitude is λ, show that

$$\cot\phi = \frac{\cot\theta_1 \,.\, \cot\theta_2 \,.\, \sin\lambda}{\sqrt{\cot^2\theta_1 + \cot^2\theta_2 - 2\cot\theta_1 \cot\theta_2 \cos\lambda}}.$$

(129) If X (latitude θ, longitude λ) be any point on the great circle joining A (latitude θ_1, longitude λ_1) and B (latitude θ_2, longitude λ_2), prove that

$$\tan\theta_1 \,.\, \sin(\lambda_2 - \lambda) + \tan\theta_2 \,.\, \sin(\lambda - \lambda_1) = \tan\theta \,.\, \sin(\lambda_2 - \lambda_1).$$

(130) A ship in latitude x is somewhere on the arc of a great circle joining A (latitude l_1) and B (latitude l_2), all the latitudes being of the same name. The ship steams on the great circle for a distance d, equal to the length of the arc AB, and is found to be again in latitude x; show that

$$\sin x = \frac{1}{2}\operatorname{cosec}\frac{d}{2}\sqrt{\sin^2 l_1 + \sin^2 l_2 - 2\sin l_1 \,.\, \sin l_2 \cos d}.$$

(131) A ship steams on a composite track from A, latitude θ_1, to B, latitude θ_2, the maximum latitude reached being 45°. If λ

be the difference of longitude between A and B, and x the distance on the parallel, show that

$$x = \frac{1}{\sqrt{2}}(\lambda - \mu),$$

where $\cos\mu = \tan\theta_1 \tan\theta_2 - \sec\theta_1 \sec\theta_2 \sqrt{\cos 2\theta_1 . \cos 2\theta_2}$.

(132) Rigel was on the prime vertical east of the meridian at 8 h. 6 m. 12 s. by chronometer, and on the prime vertical west of the meridian when the same chronometer showed 6 h. 12 m. 24 s. Find the latitude of the place and the sidereal time at which Rigel will set, having given Rigel's R.A. 5 h. 9 m. 42 s., declination 8° 19′ S, and rate of chronometer 3·8 s. gaining.

(133) At a place in south latitude the mean solar interval between the observed altitudes of Canopus (declination 52° 38′ 30″ S) was 3 h. 58 m. 1 s. Find the latitude, the true altitude of the star at each observation being 58° 26′.

(134) In latitude 44° N, find the angle which the path of the setting sun makes with the horizon when the declination is 8° S.

(135) In latitude 48° N the sun rose at 18 h. 46 m. 34 s., crossed the meridian at 1 h. 29 m. 10 s., and set at 8 h. 12 m. 42 s. local sidereal time. Required the sun's change of declination in a mean solar hour.

(136) At a place in latitude 28° N, the sun and Sirius (R.A. 6 h. 40 m. 44 s., declination 16° 35′ S) rose at the same time. Find the sun's longitude.

(137) In latitude 30° N, when the sun's declination is 20° N, find the ship apparent time when the angle between the hour circle and the circle of altitude passing through the sun is a maximum.

(138) Calculate the limits of south latitude between which it is possible for Fomalhaut and Altair to be on the same azimuth circle some time during the 24 hours, both stars being above the horizon.

(139) Find the minimum duration of twilight in latitude 50° N, and the time of year when it occurs.

(140) Find the time occupied by the sun in setting at a place in latitude 40° N, on the day when evening twilight is a minimum. Given sun's semi-diameter 16′.

(141) In latitude 36° 50′ S, when the sun's declination is 17° 30′ S, find the shortest time in which the sun rises through 1° of altitude. Find also the altitudes between which this occurs.

(142) Two stars whose declinations are d_1 and d_2 are on the prime vertical east of the meridian at the same time. Prove that the latitude of the place of observation is 45°.

And if the sidereal interval between the times at which the stars rise be H, prove that

$$2\cos\frac{H}{4}=\sqrt{(1+\tan d_1)(1+\tan d_2)}+\sqrt{(1-\tan d_1)(1-\tan d_2)}.$$

(143) At a place in latitude l the time of setting of a star whose R.A. and Dec. were α and δ, took place H sidereal hours before the time of setting of a star whose R.A. and Dec. were $\alpha+\alpha'$ and δ'. Prove that

$$\cot^2 l=\operatorname{cosec}^2(H\sim\alpha')\{\tan^2\delta+\tan^2\delta'-2\tan\delta\,.\,\tan\delta'\,.\,\cos(H\sim\alpha')\}.$$

(144) If the time of the place of observation is to be obtained from the altitude of the sun, and the latitude is not accurately known, show that the most advantageous time for taking the observation would be when the sun's altitude is

$$\sin^{-1}\left(\frac{\sin d}{\sin l}\right)\ \text{or}\ \sin^{-1}\left(\frac{\sin l}{\sin d}\right),$$

according as $l >$ or $< d$, where l = latitude, d = sun's declination, and l and d are of the same name.

(145) Two stars whose declinations are δ and δ' rise simultaneously, but the star whose declination is δ remains above the horizon twice as long as the other; prove that the latitude of the place of observation is given by the equation

$$\cot^2 l\pm\cot l\,.\,\tan\delta=2\tan^2\delta'.$$

(146) If α and α' be the differences between the right ascensions of the sun and a star when the sun's declination is δ and δ' respectively, and x be the right ascension of the star, then, reckoning α and α' as positive or negative according as x is $>$ or $<$ the sun's right ascension, show that

$$\tan x=\frac{\sin\alpha\,.\,\tan\delta'-\sin\alpha'\,.\,\tan\delta}{\cos\alpha\,.\,\tan\delta'-\cos\alpha'\,.\,\tan\delta}.$$

(147) If a star be, at the same instant, on the horizon of two places A and B, which are not at the extremities of a diameter of the earth; prove that the declination of the star is equal in magnitude to the co-latitude of the vertex of the great circle joining A and B.

Show also how to determine whether the declination is N or S.

(148) Show that if the altitude of a heavenly body be observed and an hour angle assumed, the corresponding latitude and azimuth may be determined from the equations

$$\sin A = \frac{\sin p}{\sin z} . \sin h,$$

$$\tan \frac{\theta}{2} = \frac{\sin z}{2 \cos \frac{1}{2}(p+z) . \cos \frac{1}{2}(p-z)} . \frac{\sin (A+h)}{\sin h},$$

where A = azimuth, p = polar distance, h = hour angle, θ = co-latitude.

(149) Two stars whose declinations are d and 0 respectively rise at the same instant at a place latitude l. If H be the sidereal interval between the instants at which they set, prove that

$$\sin \frac{H}{2} = \tan l . \tan d.$$

(150) If h and $h+\dot{h}$, p and $\dot{p}$ be the westerly hour angles and polar distances respectively of the sun at sunrise on two successive days, show that

$$\dot{h} = \frac{2\dot{p}}{15} \cot h . \operatorname{cosec} 2p \text{ approximately.}$$

(151) If, at a place in latitude l, the intervals between two successive transits of a north circumpolar star, of declination d, over a circle of altitude whose azimuth is NαW, where α is a very small angle, be 12 h. $-h$ and 12 h. $+h$ respectively, show that

$$h = \frac{2\alpha}{15} \cos l . \cos d \text{ approximately.}$$

(152) If the inclination of the line of collimation in a sextant is 40′ and the angle read off is 126° 30′, find the true angle.

NAUTICAL ALMANAC.

APRIL, 1895—AT APPARENT NOON.

Day of the Week.	Day of the Month.	THE SUN'S		Equation of Time, *to be* added to / subt. from *Apparent Time.*	Var. in 1 hour.
		Apparent Declination.	Var. in 1 hour.		
		° ′ ″	″	m. s.	s.
Mon.	1	N 4 33 23·4	57·85	3 56·90	0·754
Tues.	2	4 56 29·3	57·64	3 38·86	0·749
Wed.	3	5 19 29·9	57·40	3 20·93	0·745
Thur.	4	5 42 24·6	57·15	3 3·12	0·739
Frid.	5	6 5 13·2	56·89	2 45·46	0·732
Sat.	6	6 27 55·4	56·62	2 27·97	0·725
Sun.	7	6 50 30·9	56·33	2 10·66	0·717
Mon.	8	7 12 59·3	56·03	1 53·55	0·708
Tues.	9	7 35 20·4	55·72	1 36·68	0·698
Wed.	10	7 57 33·8	55·39	1 20·07	0·686
Thur.	11	8 19 39·3	55·06	1 3·73	0·674
Frid.	12	8 41 36·5	54·71	0 47·70	0·661
Sat.	13	9 3 25·3	54·35	0 31·98	0·648
Sun.	14	9 25 5·1	53·97	0 16·60	0·634
Mon.	15	9 46 35·7	53·58	0 1·57	0·619
Tues.	16	10 7 56·7	53·17	0 13·09	0·603
Wed.	17	10 29 7·9	52·75	0 27·37	0·587
Thur.	18	10 50 8·8	52·32	0 41·24	0·570
Frid.	19	11 10 59·1	51·87	0 54·71	0·553
Sat.	20	11 31 38·5	51·41	1 7·76	0·535
Sun.	21	11 52 6·6	50·93	1 20·37	0·516
Mon.	22	12 12 23·1	50·44	1 32·53	0·497
Tues.	23	12 32 27·6	49·93	1 44·23	0·478
Wed.	24	12 52 19·8	49·41	1 55·46	0·458
Thur.	25	13 11 59·3	48·87	2 6·21	0·438
Frid.	26	13 31 25·8	48·33	2 16·48	0·418
Sat.	27	13 50 39·0	47·76	2 26·25	0·397
Sun.	28	14 9 38·4	47·18	2 35·53	0·376
Mon.	29	14 28 23·8	46·59	2 44·30	0·355
Tues.	30	14 46 54·9	45·99	2 52·57	0·334
Wed.	31	N 15 5 11·2	45·36	3 0·33	0·313

APRIL, 1895—AT MEAN NOON.

Day of the Week.	Day of the Month.	THE SUN'S Apparent Declination.	THE SUN'S Semi-diameter.*	Equation of Time, to be subt. from / added to Mean Time.	Sidereal Time.
		° ′ ″	′ ″	m. s.	h. m. s.
Mon.	1	N 4 33 19·6	16 2·0	3 56·95	0 38 22·47
Tues.	2	4 56 25·8	16 1·7	3 38·91	0 42 19·02
Wed.	3	5 19 26·7	16 1·4	3 20·97	0 46 15·58
Thur.	4	5 42 21·7	16 1·2	3 3·16	0 50 12·13
Frid.	5	6 5 10·6	16 0·9	2 45·50	0 54 8·68
Sat.	6	6 27 53·1	16 0·6	2 28·00	0 58 5·24
Sun.	7	6 50 28·8	16 0·4	2 10·68	1 2 1·79
Mon.	8	7 12 57·5	16 0·1	1 53·57	1 5 58·35
Tues.	9	7 35 18·9	15 59·8	1 36·70	1 9 54·90
Wed.	10	7 57 32·6	15 59·5	1 20·08	1 13 51·46
Thur.	11	8 19 38·3	15 59·3	1 3·75	1 17 48·01
Frid.	12	8 41 35·8	15 59·0	0 47·71	1 21 44·56
Sat.	13	9 3 24·8	15 58·7	0 31·99	1 25 41·12
Sun.	14	9 25 4·9	15 58·5	0 16·60	1 29 37·67
Mon.	15	9 46 35·7	15 58·2	0 1·57	1 33 34·23
Tues.	16	10 7 56·9	15 57·9	0 13·09	1 37 30·78
Wed.	17	10 29 8·3	15 57·6	0 27·37	1 41 27·34
Thur.	18	10 50 9·4	15 57·4	0 41·25	1 45 23·89
Frid.	19	11 10 59·9	15 57·1	0 54·72	1 49 20·45
Sat.	20	11 31 39·5	15 56·9	1 7·77	1 53 17·00
Sun.	21	11 52 7·8	15 56·6	1 20·38	1 57 13·56
Mon.	22	12 12 24·4	15 56·3	1 32·54	2 1 10·11
Tues.	23	12 32 29·1	15 56·1	1 44·24	2 5 6·67
Wed.	24	12 52 21·4	15 55·8	1 55·47	2 9 3·22
Thur.	25	13 12 1·0	15 55·6	2 6·23	2 12 59·78
Frid.	26	13 31 27·7	15 55·3	2 16·50	2 16 56·33
Sat.	27	13 50 40·9	15 55·1	2 26·27	2 20 52·89
Sun.	28	14 9 40·4	15 54·8	2 35·55	2 24 49·44
Mon.	29	14 28 25·9	15 54·6	2 44·32	2 28 46·00
Tues.	30	14 46 57·0	15 54·4	2 52·59	2 32 42·55
Wed.	31	N 15 5 13·4	15 54·1	3 0·34	2 36 39·11

* The Semi-diameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

APRIL, 1895—MEAN TIME.

THE MOON'S

Day of the Month.	Semi-diameter.		Horizontal Parallax.		Meridian Passage.
	Noon.	*Midnight.*	*Noon.*	*Midnight.*	*Upper.*
	′ ″	′ ″	′ ″	′ ″	h. m.
1	15 49·1	15 53·8	57 57·3	58 14·6	5 5·2
2	15 58·6	16 3·2	58 31·9	58 48·9	6 6·1
3	16 7·7	16 12·0	59 5·4	59 21·1	7 6·3
4	16 15·9	16 19·5	59 35·6	59 48·6	8 4·0
5	16 22·5	16 24·9	59 59·6	60 8·3	8 58·5
6	16 26·5	16 27·3	60 14·3	60 17·2	9 50·2
7	16 27·2	16 26·2	60 16·9	60 13·0	10 40·1
8	16 24·1	16 21·2	60 5·6	59 54·8	11 29·4
9	16 17·3	16 12·6	59 40·6	59 23·5	12 19·3
10	16 7·3	16 1·3	59 3·8	58 41·9	13 10·8
11	15 54·9	15 48·2	58 18·4	57 53·9	14 4·3
12	15 41·3	15 34·5	57 28·8	57 3·7	14 59·7
13	15 27·8	15 21·3	56 39·1	56 15·4	15 55·7
14	15 15·2	15 9·6	55 53·1	55 32·5	16 50·8
15	15 4·5	15 0·0	55 13·8	54 57·4	17 43·6
16	14 56·1	14 53·0	54 43·2	54 31·6	18 33·1
17	14 50·5	14 48·8	54 22·6	54 16·1	19 19·3
18	14 47·7	14 47·3	54 12·2	54 10·8	20 2·5
19	14 47·6	14 48·5	54 11·9	54 15·3	20 43·6
20	14 50·1	14 52·2	54 20·9	54 28·6	21 23·6
21	14 54·7	14 57·8	54 38·0	54 49·1	22 3·5
22	15 1·2	15 4·9	55 1·7	55 15·4	22 44·5
23	15 8·9	15 13·1	55 30·0	55 45·4	23 27·7
24	15 17·4	15 21·8	56 1·2	56 17·2	* *
25	15 26·2	15 30·5	56 33·3	56 49·1	0 14·3
26	15 34·8	15 38·9	57 4·7	57 19·8	1 5·1
27	15 42·8	15 46·6	57 34·3	57 48·1	2 0·4
28	15 50·2	15 53·6	58 1·3	58 13·7	2 59·5
29	15 56·8	15 59·7	58 25·3	58 36·1	4 0·5
30	16 2·4	16 4·9	58 46·1	58 55·2	5 0·9
31	16 7·2	16 9·2	59 3·5	59 10·7	5 58·7

APRIL, 1895—MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
MONDAY 1.				
	h. m. s.	s.	° ′ ″	″
0	5 31 16·95	25·672	N 28 33 3·0	15·84
1	5 33 51·08	25·704	28 34 32·7	14·04
2	5 36 25·40	25·736	28 35 51·5	12·23
3	5 38 59·91	25·765	28 36 59·5	10·42
4	5 41 34·58	25·793	28 37 56·5	8·59
5	5 44 9·42	25·819	28 38 42·6	6·77
6	5 46 44·41	25·844	28 39 17·8	4·94
7	5 49 19·55	25·868	28 39 41·9	3·09
8	5 51 54·83	25·891	28 39 54·9	1·25
9	5 54 30·24	25·912	28 39 56·9	0·59
10	5 57 5·77	25·931	28 39 47·8	2·45
11	5 59 41·41	25·948	28 39 27·5	4·31
12	6 2 17·15	25·964	28 38 56·1	6·17
13	6 4 52·98	25·979	28 38 13·5	8·03
14	6 7 28·90	25·992	28 37 19·7	9·89
15	6 10 4·89	26·004	28 36 14·8	11·76
16	6 12 40·95	26·015	28 34 58·6	13·64
17	6 15 17·07	26·023	28 33 31·1	15·52
18	6 17 53·23	26·031	28 31 52·4	17·38
19	6 20 29·44	26·037	28 30 2·5	19·26
20	6 23 5·67	26·040	28 28 1·3	21·13
21	6 25 41·92	26·043	28 25 48·9	23·01
22	6 28 18·19	26·045	28 23 25·2	24·89
23	6 30 54·46	26·044	28 20 50·2	26·78
24	6 33 30·72	26·043	N 28 18 3·9	28·65
THURSDAY 4.				
0	8 35 52·76	24·563	N 22 33 27·3	111·68
1	8 38 19·99	24·514	22 22 12·8	113·14
2	8 40 46·93	24·464	22 10 49·6	114·60
3	8 43 13·56	24·413	21 59 17·6	116·04
4	8 45 39·89	24·364	21 47 37·1	117·46
5	8 48 5·93	24·314	21 35 48·1	118·88
6	8 50 31·66	24·263	21 23 50·6	120·27
7	8 52 57·09	24·213	21 11 44·9	121·64
8	8 55 22·22	24·163	20 59 30·9	123·02
9	8 57 47·04	24·112	20 47 8·7	124·37
10	9 0 11·56	24·061	20 34 38·5	125·70
11	9 2 35·77	24·010	20 22 0·3	127·02
12	9 4 59·68	23·959	20 9 14·3	128·32
13	9 7 23·28	23·908	19 56 20·5	129·60
14	9 9 46·58	23·858	19 43 19·1	130·87
15	9 12 9·58	23·808	19 30 10·1	132·13
16	9 14 32·27	23·757	19 16 53·6	133·36
17	9 16 54·66	23·707	19 3 29·8	134·58
18	9 19 16·75	23·657	18 49 58·6	135·79
19	9 21 38·54	23·607	18 36 20·3	136·98
20	9 24 0·03	23·557	18 22 34·8	138·16
21	9 26 21·22	23·507	18 8 42·4	139·31
22	9 28 42·11	23·458	17 54 43·1	140·45
23	9 31 2·71	23·409	17 40 37·0	141·58
24	9 33 23·02	23·361	N 17 26 24·2	142·68

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
SATURDAY 6.				
	h. m. s.	s.	° ′ ″	″
0	10 28 8·87	22·327	N 11 16 27·3	163·91
1	10 30 22·72	22·291	11 0 1·9	164·57
2	10 32 36·36	22·256	10 43 32·5	165·21
3	10 34 49·79	22·221	10 26 59·4	165·83
4	10 37 3·01	22·188	10 10 22·5	166·44
5	10 39 16·04	22·155	9 53 42·1	167·03
6	10 41 28·87	22·122	9 36 58·2	167·60
7	10 43 41·50	22·090	9 20 10·9	168·15
8	10 45 53·95	22·060	9 3 20·4	168·68
9	10 48 6·22	22·030	8 46 26·7	169·20
10	10 50 18·31	22·000	8 29 30·0	169·69
11	10 52 30·22	21·971	8 12 30·4	170·18
12	10 54 41·96	21·943	7 55 27·9	170·63
13	10 56 53·54	21·917	7 38 22·8	171·08
14	10 59 4·96	21·891	7 21 15·0	171·50
15	11 1 16·23	21·865	7 4 4·8	171·90
16	11 3 27·34	21·839	6 46 52·2	172·29
17	11 5 38·30	21·816	6 29 37·3	172·66
18	11 7 49·13	21·793	6 12 20·3	173·01
19	11 9 59·82	21·771	5 55 1·2	173·34
20	11 12 10·38	21·749	5 37 40·2	173·65
21	11 14 20·81	21·728	5 20 17·4	173·95
22	11 16 31·12	21·708	5 2 52·8	174·23
23	11 18 41·31	21·688	4 45 26·7	174·48
24	11 20 51·38	21·670	N 4 27 59·0	174·72
SUNDAY 14.				
0	17 43 19·84	23·896	S 28 35 30·0	6·68
1	17 45 43·15	23·873	28 36 5·3	5·10
2	17 48 6·32	23·849	28 36 31·2	3·53
3	17 50 29·34	23·824	28 36 47·6	1·95
4	17 52 52·21	23·799	28 36 54·6	0·39
5	17 55 14·93	23·773	28 36 52·3	1·16
6	17 57 37·49	23·746	28 36 40·7	2·72
7	17 59 59·88	23·717	28 36 19·7	4·27
8	18 2 22·09	23·688	28 35 49·5	5·80
9	18 4 44·13	23·658	28 35 10·1	7·33
10	18 7 5·98	23·626	28 34 21·5	8·87
11	18 9 27·64	23·593	28 33 23·7	10·39
12	18 11 49·10	23·560	28 32 16·8	11·91
13	18 14 10·36	23·527	28 31 0·8	13·42
14	18 16 31·42	23·492	28 29 35·8	14·92
15	18 18 52·26	23·456	28 28 1·8	16·42
16	18 21 12·89	23·420	28 26 18·8	17·91
17	18 23 33·30	23·383	28 24 26·9	19·39
18	18 25 53·48	23·344	28 22 26·1	20·87
19	18 28 13·43	23·306	28 20 16·5	22·33
20	18 30 33·15	23·267	28 17 58·1	23·79
21	18 32 52·63	23·227	28 15 31·0	25·25
22	18 35 11·87	23·185	28 12 55·1	26·69
23	18 37 30·85	23·143	28 10 10·7	28·13
24	18 39 49·59	23·102	S 28 7 17·6	29·56

JUNE, 1895—AT APPARENT NOON.

Day of the Week.	Day of the Month.	THE SUN'S Apparent Declination.	THE SUN'S Var. in 1 hour.	Equation of Time, *to be subt. from* / *added to* Apparent Time.	Var. in 1 hour.
		° ′ ″	″	m. s.	s.
Sat.	1	N 22 3 54·8	20·39	2 26·69	0·373
Sun.	2	22 11 52·6	19·42	2 17·54	0·389
Mon.	3	22 19 27·1	18·45	2 8·03	0·404
Tues.	4	22 26 38·3	17·48	1 58·16	0·418
Wed.	5	22 33 26·0	16·49	1 47·95	0·432
Thur.	6	22 39 50·0	15·50	1 37·42	0·445
Frid.	7	22 45 50·2	14·51	1 26·58	0·458
Sat.	8	22 51 26·6	13·51	1 15·44	0·470
Sun.	9	22 56 38·9	12·51	1 4·03	0·481
Mon.	10	23 1 27·1	11·50	0 52·36	0·491
Tues.	11	23 5 51·1	10·49	0 40·45	0·501
Wed.	12	23 9 50·7	9·48	0 28·32	0·510
Thur.	13	23 13 25·9	8·46	0 15·99	0·518
Frid.	14	23 16 36·6	7·43	0 3·48	0·525
Sat.	15	23 19 22·6	6·40	0 9·19	0·531
Sun.	16	23 21 43·9	5·37	0 22·00	0·536
Mon.	17	23 23 40·5	4·34	0 34·92	0·540
Tues.	18	23 25 12·3	3·31	0 47·93	0·544
Wed.	19	23 26 19·3	2·28	1 1·01	0·546
Thur.	20	23 27 1·5	1·24	1 14·14	0·547
Frid.	21	23 27 18·8	0·20	1 27·28	0·547
Sat.	22	23 27 11·2	0·83	1 40·41	0·546
Sun.	23	23 26 38·8	1·87	1 53·49	0·544
Mon.	24	23 25 41·5	2·90	2 6·50	0·540
Tues.	25	23 24 19·5	3·93	2 19·41	0·535
Wed.	26	23 22 32·7	4·96	2 32·18	0·529
Thur.	27	23 20 21·2	5·99	2 44·78	0·521
Frid.	28	23 17 45·2	7·01	2 57·20	0·513
Sat.	29	23 14 44·6	8·04	3 9·41	0·504
Sun.	30	23 11 19·5	9·05	3 21·39	0·494
Mon.	31	N 23 7 30·1	10·06	3 33·12	0·483

JUNE, 1895—AT MEAN NOON.

Day of the Week.	Day of the Month.	THE SUN'S Apparent Declination.	THE SUN'S Semi-diameter.*	Equation of Time, to be added to / subt. from Mean Time.	Sidereal Time.
		° ′ ″	′ ″	m. s.	h. m. s.
Sat.	1	N 22 3 55·6	15 48·2	2 26·67	4 38 52·38
Sun.	2	22 11 53·3	15 48·1	2 17·53	4 42 48·94
Mon.	3	22 19 27·8	15 48·0	2 8·02	4 46 45·50
Tues.	4	22 26 38·9	15 47·9	1 58·15	4 50 42·06
Wed.	5	22 33 26·5	15 47·7	1 47·94	4 54 38·61
Thur.	6	22 39 50·4	15 47·6	1 37·41	4 58 35·17
Frid.	7	22 45 50·6	15 47·5	1 26·57	5 2 31·73
Sat.	8	22 51 26·9	15 47·4	1 15·43	5 6 28·29
Sun.	9	22 56 39·3	15 47·3	1 4·02	5 10 24·85
Mon.	10	23 1 27·3	15 47·2	0 52·36	5 14 21·41
Tues.	11	23 5 51·2	15 47·1	0 40·45	5 18 17·96
Wed.	12	23 9 50·8	15 47·0	0 28·32	5 22 14·52
Thur.	13	23 13 26·0	15 46·9	0 15·99	5 26 11·08
Frid.	14	23 16 36·6	15 46·8	0 3·48	5 30 7·64
Sat.	15	23 19 22·6	15 46·7	0 9·19	5 34 4·20
Sun.	16	23 21 43·9	15 46·6	0 21·99	5 38 0·76
Mon.	17	23 23 40·5	15 46·6	0 34·91	5 41 57·31
Tues.	18	23 25 12·3	15 46·5	0 47·92	5 45 53·87
Wed.	19	23 26 19·3	15 46·4	1 1·00	5 49 50·43
Thur.	20	23 27 1·5	15 46·4	1 14·13	5 53 46·99
Frid.	21	23 27 18·8	15 46·3	1 27·27	5 57 43·55
Sat.	22	23 27 11·2	15 46·2	1 40·39	6 1 40·11
Sun.	23	23 26 38·8	15 46·2	1 53·47	6 5 36·67
Mon.	24	23 25 41·6	15 46·2	2 6·48	6 9 33·23
Tues.	25	23 24 19·6	15 46·1	2 19·38	6 13 29·78
Wed.	26	23 22 32·9	15 46·1	2 32·15	6 17 26·34
Thur.	27	23 20 21·5	15 46·1	2 44·76	6 21 22·90
Frid.	28	23 17 45·5	15 46·1	2 57·18	6 25 19·46
Sat.	29	23 14 45·0	15 46·0	3 9·38	6 29 16·02
Sun.	30	23 11 20·0	15 46·0	3 21·36	6 33 12·58
Mon.	31	N 23 7 30·7	15 46·0	3 33·09	6 37 9·14

* The Semi-diameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

JUNE, 1895—MEAN TIME.

THE MOON'S

Day of the Month.	Semi-diameter.		Horizontal Parallax.		Meridian Passage.
	Noon.	*Midnight.*	*Noon.*	*Midnight.*	*Upper.*
	′ ″	′ ″	′ ″	′ ″	h. m.
1	16 6·1	16 3·9	58 59·6	58 51·3	7 17·5
2	16 1·3	15 58·6	58 42·1	58 32·0	8 4·4
3	15 55·6	15 52·3	58 20·9	58 8·9	8 52·4
4	15 48·8	15 45·1	57 56·1	57 42·5	9 42·5
5	15 41·1	15 37·0	57 28·1	57 13·0	10 35·2
6	15 32·8	15 28·4	56 57·4	56 41·4	11 30·4
7	15 24·0	15 19·5	56 25·1	56 8·8	12 26·7
8	15 15·1	15 10·8	55 52·7	55 37·0	13 22·3
9	15 6·7	15 2·9	55 22·0	55 7·8	14 15·6
10	14 59·3	14 56·1	54 54·9	54 43·3	15 5·3
11	14 53·4	14 51·2	54 33·3	54 25·1	15 51·4
12	14 49·5	14 48·4	54 18·9	54 14·9	16 34·3
13	14 48·0	14 48·2	54 13·2	54 13·9	17 14·9
14	14 49·0	14 50·6	54 17·1	54 22·9	17 54·4
15	14 52·9	14 55·8	54 31·2	54 42·0	18 33·7
16	14 59·4	15 3·7	54 55·3	55 10·9	19 14·3
17	15 8·6	15 13·9	55 28·7	55 48·4	19 57·5
18	15 19·8	15 26·0	56 9·9	56 32·7	20 44·6
19	15 32·5	15 39·2	56 56·4	57 20·8	21 36·6
20	15 45·8	15 52·3	57 45·2	58 9·2	22 34·1
21	15 58·6	16 4·5	58 32·2	58 53·8	23 35·9
22	16 9·9	16 14·6	59 13·4	59 30·7	* *
23	16 18·5	16 21·6	59 45·1	59 56·5	0 39·8
24	16 23·9	16 25·2	60 4·7	60 9·5	1 42·5
25	16 25·6	16 25·1	60 11·0	60 9·3	2 41·6
26	16 23·9	16 21·9	60 4·7	59 57·4	3 36·4
27	16 19·3	16 16·1	59 47·8	59 36·2	4 27·3
28	16 12·5	16 8·6	59 23·0	59 8·5	5 15·5
29	16 4·4	16 0·0	58 53·2	58 37·2	6 2·6
30	15 55·5	15 51·0	58 20·8	58 4·2	6 50·0
31	15 46·5	15 42·0	57 47·6	57 31·2	7 38·9

JUNE, 1895—MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

MONDAY 3.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	h. m. s.	s.	° ′ ″	″
0	13 21 47·33	21·069	S 11 44 52·6	151·14
1	13 23 53·83	21·098	11 59 57·5	150·49
2	13 26 0·51	21·128	12 14 58·5	149·82
3	13 28 7·37	21·158	12 29 55·3	149·13
4	13 30 14·41	21·188	12 44 48·0	148·43
5	13 32 21·63	21·220	12 59 36·4	147·71
6	13 34 29·05	21·252	13 14 20·5	146·98
7	13 36 36·66	21·284	13 29 0·2	146·24
8	13 38 44·46	21·317	13 43 35·4	145·48
9	13 40 52·47	21·352	13 58 6·0	144·72
10	13 43 0·68	21·386	14 12 32·0	143·94
11	13 45 9·10	21·421	14 26 53·3	143·14
12	13 47 17·73	21·456	14 41 9·7	142·33
13	13 49 26·57	21·492	14 55 21·3	141·51
14	13 51 35·63	21·528	15 9 27·8	140·67
15	13 53 44·91	21·565	15 23 29·3	139·83
16	13 55 54·41	21·602	15 37 25·7	138·97
17	13 58 4·13	21·639	15 51 16·9	138·09
18	14 0 14·08	21·677	16 5 2·8	137·20
19	14 2 24·26	21·716	16 18 43·3	136·30
20	14 4 34·67	21·755	16 32 18·4	135·38
21	14 6 45·32	21·794	16 45 47·9	134·45
22	14 8 56·20	21·833	16 59 11·8	133·51
23	14 11 7·32	21·874	17 12 30·0	132·55
24	14 13 18·69	21·914	S 17 25 42·4	131·58

TUESDAY 11.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	h. m. s.	s.	° ′ ″	″
0	20 40 36·57	20·443	S 21 41 21·6	96·73
1	20 42 39·05	20·385	21 31 38·5	97·62
2	20 44 41·19	20·329	21 21 50·1	98·50
3	20 46 43·00	20·273	21 11 56·5	99·36
4	20 48 44·47	20·217	21 1 57·8	100·21
5	20 50 45·60	20·161	20 51 54·0	101·05
6	20 52 46·40	20·105	20 41 45·2	101·88
7	20 54 46·86	20·050	20 31 31·4	102·71
8	20 56 47·00	19·996	20 21 12·7	103·52
9	20 58 46·81	19·942	20 10 49·2	104·32
10	21 0 46·30	19·888	20 0 20·9	105·11
11	21 2 45·47	19·835	19 49 47·9	105·88
12	21 4 44·32	19·782	19 39 10·3	106·65
13	21 6 42·85	19·729	19 28 28·1	107·42
14	21 8 41·07	19·677	19 17 41·3	108·17
15	21 10 38·97	19·625	19 6 50·1	108·91
16	21 12 36·57	19·574	18 55 54·4	109·64
17	21 14 33·86	19·523	18 44 54·4	110·36
18	21 16 30·84	19·473	18 33 50·1	111·08
19	21 18 27·53	19·423	18 22 41·5	111·78
20	21 20 23·92	19·374	18 11 28·7	112·47
21	21 22 20·02	19·325	18 0 11·8	113·15
22	21 24 15·82	19·277	17 48 50·9	113·83
23	21 26 11·34	19·229	17 37 25·9	114·50
24	21 28 6·57	19·182	S 17 25 56·9	115·16

WEDNESDAY 12.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	h. m. s.	s.	° ′ ″	″
0	21 28 6·57	19·182	S 17 25 56·9	115·16
1	21 30 1·52	19·135	17 14 24·0	115·80
2	21 31 56·19	19·089	17 2 47·3	116·43
3	21 33 50·59	19·044	16 51 6·8	117·07
4	21 35 44·72	18·999	16 39 22·5	117·69
5	21 37 38·58	18·954	16 27 34·5	118·30
6	21 39 32·17	18·910	16 15 42·9	118·90
7	21 41 25·50	18·868	16 3 47·7	119·50
8	21 43 18·58	18·826	15 51 48·9	120·08
9	21 45 11·41	18·783	15 39 46·7	120·65
10	21 47 3·98	18·742	15 27 41·1	121·22
11	21 48 56·31	18·701	15 15 32·1	121·78
12	21 50 48·39	18·661	15 3 19·7	122·33
13	21 52 40·24	18·622	14 51 4·1	122·88
14	21 54 31·85	18·583	14 38 45·2	123·42
15	21 56 23·23	18·544	14 26 23·1	123·94
16	21 58 14·38	18·507	14 13 57·9	124·46
17	22 0 5·31	18·470	14 1 29·6	124·97
18	22 1 56·02	18·433	13 48 58·3	125·47
19	22 3 46·51	18·398	13 36 24·0	125·97
20	22 5 36·79	18·363	13 23 46·7	126·45
21	22 7 26·87	18·329	13 11 6·6	126·93
22	22 9 16·74	18·295	12 58 23·6	127·40
23	22 11 6·41	18·262	12 45 37·8	127·87
24	22 12 55·88	18·229	S 12 32 49·2	128·33

DECEMBER, 1895—AT APPARENT NOON.

Day of the Week.	Day of the Month.	THE SUN'S Apparent Declination.	Var. in 1 hour.	Equation of Time, *to be subt. from* / *added to* Apparent Time.	Var. in 1 hour.
		° ′ ″	″	m. s.	s.
Sun.	1	S 21 49 30·9	23·29	10 50·47	0·936
Mon.	2	21 58 37·3	22·24	10 27·68	0·963
Tues.	3	22 7 18·4	21·18	10 4·26	0·988
Wed.	4	22 15 33·9	20·11	9 40·24	1·013
Thur.	5	22 23 23·6	19·03	9 15·64	1·037
Frid.	6	22 30 47·2	17·93	8 50·47	1·060
Sat.	7	22 37 44·4	16·83	8 24·77	1·082
Sun.	8	22 44 15·0	15·72	7 58·55	1·102
Mon.	9	22 50 18·9	14·60	7 31·85	1·122
Tues.	10	22 55 55·8	13·47	7 4·68	1·141
Wed.	11	23 1 5·4	12·33	6 37·08	1·159
Thur.	12	23 5 47·6	11·19	6 9·07	1·175
Frid.	13	23 10 2·3	10·04	5 40·69	1·189
Sat.	14	23 13 49·3	8·88	5 11·98	1·202
Sun.	15	23 17 8·4	7·71	4 42·97	1·214
Mon.	16	23 19 59·6	6·55	4 13·71	1·224
Tues.	17	23 22 22·7	5·38	3 44·23	1·232
Wed.	18	23 24 17·7	4·20	3 14·56	1·239
Thur.	19	23 25 44·4	3·02	2 44·75	1·244
Frid.	20	23 26 42·9	1·85	2 14·84	1·248
Sat.	21	23 27 13·1	0·67	1 44·86	1·250
Sun.	22	23 27 15·0	0·51	1 14·85	1·250
Mon.	23	23 26 48·5	1·69	0 44·85	1·249
Tues.	24	23 25 53·8	2·87	0 14·89	1·247
Wed.	25	23 24 30·7	4·05	0 15·01	1·244
Thur.	26	23 22 39·4	5·22	0 44·80	1·238
Frid.	27	23 20 20·0	6·40	1 14·45	1·232
Sat.	28	23 17 32·4	7·57	1 43·92	1·224
Sun.	29	23 14 16·8	8·73	2 13·20	1·215
Mon.	30	23 10 33·3	9·89	2 42·25	1·205
Tues.	31	23 6 22·0	11·05	3 11·04	1·194
Wed.	32	S 23 1 43·1	12·19	3 39·54	1·181

DECEMBER, 1895—AT MEAN NOON.

Day of the Week.	Day of the Month.	THE SUN'S Apparent Declination.	THE SUN'S Semi-diameter.*	Equation of Time, *to be added to* / *subt. from Mean Time.*	Sidereal Time.
		° ′ ″	′ ″	m. s.	h. m. s.
Sun.	1	S 21 49 35·1	16 15·8	10 50·30	16 40 22·17
Mon.	2	21 58 41·2	16 15·9	10 27·51	16 44 18·73
Tues.	3	22 7 22·0	16 16·1	10 4·09	16 48 15·28
Wed.	4	22 15 37·2	16 16·2	9 40·07	16 52 11·84
Thur.	5	22 23 26·5	16 16·3	9 15·47	16 56 8·40
Frid.	6	22 30 49·8	16 16·5	8 50·31	17 0 4·96
Sat.	7	22 37 46·7	16 16·6	8 24·62	17 4 1·52
Sun.	8	22 44 17·1	16 16·7	7 58·41	17 7 58·08
Mon.	9	22 50 20·7	16 16·8	7 31·71	17 11 54·64
Tues.	10	22 55 57·3	16 16·9	7 4·54	17 15 51·20
Wed.	11	23 1 6·8	16 17·0	6 36·95	17 19 47·75
Thur.	12	23 5 48·8	16 17·1	6 8·95	17 23 44·31
Frid.	13	23 10 3·3	16 17·2	5 40·58	17 27 40·87
Sat.	14	23 13 50·1	16 17·3	5 11·88	17 31 37·43
Sun.	15	23 17 9·0	16 17·4	4 42·88	17 35 33·99
Mon.	16	23 20 0·0	16 17·5	4 13·62	17 39 30·55
Tues.	17	23 22 23·0	16 17·6	3 44·15	17 43 27·11
Wed.	18	23 24 17·9	16 17·6	3 14·49	17 47 23·67
Thur.	19	23 25 44·6	16 17·7	2 44·70	17 51 20·23
Frid.	20	23 26 43·0	16 17·8	2 14·80	17 55 16·79
Sat.	21	23 27 13·1	16 17·8	1 44·83	17 59 13·34
Sun.	22	23 27 15·0	16 17·9	1 14·83	18 3 9·90
Mon.	23	23 26 48·5	16 18·0	0 44·83	18 7 6·46
Tues.	24	23 25 53·8	16 18·0	0 14·88	18 11 3·02
Wed.	25	23 24 30·7	16 18·1	0 15·00	18 14 59·58
Thur.	26	23 22 39·5	16 18·1	0 44·78	18 18 56·14
Frid.	27	23 20 20·1	16 18·2	1 14·42	18 22 52·70
Sat.	28	23 17 32·6	16 18·2	1 43·89	18 26 49·26
Sun.	29	23 14 17·1	16 18·2	2 13·16	18 30 45·82
Mon.	30	23 10 33·7	16 18·2	2 42·20	18 34 42·38
Tues.	31	23 6 22·6	16 18·2	3 10·98	18 38 38·94
Wed.	32	S 23 1 43·9	16 18·2	3 39·47	18 42 35·50

* The semi-diameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

DECEMBER, 1895—MEAN TIME.

THE MOON'S

Day of the Month.	Semi-diameter.		Horizontal Parallax.		Meridian Passage.
	Noon.	*Midnight.*	*Noon.*	*Midnight.*	*Upper.*
	′ ″	′ ″	′ ″	′ ″	h. m.
1	15 27·0	15 31·8	56 36·2	56 53·7	11 30·0
2	15 36·5	15 40·9	57 10·9	57 27·3	12 27·6
3	15 45·1	15 49·1	57 42·8	57 57·3	13 27·4
4	15 52·7	15 56·0	58 10·5	58 22·4	14 27·0
5	15 58·8	16 1·3	58 32·9	58 42·1	15 24·2
6	16 3·5	16 5·3	58 50·0	58 56·6	16 18·2
7	16 6·8	16 7·9	59 2·0	59 6·2	17 9·0
8	16 8·7	16 9·3	59 9·2	59 11·2	17 57·4
9	16 9·6	16 9·5	59 12·2	59 12·0	18 44·9
10	16 9·2	16 8·5	59 10·7	59 8·3	19 32·9
11	16 7·5	16 6·1	59 4·5	58 59·5	20 22·6
12	16 4·3	16 2·2	58 53·0	58 45·1	21 15·1
13	15 59·6	15 56·6	58 35·6	58 24·6	22 10·8
14	15 53·2	15 49·3	58 12·1	57 58·1	23 9·0
15	15 45·2	15 40·7	57 42·8	57 26·3	* *
16	15 35·9	15 31·0	57 9·0	56 51·0	0 7·9
17	15 26·0	15 21·0	56 32·6	56 14·2	1 5·4
18	15 16·0	15 11·2	55 56·0	55 38·5	1 59·4
19	15 6·7	15 2·5	55 21·9	55 6·5	2 49·1
20	14 58·7	14 55·4	54 52·6	54 40·5	3 34·7
21	14 52·7	14 50·5	54 30·4	54 22·5	4 17·0
22	14 49·0	14 48·1	54 16·9	54 13·8	4 57·0
23	14 48·0	14 48·6	54 13·4	54 15·6	5 36·0
24	14 49·9	14 52·0	54 20·4	54 27·9	6 15·0
25	14 54·7	14 58·1	54 38·0	54 50·5	6 55·4
26	15 2·2	15 6·8	55 5·3	55 22·2	7 38·5
27	15 11·9	15 17·5	55 41·0	56 1·3	8 25·3
28	15 23·4	15 29·5	56 22·9	56 45·3	9 16·7
29	15 35·7	15 41·9	57 8·0	57 30·8	10 12·8
30	15 47·9	15 53·7	57 53·0	58 14·2	11 12·6
31	15 59·1	16 4·0	58 34·0	58 51·9	12 13·8
32	16 8·3	16 11·9	59 7·7	59 20·9	13 13·9

DECEMBER, 1895—MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.	Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	MONDAY 2.					MONDAY 23.			
	h. m. s.	s.	° ′ ″	″		h. m. s.	s.	° ′ ″	″
0	4 42 46·00	24·738	N 27 17 31·4	41·48	0	23 34 14·96	17·381	S 1 5 17·3	139·53
1	4 45 14·58	24·788	27 21 35·5	39·88	1	23 35 59·24	17·379	0 51 20·0	139·57
2	4 47 43·45	24·837	27 25 29·9	38·26	2	23 37 43·51	17·378	0 37 22·5	139·60
3	4 50 12·62	24·885	27 29 14·6	36·63	3	23 39 27·77	17·378	0 23 24·8	139·63
4	4 52 42·07	24·932	27 32 49·5	34·98	4	23 41 12·04	17·378	S 0 9 26·9	139·65
5	4 55 11·80	24·978	27 36 14·4	33·33	5	23 42 56·31	17·379	N 0 4 31·0	139·66
6	4 57 41·80	25·022	27 39 29·5	31·68	6	23 44 40·59	17·381	0 18 29·0	139·67
7	5 0 12·06	25·065	27 42 34·6	30·01	7	23 46 24·88	17·383	0 32 27·0	139·67
8	5 2 42·58	25·107	27 45 29·6	28·33	8	23 48 9·19	17·387	0 46 25·0	139·66
9	5 5 13·34	25·147	27 48 14·5	26·64	9	23 49 53·53	17·392	1 0 22·9	139·64
10	5 7 44·34	25·186	27 50 49·3	24·94	10	23 51 37·90	17·398	1 14 20·7	139·63
11	5 10 15·57	25·224	27 53 13·8	23·23	11	23 53 22·30	17·403	1 28 18·4	139·61
12	5 12 47·03	25·261	27 55 28·1	21·52	12	23 55 6·74	17·410	1 42 16·0	139·58
13	5 15 18·70	25·296	27 57 32·1	19·81	13	23 56 51·22	17·418	1 56 13·3	139·53
14	5 17 50·58	25·329	27 59 25·8	18·08	14	23 58 35·76	17·427	2 10 10·4	139·49
15	5 20 22·65	25·361	28 1 9·1	16·35	15	0 0 20·34	17·435	2 24 7·2	139·44
16	5 22 54·91	25·393	28 2 42·0	14·61	16	0 2 4·98	17·445	2 38 3·7	139·38
17	5 25 27·36	25·422	28 4 4·4	12·86	17	0 3 49·68	17·457	2 51 59·8	139·32
18	5 27 59·97	25·448	28 5 16·3	11·10	18	0 5 34·46	17·468	3 5 55·5	139·25
19	5 30 32·74	25·475	28 6 17·6	9·34	19	0 7 19·30	17·480	3 19 50·8	139·18
20	5 33 5·67	25·500	28 7 8·4	7·57	20	0 9 4·22	17·493	3 33 45·6	139·09
21	5 35 38·74	25·523	28 7 48·5	5·80	21	0 10 49·22	17·508	3 47 39·9	139·01
22	5 38 11·94	25·544	28 8 18·0	4·03	22	0 12 34·21	17·523	4 1 33·7	138·91
23	5 40 45·27	25·565	28 8 36·9	2·25	23	0 14 19·49	17·538	4 15 26·8	138·79
24	5 43 18·72	25·584	N 28 8 45·0	0·46	24	0 16 4·77	17·555	N 4 29 19·2	138·68

JUPITER, 1895—MEAN TIME.

Month and Day.	*Apparent* Right Ascension. *Noon.*	*Apparent* Declination. *Noon.*	Meridian Passage.
	h. m. s.	° ′ ″	h. m.
April 14	6 2 18·78	N 23 29 0·7	4 32·1
15	6 2 57·57	23 29 4·4	4 28·8
16	6 3 36·87	23 29 7·5	4 25·5
December 1	8 46 40·89	N 18 29 12·0	16 3·6
2	8 46 36·10	18 29 43·6	15 59·6
3	8 46 30·50	18 30 18·3	15 55·6

APRIL, 1895—MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.	Noon.	P.L. of diff.	IIIh.	P.L. of diff.	VIh.	P.L. of diff.	IXh.	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
1	Sun W	72 14 19	2808	73 48 37	2797	75 23 9	2786	76 57 55	2775
14	Sun E	117 49 26	3166	116 22 36	3181	114 56 4	3197	113 29 51	3212
15	Sun E	106 23 5	3282	104 58 32	3294	103 34 14	3307	102 10 10	3319

JUNE.

3	Pollux W	91 50 32	2419	93 33 40	2425	95 16 39	2431	96 59 29	2438
3	Antares E	44 57 4	2415	43 13 50	2421	41 30 45	2428	39 47 50	2434

DECEMBER.

2	αPegasi W	79 41 26	2788	81 16 9	2775	82 51 9	2763	84 26 25	2752
2	Jupiter E	56 19 7	2556	54 39 12	2545	52 59 2	2536	51 18 39	2526
23	Sun W	82 12 13	3480	83 32 59	3479	84 53 47	3478	86 14 36	3476

APRIL, 1895—MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.	*Midnight.*	P.L. of diff.	XV^h	P.L. of diff.	XVIII^h	P.L. of diff.	XXI^h	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
1	Sun W	78 32 56	2763	80 8 12	2753	81 43 42	2741	83 19 27	2730
14	Sun E	112 3 56	3227	110 38 19	3241	109 12 58	3255	107 47 54	3268
15	Sun E	100 46 20	3330	99 22 43	3341	97 59 19	3352	96 36 7	3361
	JUNE.								
3	Pollux W	98 42 10	2445	100 24 41	2452	102 7 2	2459	103 49 13	2466
3	Antares E	38 5 4	2441	36 22 28	2448	34 40 1	2455	32 57 44	2463
	DECEMBER.								
2	α Pegasi W	86 1 56	2741	87 37 41	2732	89 13 39	2722	90 49 50	2713
2	Jupiter E	49 38 2	2516	47 57 11	2507	46 16 8	2497	44 34 51	2489
23	Sun W	87 35 27	3472	88 56 22	3470	90 17 20	3467	91 38 21	3462

TABLES

USED IN DETERMINING THE LATITUDE BY OBSERVATIONS OF THE POLE STAR OUT OF THE MERIDIAN.

TABLE I.

Containing the *First* Correction.

Argument:—Sidereal Time of Observation.

Sidereal Time.	Correction.	Sidereal Time.	Sidereal Time.	Correction.	Sidereal Time.
h. m.	° ′ ″	h. m.	h. m.	° ′ ″	h. m.
0 0	−1 10 29+	12 0	6 0	−0 25 39+	18 0
10	1 11 32	10	10	0 22 33	10
20	1 12 26	20	20	0 19 25	20
30	1 13 13	30	30	0 16 14	30
40	1 13 52	40	40	0 13 1	40
50	1 14 22	50	50	0 9 47	50
1 0	1 14 43	13 0	7 0	0 6 32	19 0
10	1 14 56	10	10	−0 3 16+	10
20	1 15 0	20	20	0 0 0	20
30	1 14 56	30	30	+0 3 16−	30
40	1 14 43	40	40	0 6 32	40
50	1 14 22	50	50	0 9 47	50
2 0	1 13 52	14 0	8 0	0 13 1	20 0
10	1 13 13	10	10	0 16 14	10
20	1 12 26	20	20	0 19 25	20
30	1 11 32	30	30	0 22 33	30
40	1 10 29	40	40	0 25 39	40
50	1 9 17	50	50	0 28 42	50
3 0	1 7 58	15 0	9 0	0 31 42	21 0
10	1 6 32	10	10	0 34 38	10
20	1 4 57	20	20	0 37 30	20
30	1 3 15	30	30	0 40 18	30
40	1 1 26	40	40	0 43 1	40
50	0 59 30	50	50	0 45 39	50
4 0	0 57 27	16 0	10 0	0 48 13	22 0
10	0 55 18	10	10	0 50 40	10
20	0 53 2	20	20	0 53 2	20
30	0 50 40	30	30	0 55 18	30
40	0 48 13	40	40	0 57 27	40
50	0 45 39	50	50	0 59 30	50
5 0	0 43 1	17 0	11 0	1 1 26	23 0
10	0 40 18	10	10	1 3 15	10
20	0 37 30	20	20	1 4 57	20
30	0 34 38	30	30	1 6 32	30
40	0 31 42	40	40	1 7 58	40
50	0 28 42	50	50	1 9 17	50
6 0	−0 25 39+	18 0	12 0	+1 10 29−	24 0

TABLE II.

Containing the *Second* Correction (*always to be added*).
Arguments:—Sidereal Time and Altitude.

Sidereal Time.	ALTITUDE. 0°	5°	10°	15°	20°	25°	30°	35°	Sidereal Time.
h. m.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	h. m.
0 0	0 0	0 0	0 1	0 2	0 2	0 3	0 3	0 4	12 0
30	0 0	0 0	0 0	0 1	0 1	0 1	0 1	0 2	30
1 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	13 0
30	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	30
2 0	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	14 0
30	0 0	0 0	0 1	0 1	0 2	0 2	0 3	0 3	30
3 0	0 0	0 1	0 2	0 2	0 3	0 4	0 5	0 6	15 0
30	0 0	0 1	0 2	0 4	0 5	0 7	0 8	0 10	30
4 0	0 0	0 2	0 4	0 5	0 7	0 9	0 12	0 14	16 0
30	0 0	0 2	0 5	0 7	0 10	0 12	0 15	0 19	30
5 0	0 0	0 3	0 6	0 9	0 12	0 15	0 19	0 23	17 0
30	0 0	0 3	0 7	0 10	0 14	0 18	0 22	0 27	30
6 0	0 0	0 4	0 8	0 12	0 16	0 20	0 25	0 30	18 0
30	0 0	0 4	0 8	0 13	0 17	0 22	0 27	0 33	30
7 0	0 0	0 4	0 9	0 13	0 18	0 23	0 28	0 34	19 0
30	0 0	0 4	0 9	0 13	0 18	0 23	0 28	0 34	30
8 0	0 0	0 4	0 8	0 13	0 17	0 22	0 27	0 33	20 0
30	0 0	0 4	0 8	0 12	0 16	0 21	0 26	0 31	30
9 0	0 0	0 4	0 7	0 11	0 15	0 19	0 23	0 28	21 0
30	0 0	0 3	0 6	0 9	0 13	0 16	0 20	0 24	30
10 0	0 0	0 3	0 5	0 8	0 10	0 13	0 17	0 20	22 0
30	0 0	0 2	0 4	0 6	0 8	0 10	0 13	0 16	30
11 0	0 0	0 1	0 3	0 4	0 6	0 8	0 9	0 11	23 0
30	0 0	0 1	0 2	0 3	0 3	0 5	0 6	0 7	30
12 0	0 0	0 0	0 1	0 2	0 2	0 3	0 3	0 4	24 0

TABLE III. (*for* 1895).

Containing the *Third* Correction (*always to be added*).
Arguments:—Sidereal Time and Date.

Sidereal Time.	Jan. 1.	Feb. 1.	March 1.	April 1.	May 1.	June 1.	July 1.
h.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	1 17	1 14	1 7	0 58	0 50	0 47	0 48
2	1 15	1 17	1 14	1 6	0 57	0 50	0 46
4	1 9	1 16	1 17	1 13	1 5	0 56	0 48
6	1 1	1 10	1 16	1 16	1 11	1 3	0 53
8	0 52	1 2	1 10	1 15	1 15	1 9	1 0
10	0 46	0 53	1 1	1 10	1 14	1 13	1 7
12	0 43	0 46	0 53	1 2	1 10	1 14	1 12
14	0 45	0 43	0 46	0 54	1 3	1 10	1 14
16	0 51	0 44	0 43	0 47	0 55	1 4	1 12
18	0 59	0 50	0 44	0 44	0 49	0 57	1 7
20	1 8	0 58	0 50	0 45	0 45	0 51	1 0
22	1 14	1 7	0 59	0 46	0 46	0 47	0 53
24	1 17	1 14	1 7	0 50	0 50	0 47	0 48

TABLE II.

Containing the *Second* Correction (*always to be added*).
Arguments:—Sidereal Time and Altitude.

Sidereal Time.	ALTITUDE.								Sidereal Time.
	35°	40°	45°	50°	55°	60°	65°	70°	
h. m.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	h. m.
0 0	0 4	0 5	0 6	0 7	0 8	0 10	0 12	0 16	12 0
30	0 2	0 2	0 2	0 3	0 3	0 4	0 5	0 6	30
1 0	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	13 0
30	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	30
2 0	0 1	0 1	0 1	0 2	0 2	0 3	0 3	0 4	14 0
30	0 3	0 4	0 4	0 5	0 6	0 8	0 10	0 12	30
3 0	0 6	0 7	0 9	0 10	0 13	0 15	0 19	0 24	15 0
30	0 10	0 12	0 14	0 17	0 20	0 25	0 30	0 39	30
4 0	0 14	0 17	0 20	0 24	0 29	0 35	0 43	0 56	16 0
30	0 19	0 22	0 27	0 32	0 38	0 46	0 57	1 13	30
5 0	0 23	0 28	0 33	0 39	0 47	0 57	1 10	1 30	17 0
30	0 27	0 32	0 39	0 46	0 55	1 7	1 23	1 46	30
6 0	0 30	0 36	0 43	0 52	1 2	1 15	1 33	1 59	18 0
30	0 33	0 39	0 47	0 56	1 7	1 21	1 40	2 9	30
7 0	0 34	0 41	0 49	0 58	1 10	1 24	1 44	2 14	19 0
30	0 34	0 41	0 49	0 58	1 10	1 25	1 45	2 15	30
8 0	0 33	0 40	0 48	0 57	1 8	1 22	1 42	2 11	20 0
30	0 31	0 37	0 45	0 53	1 4	1 17	1 35	2 3	30
9 0	0 28	0 34	0 40	0 48	0 58	1 10	1 26	1 51	21 0
30	0 24	0 29	0 35	0 42	0 50	1 0	1 15	1 36	30
10 0	0 20	0 24	0 29	0 34	0 41	0 50	1 2	1 19	22 0
30	0 16	0 19	0 22	0 27	0 32	0 39	0 48	1 2	30
11 0	0 11	0 14	0 16	0 19	0 23	0 28	0 34	0 44	23 0
30	0 7	0 9	0 11	0 12	0 15	0 18	0 22	0 29	30
12 0	0 4	0 5	0 6	0 7	0 8	0 10	0 12	0 16	24 0

TABLE III. (*for* 1895).

Containing the *Third* Correction (*always to be added*).
Arguments:—Sidereal Time and Date.

Sidereal Time.	July 1.	Aug. 1.	Sept. 1.	Oct. 1.	Nov. 1.	Dec. 1.	Dec. 31.
h.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0 48	0 55	1 5	1 17	1 28	1 37	1 40
2	0 46	0 48	0 54	1 4	1 15	1 26	1 34
4	0 48	0 44	0 45	0 50	0 58	1 9	1 19
6	0 53	0 45	0 40	0 39	0 42	0 49	0 59
8	1 0	0 49	0 40	0 33	0 30	0 32	0 39
10	1 7	0 57	0 45	0 35	0 26	0 23	0 25
12	1 12	1 5	0 55	0 43	0 32	0 23	0 20
14	1 14	1 12	1 6	0 56	0 45	0 34	0 26
16	1 12	1 16	1 15	1 10	1 2	0 51	0 41
18	1 7	1 15	1 20	1 21	1 18	1 11	1 1
20	1 0	1 11	1 20	1 27	1 30	1 28	1 21
22	0 53	1 3	1 15	1 25	1 34	1 37	1 35
24	0 48	0 55	1 5	1 17	1 28	1 37	1 40

ANSWERS TO EXAMPLES.

Section II.

(1)	745′ N.	(7)	23° 55′ N.
(2)	582 S.	(8)	1 42 N.
(3)	1072 S.	(9)	41 34 S.
(4)	994 N.	(10)	2 41 S.
(5)	1063 S.	(11)	10 55 S.
(6)	1566 N.	(12)	21 37 N.
(13)	838′ W.	(21)	68° 19′ W.
(14)	1664 E.	(22)	15 9 W.
(15)	1497 E.	(23)	136 2 E.
(16)	954 W.	(24)	63 4 E.
(17)	543 E.	(25)	7 38 W.
(18)	841 W.	(26)	11 26 E.
(19)	3672 W.	(27)	168 8 W.
(20)	8141 E.	(28)	172 24 E.

Section III.

(1)	$\frac{3}{4}$ pts. 8° 26′ 15″ to right of N.	(9)	$\frac{1}{2}$ pts. 5° 37′ 30″ to right of S.
(2)	$6\frac{1}{2}$ „ 73 7 30 „ „ „	(10)	$1\frac{3}{4}$ „ 19 41 15 „ left „
(3)	$2\frac{3}{4}$ „ 30 56 15 „ left „	(11)	$2\frac{1}{2}$ „ 28 7 30 „ right „
(4)	$6\frac{3}{4}$ „ 75 56 15 „ „ „	(12)	$7\frac{1}{4}$ „ 81 33 45 „ „ „
(5)	$5\frac{1}{2}$ „ 61 52 30 „ right „	(13)	$7\frac{1}{2}$ „ 84 22 30 „ left „
(6)	$4\frac{3}{4}$ „ 53 26 15 „ left „	(14)	$5\frac{3}{4}$ „ 64 41 15 „ „ „
(7)	$1\frac{3}{4}$ „ 19 41 15 „ right „	(15)	$6\frac{1}{2}$ „ 73 7 30 „ right „
(8)	$3\frac{3}{4}$ „ 42 11 15 „ left „	(16)	$6\frac{1}{4}$ „ 70 18 45 „ left „

Section IV.

(1)	S 51° 21′ W.	(7)	N 45° 34′ W.
(2)	S 46 30 E.	(8)	S 17 2 E.
(3)	S 7 9 E.	(9)	N 70 0 W.
(4)	N 82 9 E.	(10)	N 68 42 E.
(5)	S 62 11 W.	(11)	N 29 1 W.
(6)	S 87 49 W.	(12)	S 23 14 W.

Sections V., VI.

(1) 3° 30′ W; 3° 40′ W; 1° 30′ W; 1 30 E; 2 40 E.

(2) 2° 50′ E; 4° 10′ E; 4° 20′ W; 3 10 W; 4 10 E.

Sections VII., VIII.

(1) 1° 14′ E; 1° 44′ E; 2° 44′ E; 1 54 E; 1 4 E; 0 6 W; 3 26 W; 2 56 W; 2 26 W; 0 46 W; 2 24 E; 1 34 E; 1 24 E; 0 46 W; 2 56 W; 2 26 W.

(2) 0° 20′ E; 2° 0′ E; 3° 40′ E: 2 20 E; 0 30 W; 2 0 W; 3 40 W; 2 10 W.

Sections IX.-XI.

(1) $50\frac{2}{3}$ feet; distance shown would be $\frac{12}{13}$ of true distance.

(2) 198·9 miles.

(3) 10·32 knots.

(4) 1·27 feet too short; 2 knots nearly.

(5) 3·05 feet too short.

(6) 4 minutes. 2·3 feet too long.

Sections XII.-XIV.

(1) Lat., 50° 50·8′ N.
Dep., 261·2′.

(2) Lat., 46° 57·7′ N.
Dep., 177′.

(3) Lat., 0° 10·3′ N.
Dep., 14·3′.

(4) Course N 28° E.
Dist., 125′.

(5) Course S 57° W.
Dist., 174′.

(6) Course N 54° E.
Dist., 192′.

(7) Course S. 32° W.
Dist., 75′.

Section XV.

(1) S 71° W; $2\frac{2}{3}$ hours.

(2) $5\frac{2}{3}$ hours; S $5\frac{1}{2}$° E.

(3) 7·36 knots.

(4) S 56° W.

(5) N 64° E; 11·1 knots.

Section XVI.

(1) 4·8 miles.

(2) 3·2 knots.

(3) 2·9 feet too short.

(4) 14·2 miles on port tack.
10·5 „ starboard tack.

Section XVII.

(1) West; 796·6′.

(2) East; 1825′.

(3) East; 2625·5′.

(4) West; 1065·4′.

(5) 45° 18·4′ W.

(6) 59 44 E.

(7) 141 35 W.

(8) 6 28 W.

(9) 175 42·3 W.

Sections XVIII.-XX.

(1) S 55° 35′ 30″ W; 340′.

(2) S 39 12 15 E; 435.

(3) N 49 22 45 W; 547.

(4) N 48 47 0 W; 313.

(5) 25° 8′ N; 62° 40′ W.

(6) 43 41 S; 86 31 E.

(7) 41 3 N; 60 52 W.

(8) 12 51 N; 45 20 E.

Sections XXI.-XXVI.

(1) S 55° 35′ 15″ W ; 340′.
(2) S 39 10 45 E ; 435.
(3) N 49 21 45 W ; 547.
(4) N 37 7 15 E ; 4038.
(5) 25° 8′ N ; 62° 40′ 30″ W.
(6) 43 41 S ; 86 30 0 E.
(7) 41 3 N ; 60 51 0 W.
(8) 34 46 N ; 14 37 0 E.

Sections XXVII.-XXIX.

(1) Lat., - - - - - 55° 27′ N.
Long. (Traverse Table), - 136 40 W.
„ (calculation), - - 136 39 W.
„ (each course separately), 136 42 W.
(2) 29° 21′ N ; 17° 58′ W.
(3) 25 5 S ; 36 22 E.
(4) 34 48 S ; 37 11 W.
(5) 38° 43′ S ; 56° 43′ W.
(6) 46 57 S ; 54 56 W.

Miscellaneous Examples in the Sailings.

(1) N 77° 0′ E ; 1213′.
(2) N 67 0 W ; 2993.
(3) N 72 56 W ; 1268.
(4) N 30 57 W ; 648.
(5) 38° 26′.
(6) 194·15′.
(7) 43° 7′ N.
(8) No.
(9) 23·8 miles.
(10) 23° 7′ S ; 35° 35′ E.
(11) 43° 50′ N ; 569′.
(12) 1297 yards.
(13) S 14° 28′ E ; 291′ ; 52° 58′ N.
(16) S 41° E ; 287′.
(17) 7° 13′ S.
(18) 139′.
(19) S 48° W ; 81′.
(20) Half-a-mile.
(21) 27° 54′ S ; 15° 24′ S.
(22) 60° 20′ N ; 35° 13′ W ; $4\frac{3}{8}$ knots.
(23) 4·3 feet too short ; 44° 45′.
(24) Port tack NbE, 8·5 miles.
Starboard tack WbS, 1·7 miles.
(25) 9·7 miles.
(26) N 17° E ; 4 knots.
(27) SSE ; 17 miles.
(28) S 79° 50′ W ; 6·5 hours.
(29) 3·8 inches too short.
(30) 6·2 knots.
(31) N 69° 37′ 30″ W.

Sections XXXII.-XXXVII.

(1) 51° 45′ 0″ N ; 8° 11′ 0″ W.
(2) 51 30 0 N ; 9 55 30 W.
(3) 51 38 45 N ; 10 26 30 W.
(4) 51 31 0 N ; 8 35 30 W.
(5) 52 30 0 N ; 10 8 0 W.
(6) 53 50 30 N ; 10 14 30 W.
(7) 51 20 0 N ; 9 40 0 W.
(8) 51 32 0 N ; 8 36 0 W.
(9) 51° 42′ 0″ N ; 10° 34′ 30″ W.
(10) 54 3 30 N ; 10 24 0 W.
(11) 51 18 0 N ; 9 34 30 W.
(12) 51 43 0 N ; 8 14 0 W.
(13) 51 44 0 N ; 10 35 0 W.
(14) 54 1 0 N ; 10 19 0 W.
(15) 55 19 0 N ; 8 13 0 W.

Sections XLIV.-LI.

(1) N 50° 30′ E ; 3105′.
(2) 5765′ ; S 65° 45′ E.
(3) S 31° E ; N 36° E ; 67° S.
(4) 35° S ; 91° E ; S 72° E.
(5) 24° 12′ W ; 51° 13′ N.
(6) 100′ ; 324′,
(7) S 59° W ; 28° 14′ W ; 2208′.
(8) 138′.
(9) 50° S ; 637′.

Sections LXXIV., LXXV.

	h. m. s.		h. m. s.				
(1)	3 9 17.	(4)	10 55 7·3.	(6)	51° 19′ 0″.	(9)	154° 56′ 52·5″.
(2)	6 35 10.	(5)	9 49 58·6.	(7)	64 19 45.	(10)	124 7 20.
(3)	7 57 47.			(8)	141 51 45.		

Sections LXXVIII.-LXXIX.

(1)	6° 43′ 9″ N.	(3)	22° 32′ 6″ N.	(5)	22° 39′ 38″ S.
(2)	13 44 35 N.	(4)	23 26 42 N.	(6)	23 14 32 S.

Section LXXXI.

(1) 1 m. 19·47 s. + to M.T. (2) 1·01 s. − from M.T.
(3) 10 m. 56·17 s. + to M.T.

Section LXXXII.

(1) 59 m. 46·22 s. (2) 5 h. 30 m. 26·73 s. (3) 17 h. 50 m. 52·24 s.

Section LXXXIII.

(1) R.A., 22 h. 1 m. 18 s.
Dec., 13° 53′ 16·8″ S.

(2) R.A., 23 h. 42 m. 42 s.
Dec., 2′ 39″ N.

Section LXXXIV.

(1) S.D., 15′ 25·4″
H.P., 56 30·3.

(2) S.D., 14′ 53·6″.
H.P., 54 34.

(3) S.D., 16′ 9·9″.
H.P., 59 12·7.

Section LXXXVIII.

	h. m. s.		h. m.		h. m.
(1)	10 23 27.	(3)	23 47.	(5)	12 42.
(2)	0 37 25.	(4)	4 57.		

Section LXXXIX.

(1) Regulus; α Crucis; Spica.
(2) Vega; Altair; α Cygni.
(3) α Orionis; Canopus; Sirius; Castor; Procyon; Pollux; Regulus.
(4) Aldebaran. Achernar on meridian, but *below* horizon.

Section XC.

(1) 7 h. 45 m. 9·6 s. P.M. (2) 11 h. 1 m. 18·71 s. A.M.

Sections XCI., XCII.

	h. m. s.		h. m. s.		h. m. s.
(1)	8 39 2.	(2)	10 52 31.	(3)	8 57 24.
	10 50 45.		12 12 54.		
	14 12 51.		12 44 57.		

Sections XCIII., XCIV.

(1) S.M.T., 10 0 P.M.
G.M.T., 14 52 April 6th.

(2) S.M.T., 11 22 P.M.
G.M.T., 8 6 April 8th.

(3) S.M.T., 8 34 A.M.
G.M.T., 14 38 April 19th.

(4) S.M.T., 0 45 A.M.
G.M.T., 20 29 June 7th.

(5) S.M.T., 4 9 P.M.
G.M.T., 19 41 June 26th.

(6) S.M.T., 2 39 A.M.
G.M.T., 20 3 December 4th.

Section XCVII.

(1) 5° 42′ 30″ W.
(2) 62 7 0 E.
(3) 26 9 0 W.
(4) 42 28 0 W.
(5) 169 42 0 E.
(6) 71° 6′ 0″ W.
(7) 71 24 0 E.
(8) 66 56 0 W.
(9) 76 33 0 E.
(10) 17 26 30 W.
(11) 2° 47′ 0″ E.
(12) 142 30 30 E.
(13) 47 10 0 W.
(14) 148 3 0 E.

Section XCVIII.

(1) 3 h. 26 m. 47 s. fast on G.M.T.
(2) 2 h. 20 m. 45 s. slow on G.M.T.; daily rate, 6·6 s. losing.
(3) 1 h. 35 m. 22 s. fast on G.M.T.; daily rate, 2·2 s. losing.
(4) 1 h. 31 m. 51 s. fast on G.M.T.
(5) 1 h. 47 m. 56 s. slow on G.M.T.; daily rate, 2·53 s. gaining.
(6) 4 h. 10 m. 20 s. slow on G.M.T.; daily rate, 5 s. losing.

Sections XCIX.-CIII.

(1) 2 30 41 slow on G.M.T.
(2) 3 43 4 fast ,,
(3) 1 53 26 slow ,,
(4) 3 43 30 fast on G.M.T.
(5) 3 53 38 slow ,,
(6) 0 31 25 slow ,,

Section CV.

(1) 60° 56′ 30″ N.
(2) 56 14 30 N.
(3) 46° 2′ 0″ S.
(4) 50 11 0 S.

Section CVI.

(1) 61° 27′ N
(2) 65° 27′ N.
(3) 43° 40′ S.

Section CVII.

(1) 30° 14′ 0″ N.
(2) 32 48 0 S.
(3) 35 42 30 N.
(4) 25 40 0 N.
(5) 37 40 0 N.
(6) 17 4 0 N.
(7) 9 50 0 S.
(8) 49° 9′ 30″ N.
(9) 15 25 0 N.
(10) 34 20 30 N.
(11) 32 43 30 N.
(12) 20 40 30 S.
(13) 37 40 30 S.
(14) 45° 18′ 30″ N.
(15) 31 29 0 S.
(16) 42 56 0 S.
(17) 37 12 0 N.
(18) 14 54 0 N.

Sections CVIII.-CXV.

(1) 48° 17′ 0″ N.
(2) 19 53 0 S.
(3) 47 50 0 S.
(4) 53° 59′ 0″ N.
(5) 35 4 30 S.
(6) 54 27 0 N.
(7) 37° 14′ 30″ S.
(8) 41 56 0 N.

Section CXVIII.

(1) 3 h. 40 m. 50 s.
(2) 2 h. 14 m. 59 s.
(3) 6 m. 44 s.
Aldebaran.

Section CXXI.

(1) 9·2′+ to 1st altitude.
(2) 7·8 − from „
(3) 22·2 − from „
(4) 23·8′+ to 1st altitude.
(5) S 55° W.
(6) N 8° W or S 38° W.

Sections CXVI.-CXXI.

(1) 44° 50′ 0″ N.
(2) 50 26 0 N.
(3) 56 43 30 N.
(4) 32° 51′ 0″ S.
(5) 36 39 0 N.
(6) 41 42 0 N.
(7) 37° 2′ 0″ N
(8) 34 0 0 S.

Section CXXII.

(1) 48° 49′ N.
(2) 41 20 N.
(3) 47° 41′ N.
(4) 40 26 N.
(5) 31° 5′ N.
(6) 21 10 N.

Section CXXIV.

(1) 2° 10′ E.
(2) 3 23 W.
(3) 1° 50′ W.
(4) 3 3 W.
(5) 2° 53′ W.

Section CXXV.

(1) 2° 5′ E.
(2) 2 29 W.
(3) 3° 9′ E.
(4) 1 7 W.

Section CXXVI.

(1) 2° 42′ E.
(2) 1 42 W.
(3) 2° 57′ W.
(4) 2 58 W.
(5) 3° 10′ E.
(6) 2 35 W.

Sections CXXVII., CXXVIII.

(1) 9° 44′ W.
(2) S 28° 12′ W.
(3) S 24° E.

Section CXXIX.

(1) 1 h. 48 m. 24 s.
(2) 1 h. 30 m. 16 s.
(3) 70° 47′.
5 h. 3 m. 50 s.

Sections CXXX.-CXL.

(1) 44° 50′ N ; 140° 25′ 30″ W.
(2) 37 1 N ; 9 50 0 W.
(3) 53 57 N ; 6 46 30 E.
(4) 54 42 N ; 5 29 0 W.
(5) 53° 6′ N ; 5° 49′ 0″ W.
(6) 3 28 E ;
(7) 49 46 N ; 6 21 0 W.

Sections CXLI.-CXLVI.

(1) Lat. D.R., - - 23° 17′ N.
„ obs., - - 23 31 N.
Long. D.R., - - 122 55 E.
„ chron., - 123 3 E.

Course and distance.
N 38° E ; 90′.

(2) Lat. D.R., - - 24° 18′ S.
„ obs., - - 24 32 S.
Long. D.R., - - 36 40 E.
„ chron., - 36 48 E.

Course and distance.
S 39° E ; 55′.

Sections CXLI.-CXLVI. (*continued*).

Current.
N 28° E ; 16′.

True bearing and distance.
N 52° 51′ E ; 1154′.

Deviation.
3° 10′ W.

(3) Lat. D.R., - - 41° 33′ N.
„ obs., - - 41 41 N.
Long. D.R., - - 46 17 W.
„ chron., - 45 59 W.

Course and distance.
N 43° E ; 84′.

Current.
N 59° E ; 16′.

True bearing and distance.
N 61° E ; 1697′.

Deviation.
3° 8′ E.

Current.
S 28° E ; 16′.

True bearing and distance.
N 74° 20′ E ; 4087′.

Deviation.
2° 55′ W.

(4) Lat. D.R., - - 17° 6′ S.
„ obs., - - 16 55 S.
Long. D.R., - - 100 10 E.
„ chron., - 100 2 E.

Course and distance.
N 21° E ; 117′.

Current.
N 36° W ; 13′.

True bearing and distance.
N 26° E ; 679′.

Deviation.
3° 32′ E.

Sections CXLVII.-CLIV.

(1) 67° 33′ 0″ W.
(2) 70 15 0 E.
(3) 135 48 0 E.
(4) 92° 49′ 15″ E.
(5) 60 7 0 W.
(6) 15° 42′ 30″ W.
(7) 105 6 0 E.

Sections CLV.-CLVII.

(1) 76° 25′ 30″ E.
(2) 70 15 15 E.
(3) 15° 42′ 30″ W.
(4) 60 7 0 W.

Miscellaneous Examples.

(1) 7 knots ; 11 knots.
(2) 48° 27′ 45″.
(3) S 71° 34′ W ; S 18° 26′ W.
33° 44′ 30″ N ; 21° 13′ 30″ N.
(4) 310 hours 48 minutes.
(5) 50° N ; 4° 4′ W.
(6) 15·5 knots ; N 52° 3′ E ; 101 miles.
(7) 75°.
(8) 9 h. 18 m. 4 s.
(9) 3 h. 54 m. 57 s.
(10) 13 m. + to M.T.
(11) 8 h. 50 m. P.M.
(12) 0 h. 37 m. 3 s.
(13) 4 h. 40 m. 26·154 s.
(14) 1 h. 42 m. 55 s. slow.
(15) 49° 3′.
(16) 11 h. 8 m. 29 s.
(17) 9 h. 32 m. 19 s.
(18) Latitude is equal to the obliquity of the ecliptic.
Apparent noon.
(19) East ; 56 miles.
(21) 21 h. 3 m. 44·6 s.
(22) 2 m. 13 s. of our mean time.
(23) 10 m. 51·3 s. – to A.T.
(24) R.A. of poles of ecliptic, 6 h. and 18 h.
(27) 2·59 s. losing.
(28) 208 ft. ; 21·7 miles.
(29) 15° 40′ W ; 49° 16′ N.
(30) 118° 5′ 30″ E.
(31) 7 min. ; A to eastward of B.
(32) 12506 feet.

MISCELLANEOUS EXAMPLES (*continued*).

(83) 2 h. 57 m. 18 s.; 51° N.
(85) 30° 48′ N; 1 h. 42 m. 28 s.
(86) 4 h. 32 m. 30 s. P.M.
1 h. 39 m. 39 s.
(87) 10 h. 23 m. 38 s. A.M.
N 71° 46′ E.
(88) 22° 25′.
(89) 251° 10′; 23° 27′ 30″.
(40) 86° 36′ 45″.
(41) 143° 26′ 36″.
(43) 77° 30′.
(45) 9 h. 34 m. 34 s. A.M.
(46) 6¼ miles, to be added to the latitude obtained.
(51) (1) 14° 45′ 45″.
(2) 23° 6′ 15″.
(56) 10·83″.
(57) 11·22″.
(58) 59′ 32″.
(59) 49″.
(60) 2 m. 35 s. before noon.
(61) 28° 30′ E.
(63) 583 feet.
(64) 2½ minutes.
(65) 19° 16′ 15″.
(68) 45°.
(75) 17° 10′ S; 2° 10′ S.
(76) 41 s.; morning shorter than afternoon.
(77) 5° N nearly.
(79) 22° 21′ 28·9″.
(80) 21° 50′ 2·4″.
(81) 15 h. 21 m. 42·21 s.
(82) 27° 13′ 56·35″.
(83) 48° 28′ 15″.
(84) 32° 3′ 48·2″.
(85) 0° 1′ 45·03″ S.
(86) 5 h. 56·4 m.
(87) 50° 23′ N; 5° 44′ W.
(88) 16° S.
(89) 81° 42′.
(90) 23° 50′ 30″ E.
(91) 8 h. 30 m.; December 31st.
(92) 9′ 20″; 21° N.
(93) 3 h. 43 m. 4 s. fast.
(94) 700 yards.
(95) 51° 7′ N.
(96) Starboard; nearer to Great Circle.
(97) 58° 37′ S; 138° 30′ W; 159° 30′ W; 128° W.
(98) 50° 22′ N; N 74° 15′ E.
(99) 35° 47′ 30″.
(100) 8 h. 5 m. 58 s.
(101) 62° 23′.
(102) 60° 27′.
(103) 10 h. 15 m. 2 s.
(104) 2·105 m.
(105) 64° 6′ N; 25° 54′ N.
(107) 1 h. 36 m. 22 s.; 23° 28′.
(108) S 58° W.
(109) 10 h. 39 m.
(110) 48° 59′ N; or 41° 1′ N.
(111) North; 19·25 knots.
(112) 39° 43′ N.
(113) 40; 51° 40′.
(114) 58° 17′ N; 27° 28′ S.
(115) 43° 10′ N; 28° 16′ 30″ N; 12° 18′ 45″ N.
(117) 40° N.
(118) N 6° 41′ E; 22 minutes.
(119) 3·8′.
(120) N 6° W; 12·8 knots.
(121) Current S 57° E; 2·8 knots; B's course and distance made good, N 59° E; 8·6′.
(122) Distance from lighthouse 9·75′; 1st Course N 39° E; 2nd Course S 39° E.
(123) 12′.
(124) 18·25′.
(126) 158·7′ ft.; 90 ft.
(132) 31° 6′ 15″ S; 11 h. 29 m. 56 s.
(133) 29° 49′ 30″ S or 83° 8′ 30″ S.
(134) 45° 27′ 15″.
(135) 54·25″ per hour.
(136) 118° 36′.
(137) 8 h. 36 m. 20 s. A.M. or 3 h. 23 m. 40 s. P.M.
(138) 30° 5′ 30″ S and 43° 41′ 30″ S; 1 h. 50 m. 30 s. A.M.
(139) Duration 1 h. 52 m. 40 s.; Declination 6° 58′ S.
(140) 2 m. 48 s.
(141) 5 minutes approximately.
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(152) 126° 29′ 5″.

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